Model Solution Q. P. Code : 23105

1

Q.5 A] State & ~~Explain~~ prove Chapman - Kolmogorov equation.

→ - Chapman - Kolmogorov equation helps us to find the probability that the System will go from i^{th} state to j^{th} State in $(m+n)$ steps.

- It states that we have to go from i^{th} state to some intermediate K^{th} State in m steps & then from k^{th} state we go to j^{th} state in next n ~~states~~ steps.

- The probability that the system which is in i^{th} state at $n=0$ i.e $X_0 = i$ goes to j^{th} state in $m+n$ Steps i.e to $X_{m+n} = j$ is denoted by

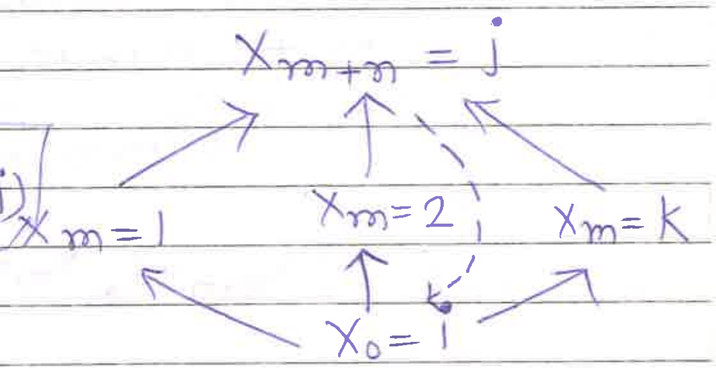
$P(X_{m+n} = j \mid X_0 = i)$ but ~~###~~ this probability is also denoted by $p^{m+n}(i,j)$

now the probability that the System goes from $X_0 = i$ to $X_m = k$ & then $X_{m+n} = j$ in n steps can be denoted by,

$$\sum_{k} P(X_{m+n} = j, X_m = k \mid X_0 = i)$$

Thus, chapman - kolmogorov equation States

$$\boxed{p^{m+n}(i,j) = \sum_{k} p^{m}(i,k)\, p^{n}(k,j)}$$



Proof - The above equation states that to go from i^{th} state to the j^{th} state in $(m+n)$ steps, we have to go from i^{th} state to some intermediate K-th

state in m steps & then from k-th state to j-th state in n steps.
By the Markov property the two probabilities are independent.
Thus, we can write

$$P(X_{m+n}=j \mid X_0=i) = \sum_{K} P(X_{m+n}=j, X_m=K \mid X_0=i)$$

using the defination of conditional probability i.e.

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$ the R.H.S

$$P(X_{m+n}=j, X_m=K \mid X_0=i) = \frac{P(X_{m+n}=j, X_m=K, X_0=i)}{P(X_0=i)}$$

$$= \frac{P(X_{m+n}=j, X_m=K, X_0=i)}{P(X_m=K, X_0=i)} \cdot \frac{P(X_m=K, X_0=i)}{P(X_0=i)}$$

$$= P(X_{m+n}=j \mid X_m=K, X_0=i) \cdot P(X_m=K \mid X_0=i)$$

but the Markov property the first probability depends only on the present state X_m & not on the past state X_0.
i.e $P[X_{m+n}=j \mid X_m=K, X_0=i] = P[X_{m+n}=j \mid X_m=K]$

$$\therefore P[X_{m+n}=j, X_m=K \mid X_0=i] = P[X_{m+n}=j \mid X_m=K] \cdot P(X_m=K \mid X_0=i)$$

$$= P^n(K,j) \cdot P^m(i,K)$$

$$= P^m(i,K) \cdot P^n(K,j)$$

Hence summing over the index k, we get

$$P(X_{m+n}=j \mid X_0=i) = \sum_{K} P^m(i,K) \cdot P^n(K,j)$$

3

Q. 6 A] Prove that if input to LTI system is WSS then the output is also WSS.

→ Let the time-invariant linear system be

$$y(t) = \int_{-\infty}^{\infty} h(u)\, x(t-u)\, du.$$

then $E[y(t)] = \int_{-\infty}^{\infty} h(u)\, E[x(t-u)]\, du$

since $x(t)$ is WSS, $E[x(t-u)]$ is constant (independent of t) say, μ_x

$$E[y(t)] = \int_{-\infty}^{\infty} h(u)\, \mu_x\, du$$

$$= \mu_x \int_{-\infty}^{\infty} h(u)\, du = \text{independent of } t$$

further, the autocorrelation function of $y(t)$ is given by,

$$R_y(t_1, t_2) = E[y(t_1) \cdot y(t_2)]$$

$$= E\left[\int_{-\infty}^{\infty} h(u_1)\, x(t_1 - u_1)\, du_1 \cdot \int_{-\infty}^{\infty} h(u_2) \cdot x(t_2 - u_2)\, du_2\right]$$

$$= \int_{-\infty}^{\infty} h(u_1)\, du_1 \int_{-\infty}^{\infty} h(u_2)\, du_2\, E[x(t_1 - u_1) \cdot x(t_2 - u_2)]$$

Since $x(t)$ is WSS, its autocorrelation is function is function of time-difference i.e difference betn $t_1 - u_1$ & $t_2 - u_2$

$$\therefore E[x(t_1 - u_1) \cdot x(t_2 - u_2)] = R_x[(t_1 - u_1) \cdot (t_2 - u_2)]$$

$$= R_x(t_1', t_2')$$

where $t_1' = t_1 - u_1$, $t_2' = t_2 - u_2$

$E[X(t_1-u_1) \cdot X(t_2-u_2)] = R_X(t_1' - t_2')$

$\because X(t)$ is WSS

$$= R_X[(t_1-t_2) - (u_1-u_2)]$$

$$\therefore R_Y(t_1, t_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(u_1) \cdot h(u_2) \cdot R_X[(t_1-t_2) - (u_1-u_2)]\, du_1\, du_2$$

$$= R_Y(\tau)$$

thus, the mean of $Y(t)$ is independent of t & the autocorrelation function of $y(t)$ is function of the time-difference.

Hence, $Y(t)$ is also WSS.

Q. 3 B] Explain strong law of large numbers & weak law of large numbers.

→ a] The strong Law of Large numbers

Let $x_1, x_2 \ldots$ be a sequence of independent & identically distributed random variables with finite mean $E[x_i] = \mu$, then

$$P\left[\lim_{n\to\infty} \overline{X}_n = \mu\right] = 1$$

or

$$P\left[\lim_{n\to\infty} |\overline{X}_n - \mu| > \epsilon\right] = 0 \quad \text{— (A)}$$

b] Weak law of Large numbers

Let $x_1, x_2 \ldots$ be sequence of independent & identically distributed random variables with the finite mean $E(x_i) = \mu$.

Let $\overline{X}_n = \dfrac{x_1 + x_2 + \cdots + x_n}{n}$, then for $\epsilon > 0$

$$\lim_{n\to\infty} P(|\overline{X}_n - \mu| < \epsilon) = 1$$

or $$\lim_{n\to\infty} P(|\overline{X}_n - \mu| > \epsilon) = 0 \quad \text{— (B)}$$

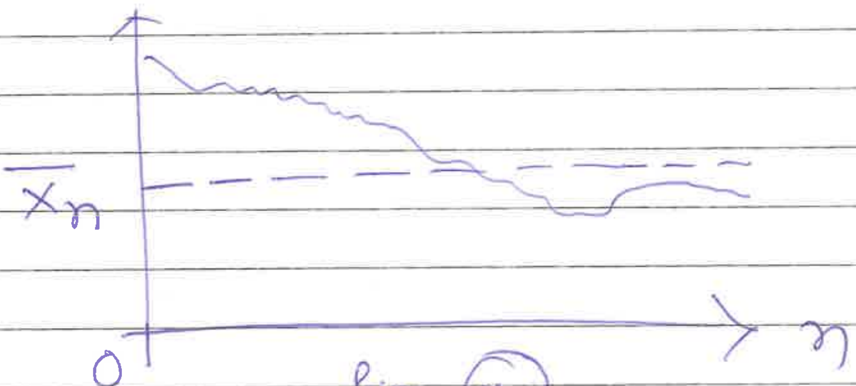


fig. (a)

In simple word it states that if we take large samples of a fixed value of n, the sample mean will be close to the population mean with high probability.
fig (a) shows how the sequence of a sample means approaches the population mean as n the size of a sample approaches infinity.

Note

from (A) & (B), the difference bet^n A & B. In weak law of large numbers (B), we take the limit of the probability as $n \to \infty$ & in strong law of large numbers (A) we consider the probability of the limit.

Q.5B] If a Random process $\{x(t)\}$ is given by $X(t) = 10\cos(100t+\theta)$ where θ is uniformly distributed over $(-\pi, \pi)$. Prove that $\{x(t)\}$ is correlation ergodic.

→ We first prove that the process is stationary.

$$E[x(t)] = \int_{-\pi}^{\pi} \frac{10}{2\pi} \cos(100t+\theta)\, d\theta$$

$$f_\theta(\theta) = \frac{1}{\pi-(-\pi)} = \frac{1}{2\pi}$$

$$E[x(t)] = \frac{5}{\pi}\left[\sin(100t+\theta)\right]_{-\pi}^{\pi}$$

$$= \frac{5}{\pi}\left[\sin(\pi+100t) - \sin(-\pi+100t)\right]$$

$$= \frac{5}{\pi}\left[\sin(\pi+100t) + \sin(\pi-100t)\right]$$

$$= \frac{5}{\pi}\left[-\sin 100t + \sin 100t\right]$$

$$\boxed{E[x(t)] = 0}$$

now,

$$R(\tau) = E[x(t) \cdot x(t+\tau)]$$

$$= E[100\cos(100t+100\tau+\theta) \cdot \cos(100t+\theta)]$$

$$R(\tau) = E\, 50\{\cos(200t+100\tau+2\theta) + \cos(100\tau)]$$

$$= 50\, E[\cos(200t+100\tau+2\theta] + 50\, E\cos(100\tau)$$

$$= 50\int_{-\pi}^{\pi} \cos(200t+100\tau+2\theta) \cdot \frac{d\theta}{2\pi} + 50\cos(100\tau)$$

$$= \frac{50}{\pi} \int_0^{\pi} \cos(200t + 100\tau + 2\theta)\, d\theta + 50\cos(100\tau)$$

$$= \frac{50}{\pi} \left[\frac{\sin(200t + 100\tau + 2\theta)}{2} \right]_0^{\pi} + 50\cos(100\tau)$$

$$= \frac{50}{2\pi} \left[\sin(4\pi + 200t + 100\tau) - \sin(200t + 100\tau) \right] + 50\cos(100\tau)$$

$$= 0 + 50\cos(100\tau)$$

$$= 50\cos(100\tau)$$ i.e function of τ.

Hence $x(t)$ is WSS.

Q. 4A] $ Explain Power Spectral density function State its important properties & prove any two of the property.

→ Defination – If $\{X(t)\}$ is continuous Stionary random process (either in the Strict Sense or in the wide sense) With autocorrelation fun.

$$R_X(\tau) = E[x(t) \cdot x(t+\tau)]$$

then the Fourier transform of $R_X(\tau)$ is called the power spectral density function or power Spectral density or power density or power spectrum of $\{X(t)\}$ & is denoted by $S_{XX}(\omega)$ or $S_X(\omega)$ or simply $S(\omega)$ & fourier transform of $R_X(\tau)$ is denoted by $F[R_X(\tau)]$, where F Stands for "fourier transform" Thus using the defination of fourier transform,

$$S_X(\omega) = F[R_X(\tau)] \quad \text{—} (1)$$

$$S_X(\omega) = \int_{-\infty}^{\infty} R_X(\tau) \cdot e^{-j\omega\tau} \, d\tau \quad \text{—} (2)$$

The power spectral density is Sometimes Written in short as PSD.
We may also define the power spectral density function in terms of the frequency f of the variable by replacing ω by $2\pi f$ in the above def^n. In this case the PSD is denoted by $S_{XX}(f)$ or by $S_X(f)$ or $S(f)$.

$$S_X(f) = F[R_X(\tau)] \quad \text{—} (3)$$

$$S_X(f) = \int_{-\infty}^{\infty} R_X(\tau) \cdot e^{-j2\pi f \tau} \, d\tau \rightarrow (4)$$

properties of PSD -

1. The value of the spectral density function at zero frequency is equal to the total area under the autocorrelation function.

proof - putting $f = 0$ in (4) we get

$$S(0) = \int_{-\infty}^{\infty} R(\tau)\, d\tau$$

The LHS is the value of the spectral density function at zero freq. & the RHS is the area under the autocorrelation function.

Hence, the result.

2. The mean square value of a wide sense stationary process is equal to the area under spectral density function.

proof - We know that,

$$R(\tau) = E[x(t) \cdot x(t+\tau)] \quad \text{--- (5)}$$

putting $\tau = 0$,

$$R(0) = E[x^2(t)]$$

but $R(\tau) = \int_{-\infty}^{\infty} S_x(f) \cdot e^{j2\pi f\tau}\, df$

putting $\tau = 0$

$$R(0) = \int_{-\infty}^{\infty} S_x(f)\, df \quad \text{--- (6)}$$

3. The spectral density function of real random process is an even function.

proof - by def^n (2) the spectral density function

$$S(\omega) = \int_{-\infty}^{\infty} R(\tau) \cdot e^{-j\omega\tau}\, d\tau$$

Changing the sign of ω

$$S(-\omega) = \int_{-\infty}^{\infty} R(\tau)\, e^{j\omega\tau}\, d\tau$$

putting $\tau = -u$, $d\tau = -du$,

when $\tau \to -\infty$, $u \to \infty$ & when $\tau \to \infty$, $u \to -\infty$

$$\therefore S(-\omega) = \int_{\infty}^{-\infty} R(-u) . e^{-j\omega u} (-du)$$

$$= \int_{-\infty}^{\infty} R(-u) . e^{-j\omega u}\, du$$

but $R(\tau)$ is an even function of τ

$R(-u) = R(u)$

$$\therefore S(-\omega) = \int_{-\infty}^{\infty} R(u) . e^{-j\omega u}\, du$$

$$= S(\omega)$$

Hence, $S(\omega)$ is an even function.

4. The spectral density fu^n of Random process $\{x(t)\}$ which may be real or complex, is a real fu^n.

5. The spectral density & the autocorrelation function of real WSS process from a fourier cosine transform pair.

Q.2B] A Random variable has the following exponential probability density function $f(x) = K \cdot e^{-|x|}$. determine the value of K & the corresponding distribution function.

→ i) Since X is continuous Random variate whose pdf is defined in $[-\infty, \infty]$ we have

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\int_{-\infty}^{\infty} K \cdot e^{-|x|} = 1 \quad \cdots\cdots \begin{cases} |x| = x & \text{if } x \geq 0 \\ |x| = -x & x \leq 0 \end{cases}$$

$$2 \int_{0}^{\infty} K \cdot e^{-x}\, dx = 1$$

$$2K \left[-e^{-x}\right]_0^{\infty} = 1$$

$$-2K [0-1] = 1$$

$$2k = 1$$

$$\boxed{k = 1/2}$$

ii) mean & variance

$$E(x) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$$

$$= \int_{-\infty}^{\infty} x \cdot 0.5\, e^{-|x|}\, dx \quad \cdots\cdots \int_{-a}^{a} f(x)\, dx = 0 \text{ if } f(x) \text{ is odd}$$

$$= 0$$

$$E[x^2] = \int_{-\infty}^{\infty} x^2 \cdot f(x)\, dx$$

$$= \int_{-\infty}^{\infty} x^2 \left(0.5 \cdot e^{-|x|}\right) dx$$

$$= 0.5 \times 2 \int_0^{\infty} x^2 e^{-x}\, dx$$

$$= \int_0^{\infty} x^2 \cdot e^{-x}\, dx = 2 \quad \cdots\cdots |x| = x \text{ if } x \geq 0$$

(B)

Q. 1A] State the three axioms of probability.

→ The axiomatic approach to probability is based on the following three postulates & on nothing else.

1) The Probability $P(A)$ of an event A is non negative number assigned to this event. i.e $P(A) \geq 0$ or $0 \leq P(A) \leq 1$

2) The Probability of the certain event equals 1. i.e $P(S) = 1$

3) If the events A & B are mutually exclusive then $P(A \cup B) = P(A) + P(B)$

Q.1.D] Define & explain moment Generating Function.

→ for finding moments of higher order we use a function that generates moments. It is called moment generating function. It is obtained as explained below. from it we can get moments of any order.

(a) Defination (discrete Random variable)
The moment generating function (m.g.f) of a discrete random variate x about 'a' denoted by $M_a(t)$ is defined by,

$$M_a(t) = E\left[e^{t(x-a)}\right]$$

$$\boxed{\therefore\ M_a(t) = \sum P_i e^{t(x_i - a)}} \quad \text{—(1)}$$

The m.g.f. is fun of the real parameter 't'.
expanding eqn (1),

$$M_a(t) = \sum p_i \left[1 + \frac{t}{1!}(x_i - a) + \frac{t^2}{2!}(x_i - a)^2 + \frac{t^3}{3!}(x_i - a)^3 + \cdots\right]$$

$$= \sum p_i + \frac{t}{1!}\sum p_i (x_i - a) + \frac{t^2}{2!}\sum p_i (x_i - a)^2 + \cdots$$

— (1a)

but $\sum p_i (x_i - a)^r$ is r^{th} moment of X about 'a' i.e μ_r'. Hence, we have

$$M_a(t) = 1 + \mu_1' \cdot \frac{t}{1!} + \mu_2' \cdot \frac{t^2}{2!} + \mu_3' \cdot \frac{t^3}{3!} + \cdots + \mu_r' \frac{t^r}{r!}$$

Coefficient of $(t^r/r!)$ is the rth moment of X about a. i.e μ_r' In this way $M_a(t)$ generates moments. this.

b] definatn [Continuous Random Variable] -
The moment generating fun of continuous random variate X about 'a' denoted $M_a(t)$ is defined by, $M_a(t) = E[e^{t(x-a)}]$

$$M_a(t) = \int_{-\infty}^{\infty} e^{t(x-a)} \cdot f(x)\, dx \quad \text{[where } f(x) \text{ is pdf]} \quad -(3)$$

expanding the exponential in (3) we get

$$M_a(t) = \int_{-\infty}^{\infty} f(x) \cdot \left[1 + \frac{t}{1}(x-a) + \frac{t^2}{2!}(x-a)^2 + \frac{t^3}{3!}(x-a)^3 + \cdots\right] dx$$

$$= \int_{-\infty}^{\infty} f(x)\, dx + \frac{t}{1!}\int_{-\infty}^{\infty} (x-a) f(x)\, dx$$

$$+ \frac{t^2}{2!}\int_{-\infty}^{\infty} (x-a)^2 f(x)\, dx + \cdots$$

but $\int_{-\infty}^{\infty} (x-a)^r f(x)\, dx$ is rth - moment μ_r' at 'a' point

$$M_a(t) = 1 + \mu_1' \cdot \frac{t}{1!} + \mu_2' \cdot \frac{t^2}{2!} + \mu_3' \frac{t^3}{3!} + \cdots \mu_r \frac{t^r}{r!}$$

the coefficient of $(t^r/r!)$ is the r^{th} moment of X about a i.e μ_r'. #

The mean $= E(x) = 0$

Variance $= E[x^2] - [E(x)]^2 = 2 - 0 = 2$

Q.3 A] A distribution has unknown mean μ & Variance 1.5. Using Central limit theorem find the size of the Sample such that the probability that difference bet^n Sample mean & the population mean will be less than 0.5 is 0.95.

→ We have. $E(x_i) = \mu$ & $Var(x_i) = 1.5$

If $\overline{X}$ is the sample mean then

$$Z = \frac{\overline{X}-\mu}{\sigma/\sqrt{n}} \text{ is S.N.V.}$$

We have the difference bet^n the Sample mean $\overline{X}$ & the population mean μ i.e $|\overline{X}-\mu| = 0.5$

$$|Z| = \left|\frac{\overline{X}-\mu}{\sigma/\sqrt{n}}\right| = \left|\frac{0.5}{\sqrt{1.5}/\sqrt{n}}\right|$$

$$= |0.4082\sqrt{n}|$$

we want this probability to be 0.95

We know from the table that $P(|Z|) = 0.95$ when $Z = 1.96$

$\therefore 0.4082\sqrt{n} = 1.96$

$$\therefore \sqrt{n} = \frac{1.96}{0.4082}$$

$\therefore n = 23.05$

Hence, 'n' must be atleast 24.

Q.6. B] The transition Probability matrix of a Markov chain is

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0.5 & 0.4 & 0.1 \\ 2 & 0.3 & 0.4 & 0.3 \\ 3 & 0.2 & 0.3 & 0.5 \end{array}$$

find the limiting probabilities.

→ Let the limiting probabilities be $\pi = [\pi_1, \pi_2, \pi_3]$. then we have $\pi P = \pi$ such that $\Sigma \pi_i = 1$

$$[\pi_1, \pi_2, \pi_3] \begin{bmatrix} 0.5 & 0.4 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{bmatrix} = [\pi_1, \pi_2 \pi_3]$$

$$0.5\pi_1 + 0.3\pi_2 + 0.2\pi_3 = \pi_1$$
$$0.4\pi_1 + 0.4\pi_2 + 0.3\pi_3 = \pi_2$$
$$0.1\pi_1 + 0.3\pi_2 + 0.5\pi_3 = \pi_3$$
& $\pi_1 + \pi_2 + \pi_3 = 1$

from the above eq we get,

$$-0.5\pi_1 + 0.3\pi_2 + 0.2\pi_3 = 0 \quad \text{--- } ①$$
$$0.4\pi_1 - 0.6\pi_2 + 0.3\pi_3 = 0 \quad \text{--- } ②$$
$$0.1\pi_1 + 0.3\pi_2 - 0.5\pi_3 = 0 \quad \text{--- } ③$$

Putting $\pi_3 = 1 - \pi_1 - \pi_2$ in ① & ②

$$-0.5\pi_1 + 0.3\pi_2 + 0.2(1 - \pi_1 - \pi_2) = 0$$
$$-0.7\pi_1 + 0.1\pi_2 = -0.2 \quad \text{--- } ④$$
$$0.4\pi_1 - 0.6\pi_2 + 0.3(1 - \pi_1 - \pi_2) = 0$$
$$0.1\pi_1 - 0.9\pi_2 = -0.3 \quad \text{--- } ⑤$$

multiply ④ by 0.9 & ⑤ by 0.1 & add

$$-0.63\pi_1 + 0.09\pi_2 = -0.18$$

$$0.01\,\pi_1 - 0.09\,\pi_2 = -0.03$$

$$-0.62\,\pi_1 = -0.21$$

$$\pi_1 = \frac{0.21}{0.62} = 0.34$$

from (4),

$$0.1\,\pi_2 = -0.2 + 0.7\,\pi_1$$

$$= -0.2 + 0.238$$

$$-0.1\,\pi_2 = 0.038$$

$$\pi_2 = \frac{0.038}{0.1}$$

$$\pi_2 = 0.38$$

& $\pi_3 = 1 - \pi_1 - \pi_2$

$$= 1 - 0.34 - 0.38$$

$$\pi_3 = 0.28$$

$\pi_1 = 0.34$, $\pi_2 = 0.38$, $\pi_3 = 0.28$

Q.1. B State central limit theorem & its Significance.

→ Central limit theorem is a very important theorem in statistical analysis. We give below the Central limit theorem in two forms, one known as Liapounoff's Form & other known as Lindberg - Levy form.

⇒ Central limit theorem (Liapounoff's form)
If $x_1, x_2 \cdots x_n$ are independent random variates with $E[x_i] = \mu_i$ & $Var(x_i) = \sigma_i^2$ $i = 1, 2 \ldots n$ then under certain general conditions $S_n = x_1 + x_2 + \cdots + x_n$ is a normal variate with $\mu = \Sigma \mu_i$ & Variance $\sigma^2 = \Sigma \sigma_i^2$ for n tends to ∞. (meaning n is large)
A particular form of the above theorem is of interest to us. The following form of the central limit theorem is known as Lindberg - Levy theorem.

⇒ Central limit theorem (Lindberg - Levy theorem)
If $X_1, X_2 \cdots X_n$ are independently & identically distributed random variates such that $E[x_i] = \mu$ & $Var(x_i) = \sigma^2$, $i = 1, 2 \cdots n$ then $S_n = X_1 + X_2 + \cdots + X_n$ is normal variate with mean $n\mu$ & var. $n\sigma^2$ as n tends to ∞.
Corollary – From the above theorem, we get a very important corollary as follows.
If $\bar{x}$ the mean of the sample of size n, taken from the population having the mean μ & variance σ^2 i.e. of

$\bar{x} = \dfrac{x_1 + x_2 + \cdots + x_n}{n}$ then $E(\bar{x}) = \dfrac{n\mu}{n} = \mu$

& $Var(\overline{X}) = \frac{1}{n^2}(n\sigma^2) = \frac{\sigma^2}{n}$

In other words, we get the following important result as a corollary of Central limit theorem.
If $\overline{X}$ is the mean of the sample of size n drawn from the population with mean μ & std. deviatⁿ σ then $\overline{X}$ is normally distributed with mean μ & std. deviatⁿ $\sigma/\sqrt{n}$ i.e

$$Z = \frac{\overline{X}-\mu}{\sigma/\sqrt{n}} \quad \text{is S.N.V. as } n \to \infty$$

⇒ Significance of CLT →
- The CLT is regarded by some Statisticians as the most important theorem in probability theory. It is important both theoretically & practically. Its significance lies in the fact that it is applicable for any distribution of X_i's there lies its power.
- CLT deals with convergence in distribution. Convergence works better near the centre. It is called the CLT.

Q. 2 A] In a communication system a zero is transmitted with probability 0.4 & a one is transmitted with probability 0.6. due to noise in the channel a zero can be received as one with probability 0.1 & as a zero with probability 0.9, lly one can be received as zero with probability 0.1 & as a one with probability 0.9.

1) If one is observed, what is the probability that a zero was transmitted.

→ $P(T_0) = 0.4$

$P(T_1) = 1 - 0.4 = 0.6$

$P(R_1/T_0) = 0.1$

$P(R_0/T_0) = 0.9$

$P(R_0/T_1) = 0.1$

$P(R_1/T_1) = 0.9$

$P(R_1) = P(R_1/T_1) \cdot P(T_1) + P(R_1/T_0) \cdot P(T_0)$

$= 0.9\ (0.6) + 0.1 \times 0.45$

$= 0.585$

i) $P(T_0/R_1) = \dfrac{P(R_1/T_0) \cdot P(T_0)}{P(R_1)}$

$= \dfrac{0.1 \times 0.45}{0.585}$

$P(T_0/R_1) = 0.076$

Q.1. c] State various properties of autocorrelation fu^n & power spectral density function.

→ ⇒ Properties of autocorrel^n fu^n $R(\tau)$ -

1. The mean square of a random process can be obtained from the autocorrl^n fu^n $R(\tau)$.

$$R_{XX}(\tau) = E[x(t) \cdot x(t+\tau)]$$

put $\tau = 0$

$$\boxed{R_{XX}(0) = E[x^2(t)]}$$

2. $R(\tau)$ is an even fu^n.

$$R(\tau) = R(-\tau)$$

3. $R(\tau)$ is maximum at $\tau = 0$ i.e

$$|R(\tau)| \leq R(0)$$

4. $R(\tau)$ is autocorrel^n of a ~~stion~~ stationary random process $\{x(t)\}$ with no periodic component ~~&~~ with non-zero mean then.

$$\lim_{\tau \to \infty} R(\tau) = [E(x)]^2$$

⇒ properties of PSD.

1. The value of SPF at 0 freq. is equal to ~~rect~~ total area under the $R(\tau)$

$$S(0) = \int_{-\infty}^{\infty} R(\tau)\, d\tau.$$

2. The mean square value of wide-sense ~~stionare~~ stationary process is equal to the area under spectral density fu^n.

$$R(0) = \int_{-\infty}^{\infty} S_X(f)\, df.$$

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3. The SDF of real random process is an even fu^n.

$$S(\omega) = S(-\omega).$$

4. SDF of random process $\{x(t)\}$ which may be real or complex, is a real fu^n.

Q.3 C] Let $z = x/y$. determine $f_z(z)$.

→ We introduce another auxillary random variable $w = y$.

now, we have $z = x/y$ & $w = y$

i.e $z = x/y$ & $w = y$

$$x = zy = zw \ \& \ y = w$$

$$\therefore J = \begin{vmatrix} \frac{\partial x}{\partial z} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial z} & \frac{\partial y}{\partial w} \end{vmatrix} = \begin{vmatrix} w & z \\ 0 & 1 \end{vmatrix} = w$$

The joint probability density fu^n of (z, w) is

$$f_{z,w}(z,w) = f_{x,y}(x,y)\,|J|$$

$$= |w| \cdot f_{xy}(x,y)$$

Since x, y are independent Random variables, we get,

$$f_{z,w}(z,w) = |w|\, f_x(x) \cdot f_y(y)$$

but $x = zw$ & $y = w$.

$$\therefore x = yz$$

$$f_{zw}(z,w) = |y| \cdot f_x(yz) \cdot f_y(y)$$

now, the marginal pdf $f_Z(z)$ is obtained by integrating $f_{Z,W}(z,w)$ w.r.to w i.e y.

$$f_Z(z) = \int_{-\infty}^{\infty} |y| \cdot f_X(yz) \cdot f_Y(y)\, dy.$$

Q.4.c] Write short note on:

i] (a)] poisson Distribution -

poisson distribution is the limiting case of the binomial distribution under the following conditions:

i] n, the no. of trials is infinitely large i.e $n \to \infty$

ii] p, the probability of success in each trial is constant & infinitely small i.e $p \to 0$

iii] np, the average success ~~in~~ is finite say m, i.e np = m.

def^n: A random variable X is said to follow poisson distribution if the probability of X is given by

$$P(X = x) = \frac{e^{-m} \cdot m^{x}}{x!} \qquad x = 0, 1, 2 \cdots$$

& $m > 0$ is called parameter of distribut^n

→ mean = m

→ variance = m.

Q.4 c] ii) ~~(b)~~ Gaussian or normal distribut^n

def^n - A continuous random variable X is said to follow normal distribution with parameter m (called mean) & σ^2 (variance) if its probability density function is given by,

$$f(x) = \frac{1}{\sqrt{2\pi}\cdot\sigma} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2} \quad — ①$$

$$-\infty < x < \infty$$

$$-\infty < m < \infty, \quad \sigma^2 > 0$$

→ mean & variance of normal distribut^n

mean $= m$

$var(x) = \sigma^2$

from eq^n ① ~~as~~ if mean $= 0$ & std. deviat^n i.e $\sigma = 1$ then.

$$Z = \frac{x-m}{\sigma} \quad \ldots \text{(x is normal variate with parameter } m, \sigma)$$

is called Standard normal variate.

~~uniform~~