QP Code : 75457

(3 Hours) [Total Marks: 75

N.B.: (1) **All** questions are **compulsory.**
(2) **Figures** to the **right** indicate **maximum** marks.
(3) Answers to the two sections must be written in **same** answer-books.

Section I (Mathematical Methods)

1. a Obtain the Fourier series for 06
$f(x) = -1, \quad -l < x < 0$
$= 1, \quad 0 < x < l$
And show that $1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$

b Find the eigenvalues and eigenvectors. 06

$$A = \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & 0 & 1 \end{pmatrix}$$

OR

2. a State and prove Parseval's theorem 04
b i) Find the Laplace transform of $f(t) = t\,sinwt$ 04
ii) Find the inverse Laplace transform of $F(s) = \frac{1}{s(s^2+w^2)}$ 04

3. a State and prove Cauchy's theorem 07
b Write the Cauchy Riemann conditions and show that 06
$u = xe^x cosy - ye^x siny$ is harmonic and find the conjugate v.

OR

4. a Prove the Residue theorem and find the residues of 07
$f(z) = \frac{z}{(2z+1)(5-z)}$
b Evaluate $I = \int_0^{2\pi} \frac{d\theta}{5-4\,sin\theta}$ 06

5. a Solve the differential equation using Frobenius method 07
$x(x-1)y'' + (3x-1)y' + y = 0$
b Solve the one dimensional wave equation 06
$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$
with conditions $u(x,0) = 0, u(0,t) = 0, u(x,0) = f(x), \frac{\partial u(x,0)}{\partial t} = g(x)$

OR

6 a Solve the differential equation 07
$y'' + y' - 6y = 6x^3 - 3x^2 + 12x$
b Obtain the series solution 06
$2x^2y'' + 3xy' + y = 0$

SECTION II
CLASSICAL MECHANICS

7. a What are generalized coordinates? Using transformation equations derive an expression for the kinetic energy of a system. 6
b Derive Lagrange's equations of motion from Hamilton's principle. 6

OR

8	a	Taking freely falling particle as an example, obtain Lagrangian, Lagrange's equations of motion and solve them.	6
	b	A hoop is rolling without slipping down an inclined plane which is at an angle ϕ to the ground. Find the equation of motion.	6
9	a	Prove Virial theorem.	6
	b	What are Kepler's three laws of planetary motion? Derive third law.	6

OR

10	a	Discuss small oscillations about the minima of the potential function.	6
	b	Obtain an expression for differential scattering cross section.	6
11	a	Show that Poisson bracket remain invariant under canonical transformation.	7
	b	A Hamiltonian of the system is given by: $$H = q_1 p_1 - q_2 p_2 - a q_1^2 + b q_2^2$$ Show that $F = q_1 q_2$ is constant of motion.	6

OR

12	a	What are Poisson bracket? What are elementary Poisson brackets?	5
	b	Prove Jacobi's identity for Poisson bracket.	8

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