(3 Hours) [Total Marks: 75

N.B.: (1) **All** questions are **compulsory.**
(2) **Figures** to the **right** indicate **maximum** marks.
(3) Answers to the two sections must be written in the **same** answer-book.

SECTION I

QUANTUM MECHANICS I

1. Initially a free particle is represented by a wave function 12

$\Psi(x,0) = A(a+x)(a-x) \quad for\ -a \leq x \leq a$

$= 0 \quad otherwise.$

i] Calculate <x>, <p>, <x^2>, <p^2> and <H>

ii] Find the uncertainty product $\Delta x\ \Delta p_x$

OR

2. Write down Schrodinger's time dependent equation. From that derive the continuity equation: 12

$$\frac{\partial \rho}{\partial t} + \nabla \cdot J = 0$$

Identify the expression for the current. Evaluate $\vec{j}$ for $\psi(r) = e^{i\vec{k}\cdot\vec{r}}$

3. Consider the operator matrix,

$$A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

i] Is A Hermitian? 1

ii] Find its eigenvalues. 3

iii] Obtain the eigenvectors and normalize them. 4

iv] What is the matrix operation that diagonalizes A? Construct the unitary diagonalizing matrix S. 5

OR

4. (a) The initial state of the Gaussian wave packet is:

$$\Psi(x, t=0) = \frac{1}{\sqrt{a}\,(2\pi)^{1/4}} e^{ik_0 x} e^{-x^2/4a^2}$$

i] Find the momentum amplitudes for this state? 4

ii] What is the momentum probability density? 3

Use $\int_{-\infty}^{\infty} e^{-\alpha x^2 + \beta x} dx = \sqrt{\frac{\pi}{\alpha}} e^{\beta^2/4\alpha}$

(b) i] Define Hermitian adjoint and Hermitian operator. 2

ii] Show that the momentum operator is Hermitian. 2

iii] Find the normalized eigenfunction of the momentum operators. 2

5. Consider a particle in an finite potential well given by:

$V(x) = 0$ for $x < 0$ and $x > a$

$= -V_0$ for $0 < x < a$

i] Set up the Schrodinger equation in different region and solve. 5

ii] Obtain the transcendental equation and calculate energy eigenvalues from them. 5

iii] Sketch the ground state and the first excited state eigenfunctions. 3

OR

6. (a) For an harmonic oscillator

i] Define an annihilation operator. Obtain the normalized ground state wave function using it. 3

ii] find the expression for the remaining wave functions using creation Operators. 3

(b) A particle moving along positive x direction experiences a potential given by:

$V(x) = 0$ for $x<0$

$= - V_0$ for $x>0$.

Define and calculate

i] Reflection coefficient 4

ii] Transmission coefficient 3

SECTION II

NUCLEAR PHYSICS

7. (a) Explain the rotational energy spectrum for an even – even nucleus. 6

(b) Explain why the nucleus has the zero electric dipole moment? What information does the measurement of the electric quadrupole moment of the nucleus provide? 6

OR

8. (a) What experimental evidence indicates that the deuteron wave function has contribution from the terms other than $l = 0$? Explain the concept of tensor force. 6

(b) Give the expected shell model spin and parity assignments for ground states of: 6

(i) $^{7}_{3}Li$ (ii) $^{63}_{29}Cu$ (iii) $^{40}_{20}Ca$ (iv) $^{31}_{16}P$

9. (a) Why the emission of α particle is preferred over that of individual nucleon in radioactive decay of the nucleus? State Geiger – Nuttal law. 6

(b) Explain the compound nucleus hypothesis. 6

OR

10. Obtain the Breit Wigner single level resonance formula for S wave neutron scattering. 12

11. (a) Explain the principle and working of linear accelerators. Show that the length of n^{th} drift tube $\sqrt{n}$ times length of the first drift tube of the linear accelerator. 7

(b) Draw Feynman diagram for $\Delta^0 \rightarrow p^+ + \pi^-$ as a week interaction and also as a strong interaction. Which process is more probable? 6

OR

12. (a) Using the schematic diagram, show the construction of synchrotron. What are the differences between synchrotron and synchrocyclotron? 7

(b) What are semileptonic and purely hadronic processes? Give an example of each. 6

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