SE Sem-III
Biotechnology
Q.P. Code 38805 Applied Maths CBCGS R(2016)
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Synoptic Solution.

Q.1(a) $L\left[\int_0^\infty \frac{e^{-3t}-e^{-6t}}{t}dt\right]$

$$L\{e^{-3t}-e^{-6t}\} = \frac{1}{s+3} - \frac{1}{s+6}$$

$$L\left\{\int_0^\infty \frac{e^{-3t}-e^{-6t}}{t}dt\right\} = \int_s^\infty\left(\frac{1}{s+3} - \frac{1}{s+6}\right)ds$$

$$= \left(\log\frac{s+6}{s+3}\right)_s^\infty$$

$$= \log 2$$

1(b) Characteristic eq-n for $A = \begin{bmatrix}1 & 2\\ 2 & 2\end{bmatrix}$

is $|A-\lambda I| = 0$

$$\lambda^2 - 3\lambda - 2 = 0$$

$$A^2 - 3A - 2 = 0$$

$$2A^4 - 5A^3 - 7A + 6I = (A^2-3A-2I)(2A^2+A+7I) + 16A + 20I$$

$$= 16A + 20I$$

$$= \begin{bmatrix}16 & 32\\ 32 & 32\end{bmatrix} + \begin{bmatrix}20 & 0\\ 0 & 20\end{bmatrix} = \begin{bmatrix}36 & 32\\ 32 & 52\end{bmatrix}$$

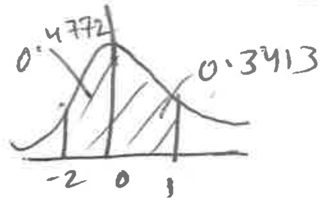
1(c) $\int_c \frac{e^{2z}}{(z+1)^4}dz$ is not analytic at $z=-1$ lies inside

$|z-1| = 3$

$$\int_c \frac{f(z)}{(z-z_0)^n}dz = \frac{2\pi i}{(n-1)!}f^{n-1}(z_0)$$

$$\int_c \frac{e^{2z}}{(z-1)^4}dz = \frac{2\pi i}{3!}f'''(z_0) = \frac{8\pi i}{3e^2}$$

2(d) S.N.V. $Z = \frac{X-M}{\sigma} = \frac{X-70}{5}$




(i) when $X = 60$, $Z = -2$; when $X = 75$, $Z = 1$

$P(60 \leq X \leq 75) = P(-2 < Z < 1)$
$= 0.8185$

(ii) No. of students getting marks b/w 65 & 70 = NP = 818

(iii) $P(X \geq 75) = P(Z \geq 1)$
$= 0.5 - 0.3413$
$= 0.1587$

No. of students getting score more then 75 = NP = 159

2(a) $\int_0^{2\pi} \frac{d\theta}{5+3\sin\theta}$ : let $e^{i\theta} = z$, $d\theta = \frac{dz}{iz}$ ; $\sin\theta = \frac{z^2-1}{2iz}$

$I = \int_c \frac{2}{3z^2+10iz-3} dz$ where $c$ is $|z|=1$

$3z^2+10iz-3 = 0 \Rightarrow (3z+i)(z+3i) = 0$

$z = -i/3$ & $z = -3i$ are poles of $f(z)$.

$z = -\frac{i}{3}$ lies inside & $z = -3i$ lies outside

Residue at $(z = -\frac{i}{3}) = \lim_{z \to -i/3} (z + i/3) \frac{2}{(3z+i)(z+3i)}$

$= \frac{1}{4i}$

$I = 2\pi i \left(\frac{1}{4i}\right) = \pi/2$

2(b) $R = 1 - \frac{6 \Sigma D^2}{n^3 - n} = -0.9$

2(c) Here $A = \begin{bmatrix} 1 & -1 & 1/2 \\ -1 & 2 & -1 \\ 1/2 & -1 & 2 \end{bmatrix}$

$A = IAI$

Linear transformation

$X = PY$

$x = u + v$

$y = u + \frac{w}{\sqrt{6}}$

$z = \sqrt{\frac{2}{3}}\, w$

The given quadratic form to $u^2 + v^2 + w^2$

Rank $= 3$

Signature $= 3 - 0 = 3$

3(a) $\lambda = 1, 2, 3$

For $\lambda = 1$ Eigenvector $X = [0, 1, 1]'$

" $\lambda = 2$ " " $X = [1, 1, 1]'$

" $\lambda = 3$ " " $X = [1, 0, 1]$

3(b) $f(z) = \frac{1}{4(z^2+1)}$ ; $C$ is $|z| = 2$

Poles of $f(z) = z \pm i$ (simple Poles)

& both lies inside $C$

$\int_C \frac{1}{4(z^2+1)}\, dz = 2\pi i$ (sum of residue inside $C$)

$= 2\pi i \, [\mathrm{Res}(z = i) + \mathrm{Res}(z = -i)]$

Res. $= 2\pi i \left[ \frac{1}{8i} - \frac{1}{8i} \right]$

$= 0$ [can be done using cauchy integral formula]

3(c) = Let $L^{-1}\{f(s)\} = \frac{1}{s-3}$ $\quad f(t) = e^{3t}$

$L^{-1}\{g(s)\} = \frac{1}{(s+4)^2}$ $\quad h(t) = t\, e^{-4t}$

Convolution theorem

$L^{-1}\{f(s) * g(s)\} = \int_0^t f(u)\, h(t-u)\, du = e^{-4t}\left[\frac{e^{7t}}{49} - \frac{t}{7} + \frac{1}{49}\right)$

4(a) Let $w = \frac{az+b}{cz+d}$ be the required bilinear transform.

Putting z & w values we get

$$a+b=0 \Rightarrow b=-a$$
$$-c+d=0 \Rightarrow d=c$$
$$c=ai, \quad d=ai$$
$$w=\frac{az-a}{aiz+ai} = \frac{-i(z-1)}{(z+1)}$$

4(b) $u = e^{-x}\cos y + xy$

$$dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy$$
$$= -u_y dx + u_x dy$$
$$u_y = -e^{-x}\sin y + x$$
$$u_x = -e^{-x}\cos y + y$$
$$\therefore dv = (e^{-x}\sin y - x)dx + (y - e^{-x}\cos y)dy$$
$$v = -e^{-x}\sin y - \left(\frac{x^2}{2} - \frac{y^2}{2}\right) + c$$

orthogonal trajectory

(c) $L(x_1, x_2, x_3, \lambda) = 196 - 24x_1 - 8x_2 - 12x_3 + 2x_1^2 + 2x_2^2 + 2x_3^2 - \lambda(x_1+x_2+x_3-11)$

$\frac{\partial L}{\partial x_i} = 0, \quad \frac{\partial L}{\partial \lambda} = 0$ Solve the Eqn

pt is $(6,2,3)$

$$\Delta_4 = \begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 4 & 0 & 0 \\ 1 & 0 & 4 & 0 \\ 1 & 0 & 0 & 4 \end{vmatrix} = -48, \quad \Delta_3 = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 4 & 0 \\ 1 & 0 & 4 \end{vmatrix} = -8$$

$\because \Delta_3, \Delta_4$ both $-$ve pt is min

$z_{min} = -98$

(5)

$5(a) = L^{-1}\left\{\log\left(\frac{s^2+a^2}{s^2+b^2}\right)\right\} = \frac{-1}{t} L^{-1}\left[\frac{d}{ds}\{\log(s^2+a^2) - \log(s^2+b^2)\}\right]$

$= \frac{2}{t}\left[\cos bt - \cos at\right]$

(b) characteristic ~~factors~~ roots are $\lambda = 1, 2, 3$

As all eigen values are different the matrix $A$ is diagonalisable

For $\lambda = 1$ Eigen values are $X = [4, 3, 2]'$

" $\lambda = 2$ " " " $X = [3, 2, 1]'$

$\lambda = 3$ " " " $X = [2, 1, 1]'$

Transforming matrix $M = \begin{bmatrix} 4 & 3 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$

& diagonal matrix $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

5(c) solution on last page.

(c) Here $f(x_1, x_2) = \cancel{2x_1^2 - 4x_2^2 + 12x_1x_2}\; 2x_1 + x_2 - x_1^2$

$h_1(x_1, x_2) = 2x_1 + 9x_2 - 98$; $\cancel{h_2(x_1, x_2) = 2x_1 + x_2 - 4}$

KT Conditions are

(i) $\frac{\partial f}{\partial x_i} - \lambda_1 \frac{\partial h_1}{\partial x_i} = 0 \quad i = 1, 2$

(ii) $h_1(x_1, x_2) \le 0$. (iii) $\lambda_1 h_1(x_1, x_2) = 0$

(iv) $\lambda_1 \ge 0$

Case I: if $\lambda_1 = 0$ we get ~~$x_1 = x_2 = 0$~~

$\Rightarrow$ Not feasible

Case - II ~~$\lambda_1 = 0, \lambda_2 \ne 0$~~

$x_1 = 0,\; x_2 = 4$ Not satisfies (5).

(c)

$f(x_1, x_2) = 2x_1^2 - 4x_2^2 + 12x_1x_2$

$h(x_1, x_2) = 2x_1 + 5x_2 - 98$

Case I $\lambda = 0$

Not feasible

Case II $\lambda \ne 0$

case III $\lambda_1 \neq 0$ & $\lambda_2 = 0$ $x_2 = \frac{14}{9}$, $x_1 = 2/3$ ; $\lambda_1 = 1/3$

$Z_{max} = \frac{22}{9}$

6(a) $u = e^{-x}(x\sin y - y\cos y)$

$$f'(z) = u_x + i v_x$$
$$= u_x - i u_y$$

$$u_x = e^{-x}[\sin y - x\sin y + y\cos y] = \phi_1(x,y)$$

$$u_y = e^{-x}[x\cos y - \cos y + y\sin y] = \phi_2(x,y)$$

Put $x = z$, $y = 0$

$$u_x(z,0) = e^{-z}(0) = 0 \;;\; u_y(z,0) = e^{-z}(z-1)$$

$$\therefore f'(z) = i e^{-z}(1-z)$$

$$f(z) = z e^{-z} + C$$

(b) $L\{\cos^2 t\} = \frac{1}{2s} + \frac{1}{2}\frac{s}{s^2+4}$

$L\{e^{-t}\cos^2 t\} = \frac{1}{2(s+1)} + \frac{1}{2}\frac{s+1}{(s+1)^2+4}$ ($1^{st}$ shifting)

$L\{\int_0^t e^{-u}\cos^2 u\} = \frac{1}{2s(s+1)} + \frac{1}{2s}\frac{(s+1)}{(s+1)^2+4}$ (Division by s)

(c) Characteristic Eqn of A is

$$\lambda^3 - 5\lambda^2 + 9\lambda - 1 = 0$$

$$A^3 - 5A^2 + 9A - I = 0$$

$$A^2 = \begin{bmatrix} -1 & 12 & -4 \\ -4 & 7 & 2 \\ 2 & -8 & 1 \end{bmatrix}, \quad A^3 = \begin{bmatrix} -13 & 92 & -2 \\ -11 & 9 & 10 \\ 10 & -22 & -3 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}; \quad A^4 = \begin{bmatrix} -55 & 104 & 24 \\ 18 & -15 & 32 \\ 32 & -40 & -5 \end{bmatrix}$$

X———X———X