Signals and control system
S.E sem IV Biomedical

1

03 Hrs
80-Marks

Answer key. Qp: 50562

Q1
(a) $x(t) = 2\cos^2(2\pi t)$

$$= 2\left[\frac{1+\cos 4\pi t}{2}\right] = 1+\cos 4\pi t$$

which is periodic, with period $\frac{2\pi}{4\pi} = 0.5$ sec

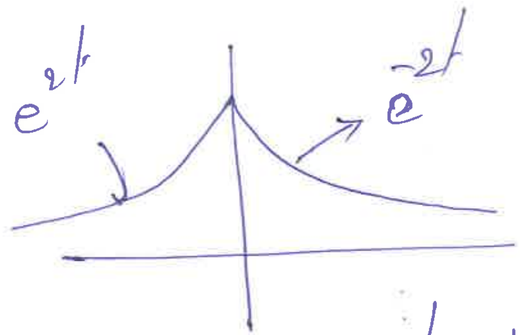
(b) $h(t) = 2e^{-t}u(t)$

since $h(t) = 0$ $t<0$ it is causal.

$$\int_{-\infty}^{\infty} |h(t)|\, dt = \int_{0}^{\infty} 2e^{-t}\, dt = 2\frac{e^{-t}}{-1}\Big]_{0}^{\infty} = 2[1] = 2$$

finite hence stable.

(c)



$$x(t) = e^{2t}u(-t) + e^{-2t}u(t).$$

$$X(\omega) = \int_{-\infty}^{0} e^{2t}\cdot e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-2t}\cdot e^{-j\omega t}\, dt$$

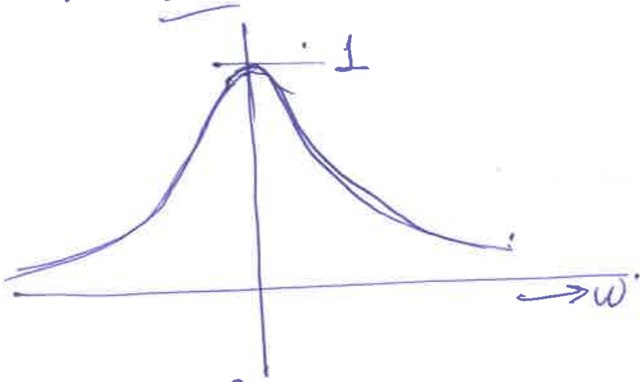
$$= \int_{-\infty}^{0} e^{(2-j\omega)t}\, dt + \int_{0}^{\infty} e^{-(2+j\omega)t}\, dt$$

$$= \frac{e^{(2-j\omega)t}}{2-j\omega}\Big]_{-\infty}^{0} + \frac{e^{-(2+j\omega)t}}{-(2+j\omega)}\Big]_{0}^{\infty}$$

02 $\frac{1}{2-j\omega} + \frac{1}{2+j\omega} = \frac{4}{4+\omega^2}$

at $\omega = 0$ $\frac{d\,X(\omega)}{d\omega} = \frac{(4+\omega^2)0 - 4(2\omega)}{(4+\omega^2)^2} = 0$

$= 0$



1d $X(s) = \frac{s+10}{s^2+2s+3}$

$x(\infty) = \lim_{s\to 0} s X(s) = \lim_{s\to 0} \frac{s(s+10)}{s^2+2s+3}$

$= \lim_{s\to 0} \frac{s^2+10s}{s^2+2s+3} = \lim_{s\to 0}$ ~~$1+10/\ldots$~~

$= 0$

$x(0) = \lim_{s\to\infty} s X(s) = \lim_{s\to\infty} \frac{1+10/s}{1+2/s+3/s^2} = 1$

1e Routh array

s^4	1	5	K
s^3	5	4	
s^2	21/5	K.	
s^1	$\frac{16.8-5K}{4.2}$		
s^0	K		

$16.8 - 5K \geq 0$

$16.8 \geq 5K$

$K = < \frac{16.8}{5} = 3.36$

$K \geq 0$ $0 < k < 3.36$

Q2

(a) $x(t) = 2\cos\left(\frac{3\pi}{4}t + \pi/6\right)$

since the signal is periodic with period $\frac{2\pi}{3\pi/4} = 8/3$.

it can't be energy signal

$$P = \frac{1}{T}\int_0^T x^2(t)\,dt = \frac{1}{8/3}\int_0^{8/3} 2^2\cos^2\left(\frac{3\pi}{4}t + \pi/6\right)dt$$

$$= \frac{4}{8/3}\int_0^{8/3}\left[\frac{1+\cos 2\left(\frac{3\pi}{4}t+\pi/6\right)}{2}\right]dt$$

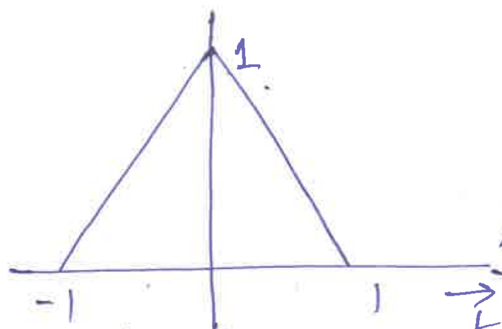
$$= \frac{12}{8}\cdot\frac{1}{2}\int_0^{8/3} T\Big]_0^{8/3} + \cancel{\sin 2(2}$$

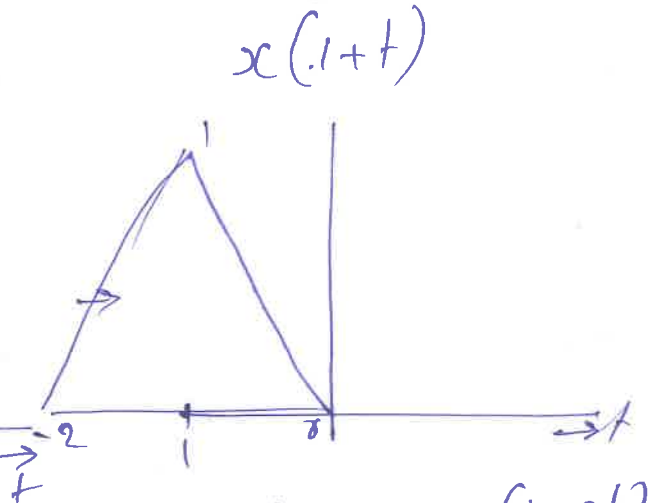
sin and cosine function over one cycle is zero.

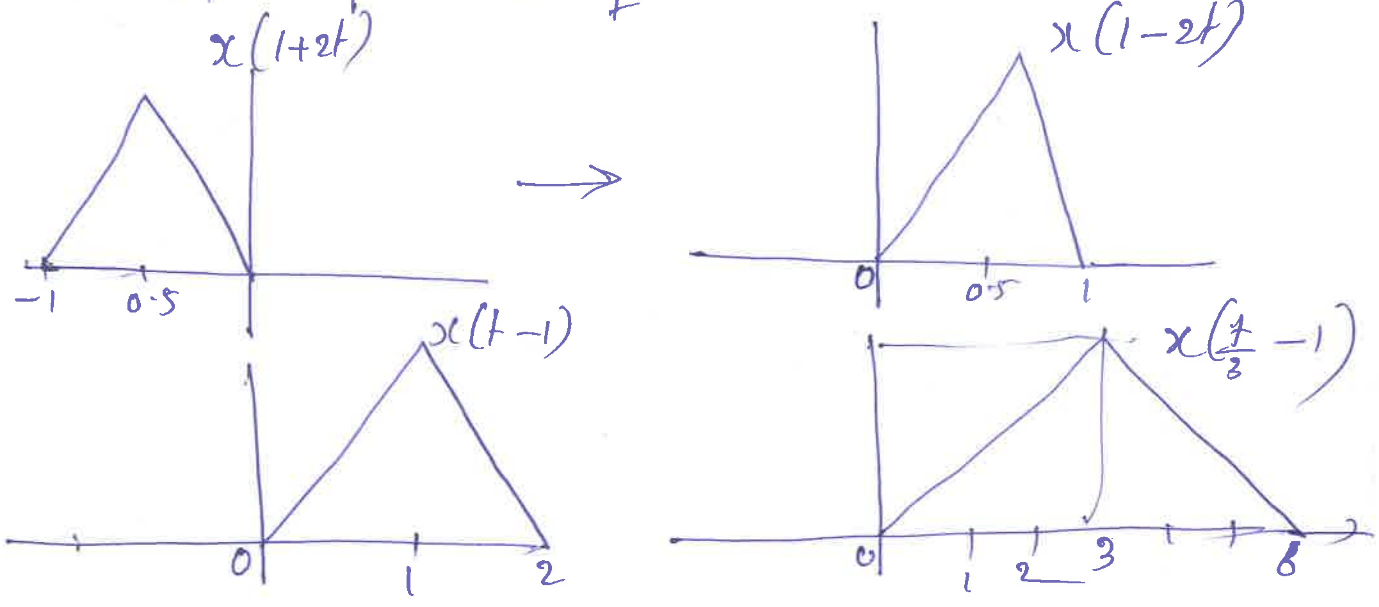
$$= \frac{12}{8}\cdot\frac{1}{2}\cdot\frac{8}{3} = \underline{\underline{2}} \text{ finite}$$

it is power signal.

(b).







2c

04

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\lambda)\, h(t-\lambda)\, d\lambda.$$

$$= \int_{-\infty}^{\infty} e^{-2\lambda} u(\lambda) .\ u(t-\lambda)\, d\lambda$$

$$= \int_{0}^{t} e^{-2\lambda} d\lambda. = \frac{e^{-2\lambda}}{-2}\Big]_0^t = \frac{1}{2}(1-e^{-2t})\ t \geq 0$$

$$= \frac{1}{2}(1-e^{-2t}) \quad t \geq 0$$

d3 (a) $y(t) = x(2t) + 3.$

Linearity

$y_1(t) = x_1(2t) + 3$ Let $x_2(t) = c\, x_1(2t),$

$y_2(t) = c\, x_1(2t) + 3 \neq c\, y_1(t)$ hence not homogeneous

hence not linear

Time invariance

$x_2(t) = x_1(t - t_0)$

$y_2(t) = x_2(2t) = x_1(2t - t_0) + 3$

replace t by $t - t_0$

$y_1(t-t_0) = c . x_1(2t - 2t_0) + 3 \neq y_2(t)$, not time invariant

3d $\quad I_{k,m} = \int_0^{2\pi/\omega_0} e^{jk\omega_0 t} \cdot e^{-jm\omega_0 t}\, dt$ (05)

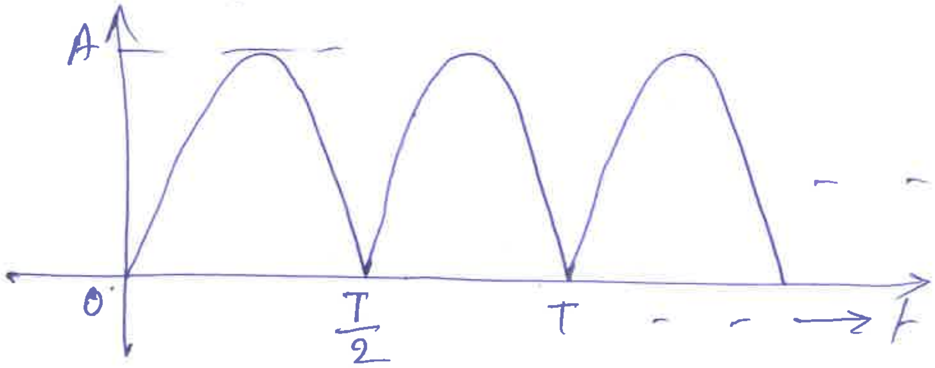
$$= \int_0^{2\pi/\omega_0} e^{j(k-m)\omega_0 t}\, dt = \left.\frac{e^{j(k-m)\omega_0 t}}{j(k-m)\omega_0}\right]_0^{2\pi/\omega_0}$$

$$= \frac{e^{j(k-m)2\pi} - 1}{j(k-m)\omega_0} = 0 \text{ when } k \neq m.$$

and when $k = m$, $\int_0^{2\pi/\omega_0} \cdot\, dt = T = 2\pi/\omega_0$.

hence orthogonal.

3e



$\omega_0 = \frac{2\pi}{T/2} = \frac{4\pi}{T}$.

~~$a_k =$~~ $A_0 = \frac{1}{T/2} \int_0^{T/2} A \sin \omega t\, dt$.

$$= \frac{1}{T/2}\left[A \frac{\cos \omega t}{-\omega}\right]_0^{T/2} = \frac{A}{\omega T/2}\left[1 - \cos \omega T/2\right]$$

$$= \frac{2A}{2\pi}\left[1 - \cos \pi\right) = \frac{2A}{\pi}$$

$$a_k = \frac{2}{T/2} \int_0^{T/2} A \sin(\omega t) \cos(k\omega_0 t)\, dt$$

$$= \frac{4A}{T} \int_0^{T/2} \sin(\omega t) \cos(k\omega_0 t)\, dt.$$

(06) $a_k = \frac{4A}{T} \int_0^{T/2} \sin(\omega t)\cos(k\omega_0 t)\,dt$

$= \frac{4A}{T\cdot 2} \int_0^{T/2} [\sin(\omega + k\omega_0)t + \sin(\omega - k\omega_0)t]\,dt$

$= \frac{2A}{T} \int_0^{T/2} [\sin\left(\frac{2\pi}{T} + k\frac{4\pi}{T}\right)t + \sin\left(\frac{2\pi}{T} - k\frac{4\pi}{T}\right)t]\,dt$

$= \frac{2A}{T}\left\{\int_0^{T/2} \sin\frac{2\pi}{T}[1+2k]t\,dt + \int_0^{T/2} \sin\frac{2\pi}{T}(1-2k)t\,dt\right]$

$= \frac{2A}{T} \cdot \left.\frac{-\cos\frac{2\pi}{T}[1+2k]t}{\frac{2\pi}{T}[1+2k]}\right|_0^{T/2} + \left.\frac{-\cos\frac{2\pi}{T}[1-2k]t}{\frac{2\pi}{T}[1-2k]}\right|_0^{T/2}$

$= \frac{2A}{T}\left[\frac{1-\cos\frac{\pi}{2}(1+2k)}{\frac{2\pi}{T}[1+2k]} + \frac{1-\cos\frac{\pi}{2}(1-2k)}{\frac{2\pi}{T}[1-2k]}\right.$

$= \frac{2A}{T}\left[\frac{1}{\frac{2\pi}{T}[1+2k]} + \frac{1}{\frac{2\pi}{T}[1-2k]}\right.$

$= \frac{2A}{T}\cdot\frac{T}{2\pi}\left[\frac{1}{1+2k} + \frac{1}{1-2k}\right] = \frac{A}{\pi}\left[\frac{1}{1+2k} + \frac{1}{1-2k}\right]$

$= \frac{A}{\pi}\frac{2}{(1-4k^2)} = \frac{2A}{\pi(1-4k^2)}$

$A_1 = \frac{2A}{-3\pi}, \quad A_2 = \frac{-2A}{15\pi}$

$B_k = \frac{4A}{T}\int_0^{T/2} \cancel{A}\sin(\omega t)\sin(k\omega_0 t)\,dt$

$= \frac{2A}{T}\int_0^{T/2} [\cos(\omega - k\omega_0)t - \cos(\omega + k\omega_0)t]\,dt$

$$\frac{2A}{T}\left[\int_0^{T/2}\left[\cos\left(\frac{2\pi}{T}-k\cdot\frac{4\pi}{T}\right)t-\cos\left(\frac{2\pi}{T}+k\frac{4\pi}{T}\right)t\right]dt\right.$$

07

$$=\frac{2A}{T}\left[\left[\frac{\sin\frac{2\pi}{T}(1-2k)t}{\frac{2\pi}{T}(1-2k)}-\frac{\sin\frac{2\pi}{T}(1+2k)t}{\frac{2\pi}{T}(1+2k)}\right]_0^{T/2}\right]$$

$$=\cancel{\frac{2A}{T}}$$

$$=\frac{A}{\pi}\left[\frac{0-0}{(1-2k)}-\cancel{\sin}\,\frac{0-0}{(1+2k)}\right]$$

$$=\underline{\underline{0}}$$

Mark should be given if students are taking cosine wave also

Q4 (a) $\left[(j\omega)^2+3j\omega+2\right]Y(\omega)=(2j\omega+1)X(\omega)$

$$H(\omega)=\frac{2\cdot j\omega+1}{(j\omega)^2+3j\omega+2}=\frac{A}{(j\omega+1)}+\frac{B}{j\omega+2}$$

$$A=\left.\frac{2\cdot j\omega+1}{j\omega+2}\right|_{j\omega=-1}=\frac{-2+1}{-1+2}=\frac{-1}{1}=-1$$

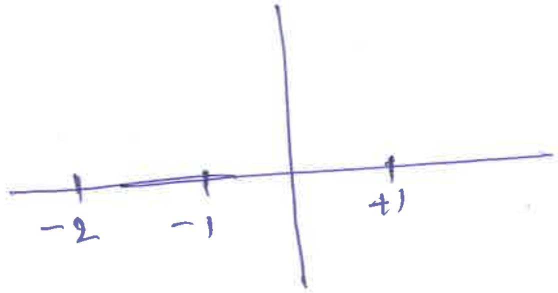
$$B=\left.\frac{2j\omega+1}{j\omega+1}\right|_{j\omega=-2}=\frac{-4+1}{-2+1}=\frac{-3}{-1}=3$$

$$H(\omega)=\frac{3}{j\omega+2}-\frac{1}{j\omega+1}$$

$$h(t)=\underline{\underline{3\cdot e^{-2t}u(t)-e^{-t}u(t)}}$$

4b (or) $\frac{-5s-7}{(s+1)(s-1)(s+2)} = \frac{1}{s+1} - \frac{2}{s-1} + \frac{1}{s+2}$

using partial fraction



when ROC $-2 < \sigma < -1$

$x(t) = e^{-2t}u(t) + 2e^{t}u(-t) - e^{-t}u(-t)$

$-1 < \sigma < 1$

$x(t) = e^{-2t}u(t) + e^{-t}u(t) + 2e^{t}u(-t)$

$\sigma > 1$

$x(t) = e^{-2t}u(t) + e^{-t}u(t) - 2e^{+t}u(t).$

$\sigma < -2$

$x(t) = -e^{-2t}u(-t) + 2e^{t}u(-t) - 2e^{-t}u(-t)$

4b e $x(t) = \frac{d^2}{dt^2} e^{-3(t-2)}u(t-2)$

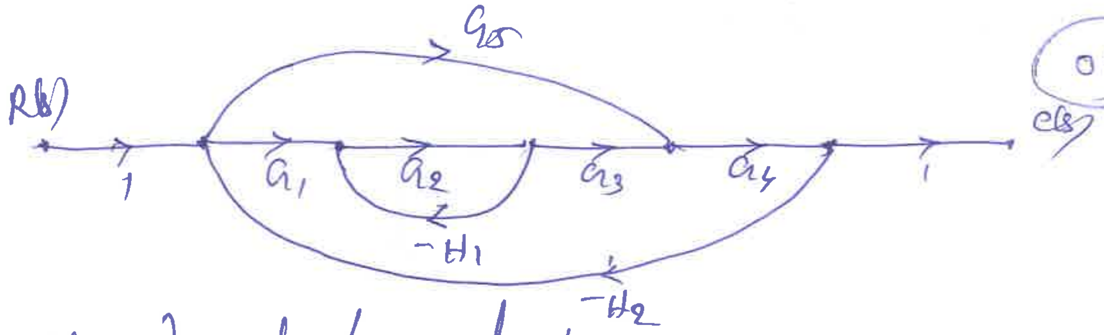
$x_1(t) = e^{-3t}u(t)$

$X_1(s) = \frac{1}{s+3}$ $\quad Re(s) > -3.$

$x_1(t-3) \longleftrightarrow e^{-3s}X_1(s) = \frac{e^{-3s}}{s+3}.$

$\frac{d^2}{dt^2}x_1(t-3) = \frac{s^2 e^{-3s}}{(s+3)}$ $\quad Re(s) > -3$

Q5 (a)



09

Number of forwardpath = 2.

$$T \cdot F = \frac{T_1 \Delta_1 + T_2 \Delta_2}{\Delta}$$

$$T_1 = G_1 G_2 G_3 G_4$$

$$T_2 = G_5 G_4.$$

individual feedback. $L_1 = -G_2 H_1$, $L_2 = -G_1 G_2 G_3 G_4 H_2$

$$L_3 = -G_5 G_4 H_2$$

L_1 and L_3 are two non touching loops

$$\Delta = 1 - [L_1 + L_2 + L_3] + L_1 L_3$$

$$\Delta_1 = 1$$

$$\Delta_2 = 1 + G_2 H_1$$

$$T \cdot F = \frac{G_1 G_2 G_3 G_4 + G_5 G_4 (1 + G_2 H_1)}{1 + G_2 H_1 + G_1 G_2 G_3 G_4 H_2 + G_5 G_4 H_2 + \cancel{G_1 G_2 G_3 G_4 H_2 G_5 G_4 H_4}\; G_2 H_1 G_5 G_4 H_2}$$

Q5 b $G(s) = \frac{16}{s(s+8)}$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s) H(s)} = \frac{16/s(s+8)}{1 + \frac{16}{s(s+8)}} = \frac{16}{s^2 + 8s + 16}$$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\delta \omega_n s + \omega_n^2}. \qquad \omega_n = 4$$

$$2\delta \omega_n = 8$$

$$\omega_n = 1$$

$$\omega_d = \omega_n \sqrt{1-\delta^2} = 0 \text{ rad/sec}$$

Q6 (10) $G(s)H(s) = \frac{k(s+5)}{s^2+4s+20}$

$N = 2-1 = 1$

number of branches terminating at ∞ is 1

Poles and zero $s = -5$ zero

$s = -2 \pm j4$

asymptotes $Q = \frac{(2q+1)180}{(P-Z)}$; $q = 0$

$Q_1 = 180$

centroid.

Breakaway point

$1 + G(s)H(s) = 0$

$1 + \frac{K(s+5)}{s^2+4s+20}$

$s^2+4s+20+Ks+K5 = 0$

$K = -\frac{s^2-4s-20}{(s+5)}$

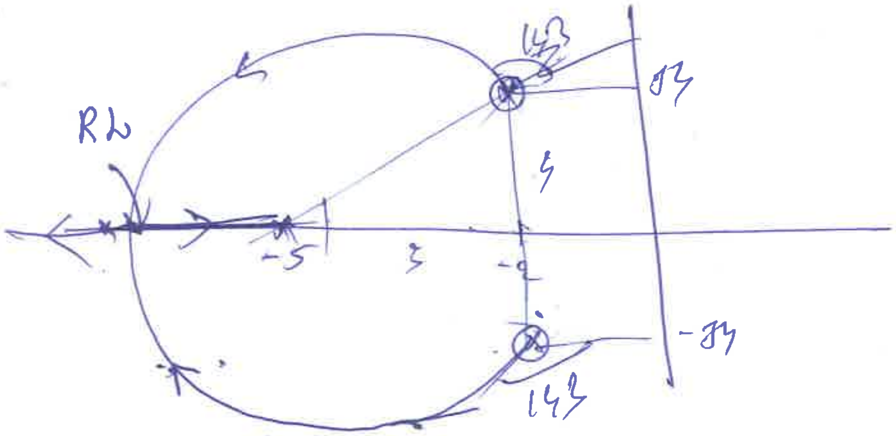
$\frac{dK}{ds} = -2s^2-14s-20+s^2+4s+20 = 0$

$-s^2-10s = 0$

$s(s+10) = 0$

$s = 0,\ s = -10.$

break away point will be $\underline{\underline{-10}}$



inter section with imaginary axis

$$s^2 + 4s + 20 + Ks + 5K = 0$$

$$s^2 + (K+4)s + 20 + 5K = 0$$

$$\begin{array}{c|cc} s^2 & 1 & 20+5K \\ & K+4 & 0 \\ & 20+5K & \end{array}$$

$K_{max} = -4.$

it is negative, no inter section

angle of departure

$$90 - 53 = 36.86$$

$$\phi_d = 180 - 36.86 = 143.13^0$$

$$\phi_a = -143.1$$

12) Q6 b

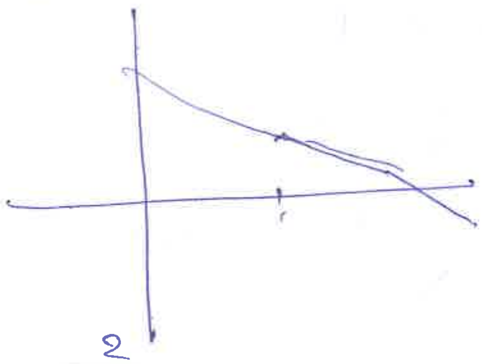
Bode plot

$$G(s)\,H(s) = \frac{80}{s(s+2)(s+20)}$$

$$= \frac{80}{40\,s(1+0.5s)(1+0.05s)}$$

$$= \frac{2}{s(1+0.5s)(1+0.05s)}$$

One pole at orgin one at 2, one at 20



$$G(j\omega)\,H(j\omega) = \frac{2}{j\omega\left(1+\frac{j\omega}{2}\right)\left(1+\frac{j\omega}{20}\right)}$$

From the graph

PM = 38°, GM = +21 dB stable