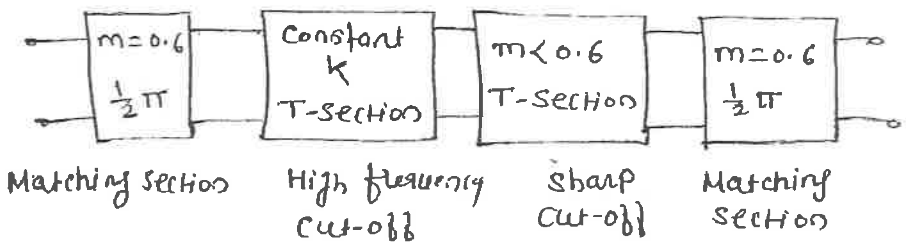
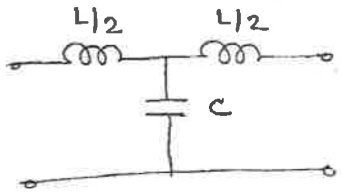
QP code: 22617

Q 2 (a) Design a low pass composite filter with cut-off frequency 3 MHz and impedance of 75 Ω. Place infinite attenuation pole at 3.08 MHz.

⇒



(i) For constant k − T section



$R_o = 75\,\Omega$

$f_c = 3\times10^{6}$ Hz

$f_\infty = 3.08$ MHz

$$L = \frac{R_o}{\pi f_c} = \frac{75}{3.14\times 3\times10^{6}} = 7.957\,\mu H$$

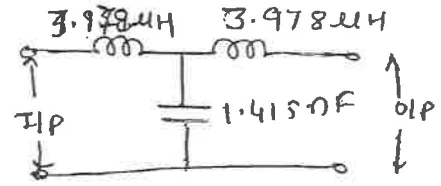
$\boxed{L = 7.957\,\mu H}$

$\therefore L/2 = 3.978\,\mu H$

$$C = \frac{1}{\pi R_o f_c} = \frac{1}{3.14\times 75\times 3\times10^{6}}$$

$\boxed{C = 1.415\ nF}$

∴ Constant k T section Low pass filter



(ii) To design m-derived section

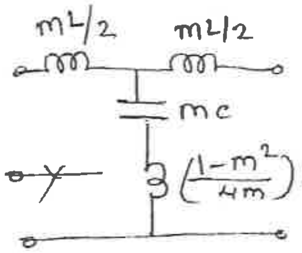
$f_c = 3\,MHz, \quad f_\infty = 3.08\,MHz$

$$\therefore \quad m = \sqrt{1 - \left(\frac{f_c}{f_\infty}\right)^2}$$

$$= \sqrt{1 - \left(\frac{3}{3.08}\right)^2}$$

$$m = 0.2264$$

(a) m derived sharp cut-off section

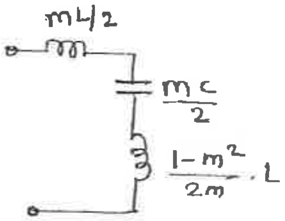


$$\frac{mL}{2} = 0.2264 \times 3.978\,\mu H = 0.9006\,\mu H$$

$$mc = 0.2264 \times 1.414\,nF = 0.3201\,nF$$

$$\left(\frac{1-m^2}{4m}\right)L = \frac{1-(0.2264)^2}{4 \times 0.2264} \times 7.957 = 8.336\,\mu H$$

(b) For matching $m = 0.6$



$$\frac{mL}{2} = 0.6 \times 3.978\,\mu H$$

$$= 2.3868\,\mu H$$

$$\frac{mc}{2} = \frac{0.6 \times 1.414\,nF}{2} = 0.4242\,nF$$

$$\frac{1-m^2}{2m} \cdot L = \frac{1-(0.6)^2}{2 \times 0.6} \times 7.957\,\mu H = 4.243\,\mu H$$

Q 3 (a) Design maximally flat low pass filter with cut-off frequency of 2 GHz, impedance of 50Ω and at least 15 dB insertion loss at 3 GHz with discrete LC components.

Solution :→ From the graph, Value of N = 5

Table gives Prototype element values as

$g_1 = 0.618$, $g_2 = 1.618$, $g_3 = 2$

$g_4 = 1.618$, $g_5 = 0.618$

$$\omega_c = 2\pi f_c = 2 \times 3.14 \times 2 \times 10^9 = 12.56 \times 10^9$$

Apply impedance and frequency scaling

$$L'_K = \frac{R_0 L_K}{\omega_c} \quad , \quad C'_K = \frac{C_K}{R_0 \omega_c}$$

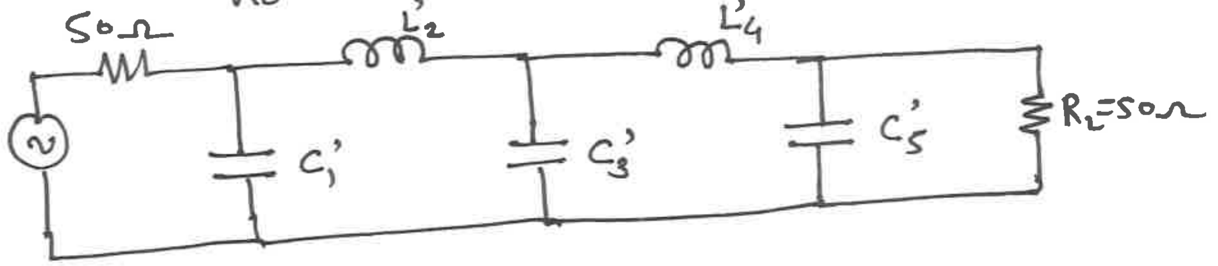
$$C'_1 = \frac{g_1}{R_0 \omega_c} = \frac{0.618}{50 \times 12.56 \times 10^9} = 0.984 \text{ PF}$$

$$L'_2 = \frac{R_0 g_2}{\omega_c} = \frac{50 \times 1.618}{12.56 \times 10^9} = 6.44 \text{ nH}$$

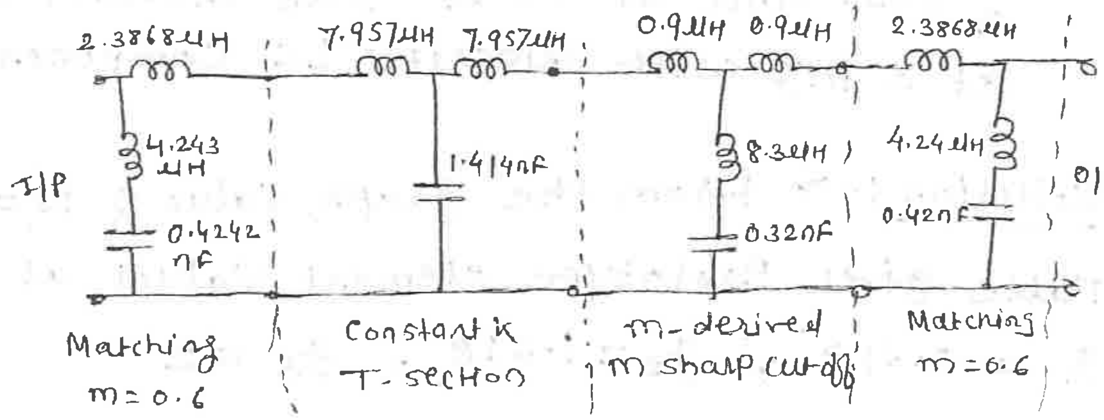
$$C'_3 = \frac{g_3}{R_0 \omega_c} = \frac{2}{50 \times 12.56 \times 10^9} = 3.184 \text{ PF}$$

$$L'_4 = \frac{R_0 g_4}{\omega_c} = \frac{50 \times 1.618}{12.56 \times 10^9} = 6.44 \text{ nH}$$

$$C_5 = \frac{g_5}{R_0 \omega_c} = \frac{0.618}{50 \times 12.56 \times 10^9} = 0.984 \text{ PF}$$



Final composite Filter



Q4 @ The Radiation Resistance of Infinitesimal dipole:---

The average poynting vector is,

$$P_{av} = \frac{1}{2} Re\,(E \times H^*)$$

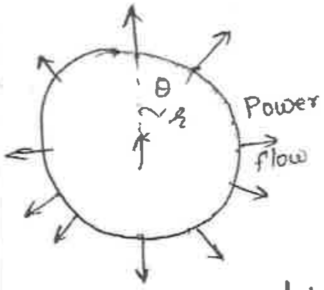
$$= \frac{1}{2} Re\,(E_\theta H_\phi^* \bar{r}$$

from eqn (13), (16), (17) E_r & H_ϕ^* are 90° out of phase & their product is purely imaginary

$$P_{av} = \frac{1}{2} Re\{E_\theta H_\phi^*\} \bar{r}$$

$$= \frac{1}{2} Re\left\{\left(\frac{I_o dl \sin\theta}{4\pi r}\right)^2 \frac{1}{\omega\epsilon}\left(j\beta^2 + \frac{\beta}{r} - \frac{j}{r^2}\right)\left(-j\beta + \frac{1}{r}\right)\right\} \bar{r}$$

$$P_{av} = \frac{1}{2}\left(\frac{I_o dl \sin\theta}{4\pi r}\right)^2 \frac{\beta^3}{\omega\epsilon}\,\bar{r} \quad (18)$$



The surface area at spherical surface is given as,

$$r^2 \sin\theta\, d\theta\, d\phi$$

∴ The total power $W = \iint P_{av}\, r^2 \sin\theta\, d\theta\, d\phi$

$$W = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \frac{1}{2}\left(\frac{I_o dl \sin\theta}{4\pi r}\right)^2 \frac{\beta^3}{\omega\epsilon}\, r^2 \sin\theta\, d\theta\, d\phi \quad (1)$$

$$W = \frac{1}{2}\left(\frac{I_o dl}{4\pi}\right)^2 \frac{\beta^3}{\omega\epsilon}\int_0^{2\pi} d\phi \int_0^{\pi} \sin^3\theta\, d\theta \quad (2)$$

Now to solve.

$$\int_0^{\pi} \sin^3\theta \, d\theta \Rightarrow \int_0^{\pi} \sin^2\theta \sin\theta \, d\theta$$

$$\sin^2\theta = 1 - \cos^2\theta .$$

$$= \int_0^{\pi} (1-\cos^2\theta) \sin\theta \, d\theta$$

put $\cos\theta = t$. $\therefore -\sin\theta \, d\theta = dt$

$$\sin\theta \, d\theta = -dt$$

$$\theta = 0 \Rightarrow t = 1$$

$$\theta = \pi \Rightarrow t = -1$$

The above integration changes

$$\int_0^{\pi} \sin^3\theta \, d\theta = \int_1^{-1} (1-t^2)(-dt)$$

$$= \int_1^{-1} (t^2-1) \, dt$$

$$= \left(\frac{t^3}{3} - t \right)_1^{-1}$$

$$= \left\{ \left(-\frac{1}{3} - (-1) \right) - \left(\frac{1}{3} - 1 \right) \right\}$$

$$= -\frac{1}{3} + 1 - \frac{1}{3} + 1 .$$

$$\int_0^{\pi} \sin^3\theta \, d\theta = 2 - \frac{2}{3}$$

$$= \frac{4}{3} \qquad \text{—— (A)}$$

$$\& \therefore \int_0^{2\pi} d\phi = [\phi]_0^{2\pi} = 2\pi \qquad \text{—— (B)}$$

from eqn A & B eq(2) becomes.

$$W = \frac{1}{2} \frac{I_0^2 dl^2}{16\pi^2} \frac{\beta^3}{\varepsilon\epsilon} 2\pi \frac{4}{3} \qquad \text{—— (4)}$$

$$\frac{\beta^3}{\omega\epsilon} = \frac{\omega^3 \mu \epsilon \sqrt{\mu\epsilon}}{\omega\epsilon} \qquad \left\{\because \beta = \omega\sqrt{\mu\epsilon}\right\}$$

$$= \omega^2 \mu \sqrt{\mu\epsilon}$$

$$= 4\pi^2 f^2 \mu \sqrt{\mu\epsilon} \qquad \left\{\because \omega = 2\pi f\right\}$$

$$f = \frac{c}{\lambda}$$

$$\therefore \frac{\beta^3}{\omega\epsilon} = 4\pi^2 \frac{c^2}{\lambda^2} \mu \sqrt{\mu\epsilon}$$

$$\left\{ \& \quad c = \frac{1}{\sqrt{\mu\epsilon}} \right\} \Rightarrow \sqrt{\mu\epsilon} = \frac{1}{c}$$

$$= 4\pi^2 \frac{c^2}{\lambda^2} \mu \times \frac{1}{c}$$

$$\frac{\beta^3}{\omega\epsilon} = \frac{4\pi^2}{\lambda^2} c \mu$$

$$c = \frac{1}{\sqrt{\mu\epsilon}} \quad \therefore 1/\sqrt{\mu\epsilon} = c$$

$$= \frac{4\pi^2}{\lambda^2} \frac{\mu}{\sqrt{\mu\epsilon}}$$

$$= \frac{4\pi^2}{\lambda^2} \sqrt{\frac{\mu}{\epsilon}}$$

Now $\sqrt{\frac{\mu}{\epsilon}} = n_o$

$$\frac{\beta^3}{\omega\epsilon} = \frac{4\pi^2}{\lambda^2} n_o$$

substitute this valle in eqn (4)

$$W = \frac{1}{2} \frac{I_o^2 dl^2}{16\pi^2} \frac{4\pi^2}{\lambda^2} \eta_o . 2\pi \frac{4}{3}$$

$$= \frac{I_o^2 dl^2}{\lambda^2 3} \pi \eta_o$$

η_o is intrinsic impedance of the medium it's valle for free space is 120π.

$$\therefore W = \frac{I_o^2 dl^2 \pi \times 120\pi}{\lambda^2 3}$$

$$= \frac{I_o^2 dl^2 \pi 40\pi}{\lambda^2}$$

$$W = 40\pi^2 I_o^2 \left(\frac{dl}{\lambda}\right)^2$$

Radiation Resistance of Hertz dipole :-

→ The power loss in a resistance carrying a peak current I_o is $\frac{1}{2} I_o^2 R_{rad}$.

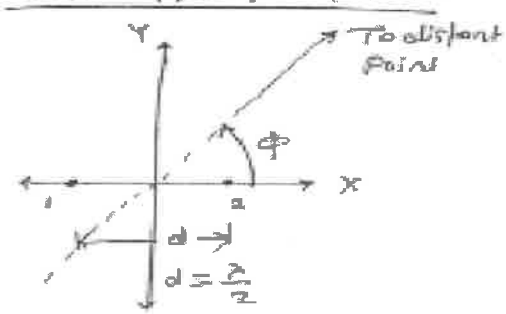
→ Equating this loss to the power radiated by the dipole w ⟹ $w = 40\pi^2 I_o^2 \left(\frac{dl}{\lambda}\right)^2 = \frac{1}{2} I_o^2 R_{rad}$

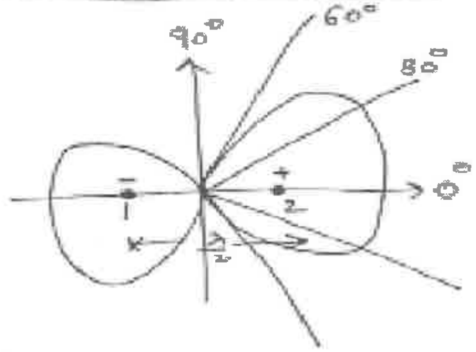
$$\Rightarrow R_{rad} = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2$$

For Hertz dipole $dl \ll \lambda$ ∴ For a 0.1λ dipole,

$R_r \simeq 8\,\Omega$ only.

Q 4 (b) Two isotropic point sources of same amplitude but opposite phase





Field due to source ① $E_0 e^{+j\psi/2}$

Field due to source ② $-E_0 e^{-j\psi/2}$ (opposite phase)

∴ Total field in the direction ϕ at a large distance r is given by

$$E = E_0 e^{+j\psi/2} - E_0 e^{-j\psi/2} \rightarrow ①$$

$$E = \frac{2j\left(E_0 e^{+j\psi/2} - E_0 e^{-j\psi/2}\right)}{2j} \quad \left(\text{divide \& multiply by } 2j\right)$$

$$\Rightarrow E = 2jE_0 \left(\frac{e^{+j\psi/2} - e^{-j\psi/2}}{2j}\right)$$

$$E = 2jE_0 \sin\frac{\psi}{2}$$

$$E = 2jE_0 \sin\left(\frac{d_r}{2}\cos\phi\right) \quad \left[\because \psi = d_r \cos\phi\right] \rightarrow ②$$

To normalize eqn ② put $2jE_0 = 1$

$$\text{eqn } ② \Rightarrow E = \sin\left(\frac{d_r}{2}\cos\phi\right) \rightarrow ③$$

since $d_r = \beta d = \frac{2\pi d}{\lambda}$

$d = \frac{\lambda}{2} \Rightarrow d_r = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = \pi$ ∴ eqn ③ $\Rightarrow \boxed{E = \sin\left(\frac{\pi}{2}\cos\phi\right)}$

E_q (3) $E = \sin\left(\frac{\pi}{2}\cos\phi\right) \rightarrow$ (4)

* The condition for maximum value of sine term is

$$\frac{\pi}{2}\cos\phi_{maximum} = \pm(2K+1)\frac{\pi}{2} \rightarrow (4a)$$

All odd multiples of $\frac{\pi}{2}$ gives maximum value of sine

where $k = 0, 1, 2, 3 \ldots$

For $K = 0$, $\frac{\pi}{2}\cos\phi_m = \pm\frac{\pi}{2} \Rightarrow \cos\phi_m = \pm 1$

$\Rightarrow \phi_m = 0^\circ$ & 180°

* The null directions ϕ_0 are given by

$$\frac{\pi}{2}\cos\phi_0 = \pm K\pi \text{ --------- } (4b)$$

for $K = 0 \Rightarrow \frac{\pi}{2}\cos\phi_0 = 0$

$\Rightarrow \phi_0 = \pm 90^\circ$

* The half power directions are given by

$$\frac{\pi}{2}\cos\phi = \pm(2K+1)\frac{\pi}{4} \text{ ------- } (4c)$$

for $K = 0 \Rightarrow \frac{\pi}{2}\cos\phi = \frac{\pi}{4}$

$\Rightarrow \cos\phi = \frac{1}{2} \Rightarrow \phi = \pm 60^\circ, \pm 120^\circ$

The two sources may be described as a simple type of End-fire array.