Solu^n for Discrete Structures (Rev)
Sem III (Comp.)

Q.P. Code: 00025227

Q.1 a) Prove by the induction that the sum of three consecutive numbers is divisible by 9.

→ Let the three consecutive numbers be, $n-1, n, n+1$.

We have to prove that

$(n-1)^3 + n^3 + (n+1)^3$ is divisible by 9.

Let $p(n) = (n-1)^3 + n^3 + (n+1)^3$

Basis of Induction: → (2M)

For $n = 1$

$0 + 1 + (2)^3 = 9$ is divisible by 9

$\therefore p(1)$ is true.

Induction Step: → (1M)

Assume $p(k)$ is true i.e.

$(k-1)^3 + k^3 + (k+1)^3$ is divisible by 9 — ①

→ $3k^3 + 6k$ is div. by 9

To prove that $p(k+1)$ is true

for $n = k+1$

$\{(k+1)-1\}^3 + (k+1)^3 + [(k+1)+1]^3 = k^3 + (k+1)^3 + (k+2)^3$

$= 3k^3 + 9k^2 + 15k + 9$

$= \underline{3k^3 + 6k} + 9(k^2 + k + 1)$ → (2M)

$(3k^3 + 6k)$ is divisible by 9 by ① and $9(k^2+k+1)$ is divisible by 9.

Hence proved.

b) Find G.F for (2½ Marks each)

i) 2, 2, 2, 2, 2, 2.

→ $\sum_{n=0}^{5} (2)^n x^n = 2^0 x^0 + 2^1 x^1 + 2^2 x^2 + 2^3 x^3 + 2^4 x^4 + 2^5 x^5$

ii) 1, 1, 1, 1, 1, 1

→ $\sum_{n=0}^{5} (1)^n x^n = 1 + x + x^2 + x^3 + x^4 + x^5 = \frac{1}{1-x}$

c) Total no. of balls $= 6 + 5 = 11$ No. of balls to draw $= 4$

(2½ M each)

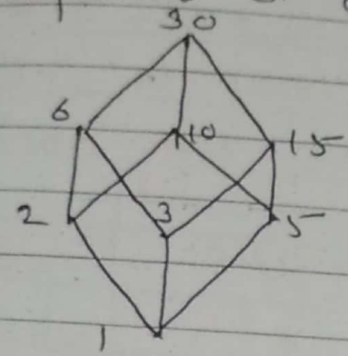
i) Any color $11C_4 = \frac{11!}{4!\,7!} = 330$

2) Same color $6C_4 + 5C_4$ (All white) (All red) $= \frac{6!}{4!\cdot 2!} + \frac{5!}{4!\,1!} = 15 + 5 = 20$

Complement of elements in D_{30}

Hasse diagram of D_{30} → 2M

Complements of all elements → 3M



Element	Complement
1	30
2	15
3	10
5	6
6	5
10	3
~~12~~	
15	2
30	1

Q.2 Defn of Isomorphism → 2M

No. of Nodes in Graph 1 = 8

——— " ——— 2 = 8

No. of Edges in Graph 1 = 12

——— " ——— 2 = 12

} 2M

Degrees of vertices in G1

A	3
B	3
C	3
D	3
E	3
F	3
G	3
H	3

Degrees of vertices in G2

A	3
B	3
C	3
D	3
E	3
F	3
G	3
H	3

— 2M

One-to-One Correspondence (Mapping) of vertices from Graph 1 to Graph 2 is

{(A, A), (B, B), (C, C), (D, D), (E, E), (F, F), (G, G), (H, H)} — 2M.

Let total no. of students = 1000
60% are girls ∴ No. of girl students = 600
∴ No. of boy students = 400

4% of 400 are taller than 1.8 = 16
1% of 600 are taller than 1.8 = 6.

From this we can tabulate

	taller than 1.8	Less than 1.8	Total
Boys	16	384	400
Girls	6	594	600
Total.	22	978	1000

probability that selected Students is a boy $= \frac{16}{22} = 0.727$

c) $\sim(p \vee (\sim p \wedge q))$

$= \sim p \wedge \sim(\sim p \wedge q)$ — DeMorgan's Law

$= \sim p \wedge (p \vee \sim q)$ — Law of Negation

$= (\sim p \wedge p) \vee (\sim p \wedge \sim q)$ — Distributive Law

$= F \vee (\sim p \wedge \sim q)$ — $\sim p \wedge p = F$

$= \sim p \wedge \sim q$ — Identity Law

Q.3 a) Multiplication Table

02M for table
1M each for each step.

$\times_7$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

i) It is closed operation

ii) Associative

let $a \times_7 (b \times_7 c) = (a \times_7 b) \times_7 c$

let $a = 1,\ b = 2,\ c = 3$

$1 \times_7 (2 \times_7 3) = (1 \times_7 2) \times_7 3$

$1 \times_7 6 = 2 \times 3$

$6 = 6$

∴ Associative.

For any element a
if $1 \times_7 a = a \times_7 1 = a$ then a is an identity
1 is identity.

iv) An element b is left inverse of a if $b * a = e$ where e is an identity element
For elements 1, 2, 3, 4, 5, 6 inverses are 1, 4, 5, 2, 3, 6 respectively.

v) Commutative if $a * b = b * a$ for all element a & b.
From the table it is seen that corresponding rows and columns are identical.
∴ Operation is commutative.

vi) The set has 6 elements hence group $(G, \times_7)$ is a finite abelian group of order 6.

Q.3 b)

$$M_R = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad M_S = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$ — 1M

$$W_0 = M_{RUS} = M_R \vee M_S = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$ — 1M

Using Warshall's Algo find till W_5

$$W_5 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$W_0 = W_1 = W_2 = W_3$
in W_4 (3,5) & (5,3) added.
$W_5 = W_4$
1M each for W, 5M

Smallest equivalence relⁿ containing R & S both is $(RUS)^{\infty}$.
$(RUS)^{\infty} = \{(1,1), (1,2), (2,1), (2,2)\ (3,3)\ (3,4)\ (3,5)\ (4,3)\ (4,4)$
$(4,5)\ (5,3)\ (5,4)\ (5,5)\}$ — 1M

$f(x) = x^2 + x + 1 \quad f: \mathbb{Z} \to \mathbb{Z}$

~~Not~~ A function is one-to-one if no two elements have same image

$f(1) = 3$ $f(-1) = 1$ $f(2) = 7$ $f(-2) = 5$ $f(3) = 13$ $f(-3) = 7$

It can be seen $f(2)$ & $f(-3)$ is same

$\therefore$ Not one-to-one.

A function is onto if every element of B is an image of some element of A.

It can be seen that negative elements of $\mathbb{Z}$ are not images of any element

$\therefore$ Not onto

$\therefore$ Not both (Bijective).

Q.4 a) $N = \{00000, 01110, 10101, 11011\}$

$\oplus$	00000	01110	10101	11011
00000	00000	01110	10101	11011
01110	01110	00000	11011	10101
10101	10101	11011	00000	01110
11011	11011	10101	01110	00000

– 2M for table
– 1M each for four steps.

i) $\forall a, b \in N$, $a \oplus b \in N$ Closed

ii) Identity = 00000 belongs to N

iii) Associative

iv) Each element is inverse of itself.

eg. $00000 \oplus 00000 = 00000 \oplus 00000 = 00000$

N is a subgroup of B^5 $\therefore$ It is a group code.

$d(00000, 01110) = 3$ $\quad d(01110, 10101) = 4$

$d(00000, 10101) = 3$ $\quad d(01110, 11011) = 3$

$d(00000, 11011) = 4$ $\quad d(10101, 11011) = 3$

$\therefore$ Min Distance = 3

The code will detect k or fewer error iff min. distance is at least $k+1$

$\therefore$ No. of errors it can detect = 2 or fewer – 1M

The code can correct k or fewer errors iff min distance is at least $2k+1$

$\therefore$ No. of errors it can correct = 1 or fewer. – 1M.

$B^3 = \{000, 001, 010, 011, 100, 101, 110, 111\}$

$e(000) = 000x_1x_2x_3$

x_1, x_2, x_3 can be calculated by

$x_k = b_1 \cdot h_{1k} + b_2 \cdot h_{2k} + \dots + b_m h_{mk}$

$\therefore e(000) = 000000$ $e(001) = 000111$ 2 M each code.

$e(010) = 010011$ $e(011) = 011100$

$e(100) = 100100$ $e(101) = 101011$

$e(110) = 110111$ $e(111) = 111000$.

Q.4 c) Let n = no. of friends = pigeons.

Let no. of months be pigeonholes $= m = 12$ — 1 M

By extended pigeonhole principle, — 1 M

$$\left\lfloor \frac{(n-1)}{m} \right\rfloor + 1 = 5$$

$$\frac{n-1}{12} = 4$$

$\therefore n = 49$ — 2 M

49 friends are required.

Q.5 a) $a * b = a + b - ab$

i) $\forall a, b \in G$ $a * b - ab \in G$ $\therefore$ closed

ii) Associative

Show $(a*b)*c = a*(b*c)$

$(a+b-ab)*c = a*(b+c-bc)$

iii) Identity $a * 0 = a + 0 - a*0$

$= a$

$\therefore$ '0' is an identity element.

iv) if $a*b$ = identity, b is inverse of a.

$a*1 = a+1 - a \times 1 = a$

$\therefore$ '1' is inverse of both a & b.

$\therefore (G, *)$ is a group. — 2 M for each step.

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b) Homogeneous eqn $a_r - 7a_{r-1} + 10a_{r-2} = 0$

Characteristic eqn $a^2 - 7a + 10 = 0$

$a = 2, 5$

Homogeneous solu: $a_r^{(h)} = A_1(2)^r + A_2(5)^r$ — 2M

Right Hands is is a polynomial

$\therefore a_r^{(P)} = P_0 + P_1 r$ — 1M

$a_r = P_0 + P_1 r$

$a_{r-1} = P_0 + P_1(r-1)$

$a_{r-2} = P_0 + P_1(r-2)$

Substituting these in given eqn we:

$(P_0 + P_1 r) - 7[P_0 + P_1(r-1)] + 10[P_0 + P_1(r-2)] = 6 + 8r$

Solving $P_0 = 8$ and $P_1 = 2$

$\therefore a_r^{(P)} = 8 + 2r$ — 2M

$a_r = a_r^{(h)} + a_r^{(P)}$

$= A_1(2)^r + A_2(5)^r + 8 + 2r$ — 1M

Using initial conditions $a_0 = 1$ and $a_1 = 2$

We get $A_1 = -9$ and $A_2 = 2$

$\therefore \boxed{a_r = -9(2)^r + 2(5)^r + 8 + 2r}$ — 2M.

Q.5 c) $\{A_1, A_2\}$ not a partition as $A_1 \cap A_2 \neq \phi$

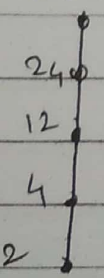
$\{A_3, A_4, A_5\}$ is a partition as

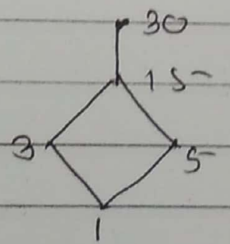
$A_3 \cup A_4 \cup A_5 = A$ and $A_3 \cap A_4 = \phi$ $A_4 \cap A_5 = \phi$

and $A_3 \cap A_5 = \phi$

6 a) i) Hasse Diagram.

Chain





4 M each
2M dia.
2M Justifica.

Not a chain.

b) $g \circ f = g(f(x)) = g(2x+3) = 6x + 13$

$f \circ g = f(g(x)) = f(3x+4) = 6x + 11$

$f \circ h = f(h(x)) = f(4x) = 8x + 3$

$g \circ f \circ h = g(8x+3) = 24x + 13$

number of vertices with odd degree = 4

Euler path.

A graph has a Euler path if it is connected and has exactly two vertices of odd degree.

$\therefore$ No Euler path — 2M

A graph has a Euler ckt. if it is connected and all vertices have Even degree.

$\therefore$ No Euler ckt — 2M.