Model Answer key.
Paper: T3425 - T.E. (Instrumentation & control)
sem V. CBSGS.
TOSOS - signals & systems qp.code: 23855

Q. ① ⓐ

① $F\{a_1 x_1(t) + a_2 x_2(t)\} = a_1 X_1(j\Omega) + a_2 X_2(j\Omega)$

② If $F[x(t)] = X(j\Omega)$ then

$F\{x(t-t_0)\} = e^{-j\Omega_0 t} X(j\Omega)$

③ $F\{x(at)\} = \frac{1}{|a|} X\left(\frac{j\Omega}{a}\right)$

④ $F\{x(-t)\} = X(-j\Omega)$

⑤ $F\{x^*(t)\} = X^*(-j\Omega)$

⑥ $F\left\{\frac{d}{dt}(x(t))\right\} = j\Omega X(j\Omega)$

ⓑ consider $x(t) = x_e(t) + x_o(t)$

$\therefore x^2(t) = x_e^2(t) + 2x_e(t) \cdot x_o(t) + x_o^2(t)$

$\int_{-\infty}^{\infty} x^2(t) = \int_{-\infty}^{\infty} x_e^2(t) + 2x_e(t) \cdot x_o(t) + x_o^2(t)\, dt$

$\int_{-\infty}^{\infty} x_e(t) \cdot x_o(t)\, dt = 0$

$\therefore$ we get $\int_{-\infty}^{\infty} x^2(t)\, dt = \int_{-\infty}^{\infty} x_e^2(t)\, dt + \int_{-\infty}^{\infty} x_o^2(t)\, dt$

ⓒ $y_1(n) = \begin{bmatrix} 2 & 0 & 1 & 1 \\ 1 & 2 & 0 & 1 \\ 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 0 \\ 1 \end{bmatrix}$

$y_1(n) = \begin{bmatrix} 3 \\ -2 \\ -1 \\ 0 \end{bmatrix}$ ⇒ convolution

(d) Initial value $x(0) = \lim_{s\to\infty} s\,X(s)$

$$\lim_{s\to\infty} \cancel{s} \cdot \frac{0.8}{\cancel{s}(s^2+0.6s+0.2)} = \frac{0.8}{1+0} = \underline{0}$$

final value $x(\infty) = \lim_{s\to 0} sF(s)$

final value $= \underline{4}$

(e) $x(t) = \mathrm{sgn}(t) = 1 \, ; \, t>0$
$= -1 \, ; \, t<0.$

$$X(j\Omega) = \frac{2}{j\Omega}$$

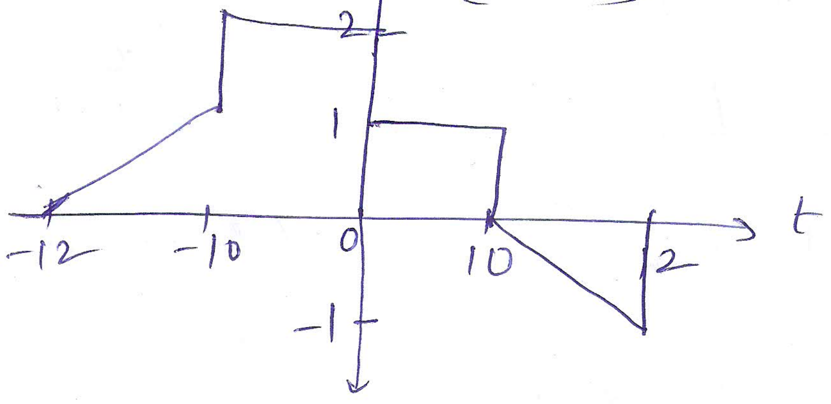
Q.2

(a) $H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2-s-6}$

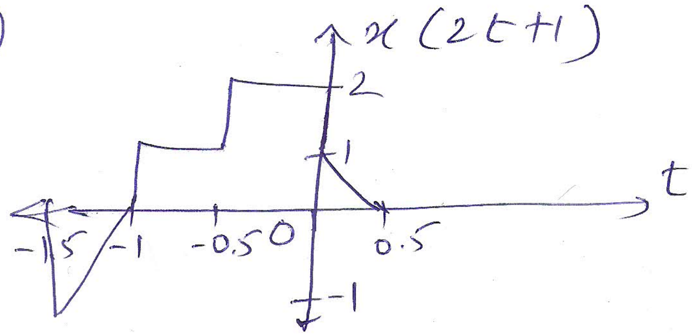
$$h(t) = \frac{1}{5}e^{3t}u(t) - \frac{1}{5}e^{-2t}u(t)$$

step response $\Rightarrow$ $y(t) = -\frac{1}{6}u(t) + \frac{1}{15}e^{3t}u(t) + \frac{1}{10}e^{-2t}u(t)$

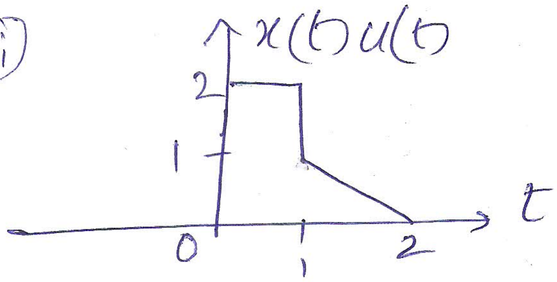
(b) (i)



Q(2) (b) (ii)



(iii)



Q(3) (a) By using convolution

$$x(t) * h(t) = y(t) = \int_{\lambda=-\infty}^{\infty} x(\lambda)\, h(t-\lambda)\, d\lambda$$

$$= \int_{\lambda=0}^{\infty} \lambda e^{-2\lambda} u(t-\lambda)\, d\lambda$$

$$= \int_{0}^{t} \lambda e^{-2\lambda}\, d\lambda$$

$$y(t) = -e^{-2t}\left[\frac{t}{2} + \frac{1}{4}\right] + \frac{1}{4}$$

(b) (i) neither energy nor power signal

E & $P = \infty$

(ii) Energy signal

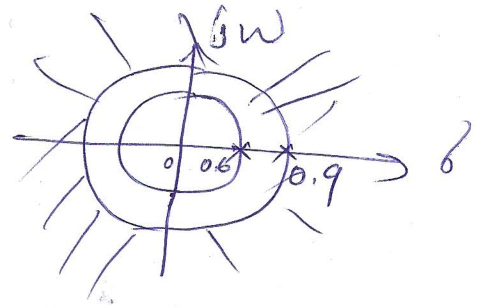
$E = \frac{A}{5}$ & power $= 0$.

(c) (i) periodic signal with fundamental period $= 4/9$ sec.

(ii) periodic signal.

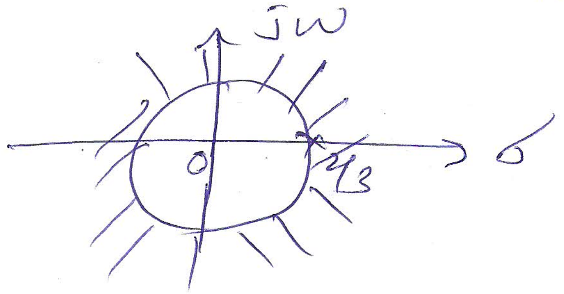
Q. 4 (a)

(i) $X[z] = \frac{z}{z-0.6} + \frac{z}{z-0.9}$



R.O.C. > 0.9

(ii) $X[z] = \frac{9}{4} \frac{z^3}{z-2/3}$



R.O.C. $> 2/3$

Q. 5 (a)

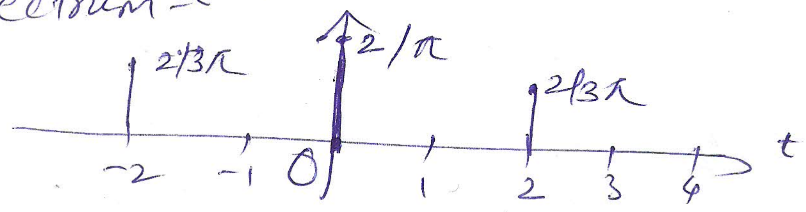
(i) $f(t) = a_0 + \sum_{n=1}^{\infty} \left[a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)\right]$

exponential form is.

$f(t) = \sum_{n=-\infty}^{\infty} f_n \cdot e^{jn\omega_0 t}$

$f(t) = \frac{2}{\pi} + \frac{2}{3\pi} \cdot e^{2jt} - \frac{2}{15} e^{4jt} + \cdots$

spectrum –



(b) (i) $x(t) = L^{-1} X(s) = \frac{1}{36}\left[25 e^{-4t/3} - 5 e^{4t/3}\right]$

(ii) $x(t) = \frac{e^{-t}}{3}\left(\sqrt{3} \sin(\sqrt{3} t) - 2\cos(\sqrt{3} t) + 2\right)$

Q 6

(a) $h[n] = \frac{-5}{4}(0.1)^n u(n) + \frac{25}{4}(0.5)^n u(n)$

(b) $x(t) = (1+t^3)(\cos^3 10t)$

even part $x_e(t) = \frac{1}{2}[x(t) + x(-t)]$

$= \frac{1}{2}\left[(1+t^3)(\cos^3 10t) + (1-t^3)(\cos^3(-10t))\right]$

Odd part $x_o(t) = \frac{1}{2}[x(t) - x(-t)]$

$= \frac{1}{2}\left[(1+t^3)(\cos^3 10t) - (1-t^3)[\cos^3 10(-t)]\right]$

(c) parseval's theorem :-

If $F[x(t)] = X(j\Omega)$ then

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\Omega)|^2 d\Omega$$