

N.B.: (1) All questions are compulsory.

(2) Figures to the right indicate maximum marks.

(3) Answers to the two sections must be written in the same answer-book.

SECTION I

QUANTUM MECHANICS I

1. Initially a free particle is represented by a wave function 12  

$$\Psi(x,0) = A(a+x)(a-x) \quad \text{for } -a \leq x \leq a$$

$$= 0 \quad \text{otherwise.}$$

- i] Calculate  $\langle x \rangle$ ,  $\langle p \rangle$ ,  $\langle x^2 \rangle$ ,  $\langle p^2 \rangle$  and  $\langle H \rangle$   
 ii] Find the uncertainty product  $\Delta x \Delta p_x$

OR

2. Write down Schrodinger's time dependent equation. From that derive the continuity equation:  

$$\frac{\partial \rho}{\partial t} + \nabla \cdot J = 0$$

Identify the expression for the current. Evaluate  $\vec{j}$  for  $\psi(r) = e^{i\vec{k}\cdot\vec{r}}$

3. Consider the operator/matrix,

$$A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

- i] Is A Hermitian? 1  
 ii] Find its eigenvalues. 3  
 iii] Obtain the eigenvectors and normalize them. 4  
 iv] What is the matrix operation that diagonalizes A? Construct the unitary diagonalizing matrix S. 5

OR

4. (a) The initial state of the Gaussian wave packet is:

$$\Psi(x,t=0) = \frac{1}{\sqrt{a} (2\pi)^{1/4}} e^{ik_0 x} e^{-x^2/4a^2}$$

- i] Find the momentum amplitudes for this state? 4  
 ii] What is the momentum probability density? 3

Use  $\int_{-\infty}^{\infty} e^{-ax^2+\beta x} dx = \sqrt{\frac{\pi}{a}} e^{\beta^2/4a}$

- (b) i] Define Hermitian adjoint and Hermitian operator. 2  
 ii] Show that the momentum operator is Hermitian. 2  
 iii] Find the normalized eigenfunction of the momentum operators. 2

5. Consider a particle in an finite potential well given by:

$$V(x) = 0 \text{ for } x < 0 \text{ and } x > a$$

$$= -V_0 \text{ for } 0 < x < a$$

- i] Set up the Schrodinger equation in different region and solve. 5  
 ii] Obtain the transcendental equation and calculate energy eigenvalues from them. 5  
 iii] Sketch the ground state and the first excited state eigenfunctions. 3

OR

6. (a) For an harmonic oscillator  
 i] Define an annihilation operator. Obtain the normalized ground state wave function using it. 3  
 ii] find the expression for the remaining wave functions using creation Operators. 3  
 (b) A particle moving along positive x direction experiences a potential given by:  
 $V(x) = 0 \text{ for } x < 0$   
 $= -V_0 \text{ for } x > 0.$   
 Define and calculate  
 i] Reflection coefficient 4  
 ii] Transmission coefficient 3

SECTION II

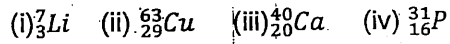
NUCLEAR PHYSICS

7. (a) Explain the rotational energy spectrum for an even – even nucleus. 6  
 (b) Explain why the nucleus has the zero electric dipole moment? What information does the measurement of the electric quadrupole moment of the nucleus provide? 6

OR

8. (a) What experimental evidence indicates that the deuteron wave function has contribution from the terms other than  $l = 0$ ? Explain the concept of tensor force. 6

- (b) Give the expected shell model spin and parity assignments for ground states of:



9. (a) Why the emission of  $\alpha$  particle is preferred over that of individual nucleon in radioactive decay of the nucleus? State Geiger – Nuttal law. 6  
 (b) Explain the compound nucleus hypothesis. 6

OR

10. Obtain the Breit Wigner single level resonance formula for S wave neutron scattering. 12

11. (a) Explain the principle and working of linear accelerators. Show that the length of  $n^{\text{th}}$  drift tube  $\sqrt{n}$  times length of the first drift tube of the linear accelerator. 7  
 (b) Draw Feynman diagram for  $\Delta^0 \rightarrow p^+ + \pi^-$  as a weak interaction and also as a strong interaction. Which process is more probable? 6

OR

12. (a) Using the schematic diagram, show the construction of synchrotron. What are the differences between synchrotron and synchrocyclotron? 7  
 (b) What are semileptonic and purely hadronic processes? Give an example of each. 6

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