QP. 24113 ①

Subject - signals & systems
Name of exam - SE (EXTC) / sem IV / CBSGS /
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Q1(a) (1) $x(t) = e^{-4t}u(t)$

→ It is an aperiodic signal. So it is energy signal

$$E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt$$

$$= \int_0^{T} (e^{-4t})^2 dt = \frac{1}{4} - \frac{0}{4} = \frac{1}{4} \text{ Joules.}$$

(2) $x(n) = \left(\frac{1}{6}\right)^n u(n)$

→ It is aperiodic signal. So, it is energy signal

$$E = \lim_{N\to\infty} \sum_{n=0}^{N} \left[\left(\frac{1}{6}\right)^n\right]^2 \cancel{= \frac{1}{1-(1/6)} = \frac{6}{5} \text{ Joules}}$$

$$= \lim_{N\to\infty} \left[\sum_{n=0}^{N} \left(\frac{1}{36}\right)^n\right] = \sum_{n=0}^{\infty} \left(\frac{1}{36}\right)^n$$

$$= \frac{1}{1-\frac{1}{36}} = \frac{36}{35}$$

Q1(b) $y(n) = a^n x(n)$

→ ① It is memoryless system.

② O/p $\propto$ present I/p. It is causal system.

③ $y_1(n) = a^n x_1(n)$

$y_2(n) = a^n x_2(n)$

$p[y_1(n)] + q[y_2(n)] = a^n[p(x) + q x_2(n)]$

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$y_3(n) = T[p x_1(n) + q x_2(n)] = a^n [p x_1(n) + q x_2(n)]$.

$y_3(n) = p y_1(n) + q y_2(n)$.

System is linear.

Q1c) $x(t) = e^{-3|t-t_0|} + e^{3|t+t_0|}$

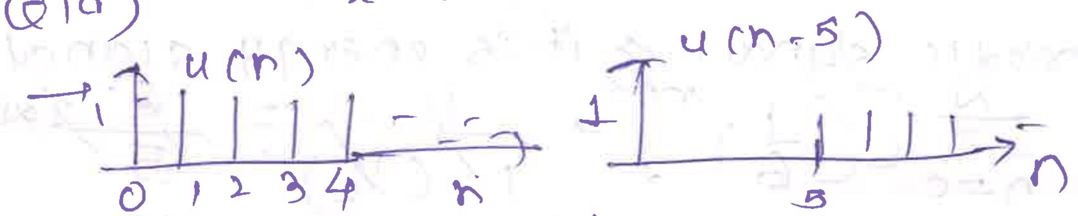
→ By time shifting property,

$X(j\omega) = F\{e^{-3|t-t_0|} + e^{3|t+t_0|}\}$.

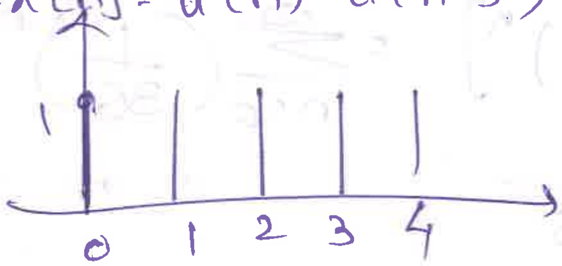
$$= \frac{3 \times 2}{3^2 + \omega^2} \times e^{-j\omega t_0} + \frac{3 \times 2}{3^2 + \omega^2} \times e^{j\omega t_0}.$$

$$= \frac{6}{3^2+\omega^2}\left(e^{-j\omega t_0} + e^{j\omega t_0}\right) = \frac{12\cos\omega t_0}{3^2 + \omega^2},$$

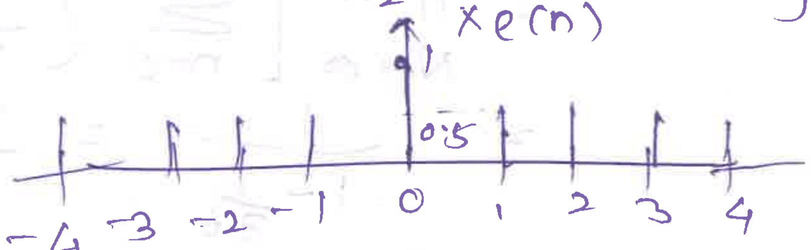
Q1d) (i) $x(n) = u(n) - u(n-5)$



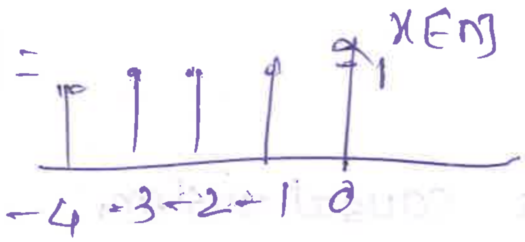
$x[n] = u(n) - u(n-5)$



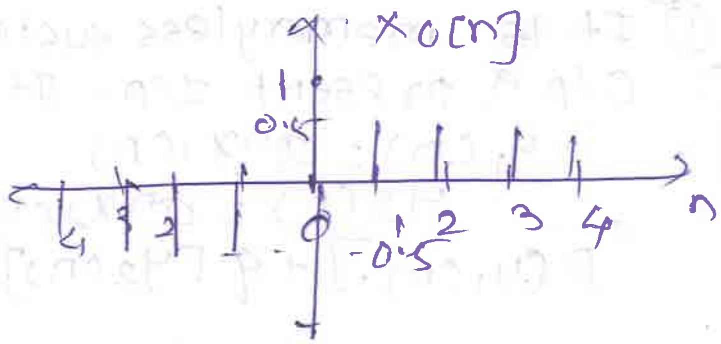
$x_e(n) = \frac{1}{2}\{x(n) + x(-n)\}$



$x[-n] =$



$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$



(ii) $x(t) = 5+7t+9t^2$

$\rightarrow$ $x(-t) = 5-7t+9t^2$

$x_e(t) = \frac{1}{2}[x(t)+x(-t)]$

$= \frac{1}{2}[(5+\cancel{7t}+9t^2)+(5-\cancel{7t}+9t^2)]$

$= \frac{1}{2}[2(5+9t^2)] = 5+9t^2$

$x_o(t) = \frac{1}{2}[x(t)-x(-t)]$

$= \frac{1}{2}[\cancel{5}+7t+\cancel{9t^2}-\cancel{5}+7t-\cancel{9t^2}]$

$= \frac{1}{2}[2(7t)] = 7t.$

Q1(e) F.T. $\rightarrow$ $X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$

where $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega$

$X(\sigma+j\omega) = \int_{-\infty}^{\infty} x(t)e^{-(\sigma+j\omega)t}\,dt.$

$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\sigma+j\omega)e^{(\sigma+j\omega)t}\,dt.$

$\sigma+j\omega$ is denoted by s.

$\frac{ds}{d\omega} = j$ or $d\omega = \frac{ds}{j}$

$\therefore$ $X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\,dt$

$x(t) = \frac{1}{2\pi j}\int_{\sigma-j\omega}^{\sigma+j\omega} X(s)e^{st}\,ds$

$X(\omega) = X(s)\big|_{s=j\omega}$

(Q2a) $a_0 = 0$, $\cancel{b_n = 0}$, $a_n = \frac{4}{T}\int_0^{T/2} x(t)\cos n\omega_0 t\,dt$

$$\cancel{a_n = \frac{4}{T}\int_0^{T/4} A\cos n\omega_0 t\,dt + \frac{4}{T}\int_{T/4}^{T/2} (-A)\cdot}$$

$a_0 = 0$, $a_n = 0$, $b_n = \frac{4}{T}\int_0^{T/2} x(t)\sin n\omega_0 t\,dt$.

$$b_n = \frac{4}{T}\int_0^{T/2} A\sin n\omega_0 t\,dt =$$

$$= \frac{4A}{T}\left[-\frac{T}{2n\pi}\cos n\pi + \frac{T}{2n\pi}\right].$$

$b_n = 0$ for even values of n

$$= \frac{4A}{T}\left[\frac{T}{2n\pi} + \frac{T}{2n\pi}\right] = \frac{4A}{n\pi} \text{ for odd values of } n$$

$b_1 = \frac{4A}{\pi}$, $b_3 = \frac{4A}{3\pi}$, $b_5 = \frac{4A}{5\pi}$ ---

Trignometric form

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos n\omega_0 t\,dt + \sum_{n=1}^{\infty} b_n\sin n\omega_0 t$$

$$x(t) = \sum_{n=\text{odd}} b_n \sin n\omega_0 t\,dt$$

$$= \frac{4A}{\pi}\left[\sin\omega_0 t + \frac{\sin 3\omega_0 t}{3} + \frac{\sin 5\omega_0 t}{5} + \cdots\right]$$

Q2 b) $H(s) = \dfrac{s+3}{s^2+6s+8}$

→ i) Impulse response 5 M

$x(t) = \delta(t)$, $X(s) = 1$

$Y(s) = H(s)X(s) = \dfrac{s+3}{s^2+6s+8}(1) = \dfrac{s+3}{(s+2)(s+4)}$

$= \dfrac{1}{2}\left(\dfrac{1}{s+2}\right) + \dfrac{1}{2}\left(\dfrac{1}{s+4}\right)$

$y(t) = \dfrac{1}{2}e^{-2t}u(t) + \dfrac{1}{2}e^{-4t}u(t)$

ii) step Response : 5 M,

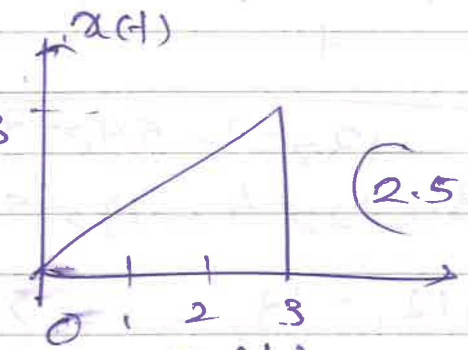
$x(t) = u(t)$ $X(s) = \dfrac{1}{s}$

$Y(s) = H(s)X(s) = \dfrac{s+3}{s(s+2)(s+4)}$

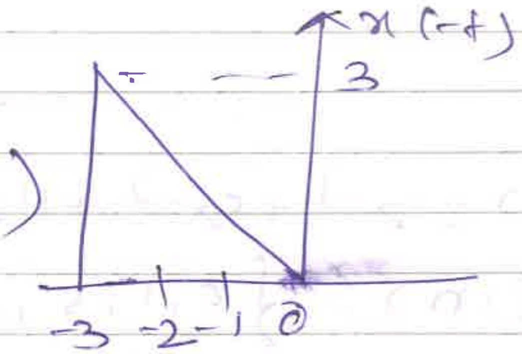
$= \dfrac{3/8}{s} - \dfrac{1/4}{s+2} - \dfrac{1/8}{s+4}$

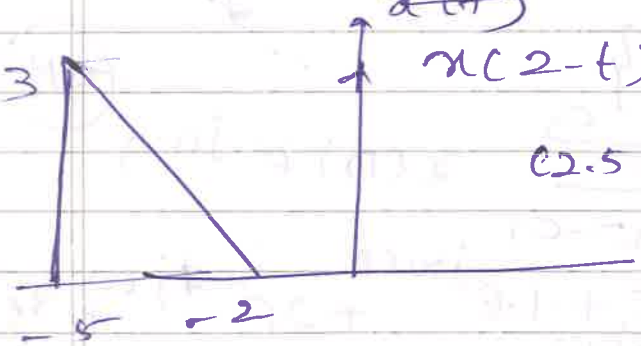
$y(t) = \dfrac{3}{8}u(t) - \dfrac{1}{4}e^{-2t}u(t) - \dfrac{1}{8}e^{-4t}u(t)$

Q3 a)

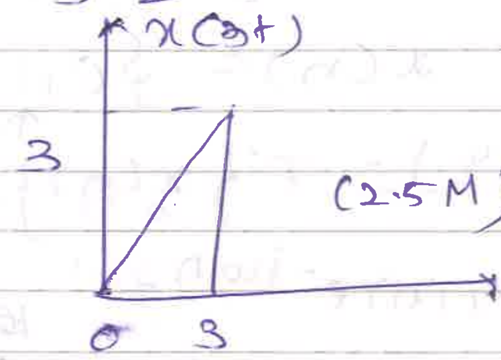


(2.5 M)



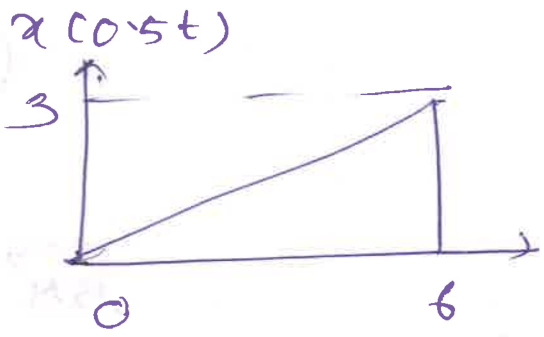


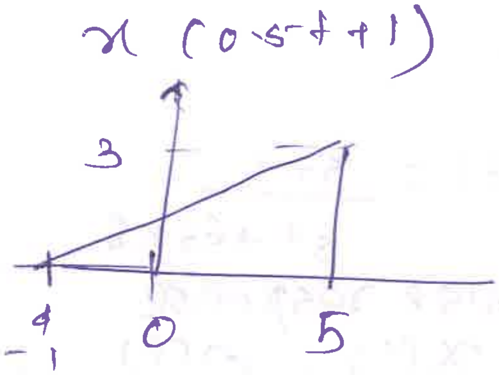
(2.5 M)



(2.5 M)

3-2





(2.5M).

Q3b) $X[z] = \dfrac{z^2+2z}{z^3-3z^2+4z+1}$ (5M)

$\rightarrow$

$$\begin{array}{r l}
 & z^{-1} + 5z^{-2} + 11z^{-3} + 12z^{-4} - 13z^{-5} \\
z^3-3z^2+4z+1 \,\big) & z^2+2z \\
 & z^2 - 3z + 4 + z^{-1} \\
 & \overline{5z - 4 - z^{-1}} \\
 & 5z - 15 + 20z^{-1} + 5z^{-2} \\
 & \overline{11 - 21z^{-1} - 5z^{-2}} \\
 & 11 - 33z^{-1} + 44z^{-2} + 11z^{-3} \\
 & \overline{12z^{-1} - 49z^{-2} - 11z^{-3}} \\
 & 12z^{-1} - 36z^{-2} + 48z^{-3} + 12z^{-4} \\
 & \overline{-13z^{-2} - 59z^{-3} - 12z^{-4}}
\end{array}$$

$X(z) = z^{-1} + 5z^{-2} + 11z^{-3} + 12z^{-4} - 13z^{-5} + \ldots$

$x(n) = \{0, 1, 5, 11, 12, -13, \ldots\}$.

Q3c) $x(n) = \{0, \underset{\uparrow}{1}, 2, 3\}$ (5M)

$X(e^{j\omega}) = F\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$

$= \sum_{n=-1}^{2} x(n) e^{-j\omega n} = 0 + 1 \cdot e^{-j\omega \times 0} + 2e^{-j\omega} + 3e^{-2j\omega}$

$= 1 + 2e^{-j\omega} + 3e^{-2j\omega}$

Q4a) $\frac{d^2y(t)}{dt^2} + \frac{6\,dy(t)}{dt} + 8y(t) = \frac{dx(t)}{dt} + x(t)$

$\frac{dy(0)}{dt} = 3,\ y(0) = 1,\ x(t) = u(t)$

$\rightarrow \because x(t) = u(t),\ X(s) = \frac{1}{s}$

Taking L.T.

$\left[s^2Y(s) - sy(0) - \frac{dy(0)}{dt}\right] + 6[sY(s) - y(0)]$

$+ 8Y(s) = [sX(s) - x(0)] + X(s)$

$s^2Y(s) + 6sY(s) + 8Y(s) = sX(s) + X(s)$

$H(s) = \frac{Y(s)}{X(s)} = \frac{s+1}{s^2+6s+8}$

$[s^2Y(s) - s - 3] + 6[sY(s) - 1] + 8Y(s) = sX(s) + X(s)$

$Y(s) = \frac{s^2+10s+1}{s(s^2+6s+8)} = \frac{s^2+10s+1}{s(s+4)(s+2)}$

$Y(s) = \frac{1/8}{s} + \frac{15/4}{s+2} - \frac{23/8}{s+4}$

$y(t) = \frac{1}{8}u(t) + \frac{15}{4}e^{-2t}u(t) - \frac{23}{8}e^{-4t}u(t)$

Q4b) $y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n) + x(n-1)$

$\rightarrow Y(z) - \frac{3}{4}[z^{-1}Y(z) + Y(-1)] + \frac{1}{8}[z^{-2}Y(z)$

$+ z^{-1}Y(-1) + Y(-2)] = X[z] + z^{-1}X[z] + X[-1]$

$Y[z] - \frac{3}{4}z^{-1}Y[z] + \frac{1}{8}z^{-2}Y[z] = X[z] + z^{-1}X[z]$

$Y[z]\left[1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}\right] = X[z]\,[1 + z^{-1}]$

For impulse response, ~~$x[z]=1$~~

$$H[z] = \frac{Y[z]}{X[z]}$$

$$= \frac{1+z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}} = \frac{z^2+z}{z^2-\frac{3}{4}z+\frac{1}{8}}$$

$$= \frac{z(z+1)}{(z-1/2)(z-1/4)} = \frac{A}{z-1/2} + \frac{B}{z-1/4}$$

$$\cancel{H(z) = \frac{3}{z-1/2} - \frac{5}{4(z-1/4)}} \quad \frac{3}{z-1/2} - \frac{5}{z-1/4}$$

$$\cancel{= 3\left(\tfrac{1}{2}\right)^n u(n) - \tfrac{5}{4}\left(\tfrac{1}{4}\right)^n}$$

$$\frac{H(z)}{z} = \frac{3}{(z-1/2)} - \frac{5}{(z-1/4)}$$

$$H(z) = 3\left(\frac{z}{z-1/2}\right) - 5\left(\frac{z}{z-1/4}\right) = 3\left(\tfrac{1}{2}\right)^n u(n) - 5\left(\tfrac{1}{4}\right)^n u(n)$$

Q5 a) $x(t) = e^{-at}u(t)$ 5M

$$\rightarrow F[e^{-at}u(-t)] = \int_{-\infty}^{\infty} e^{-at}\, u(-t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{0} e^{at} e^{-j\omega t}\, dt = \int_{-\infty}^{0} e^{at} e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{0} e^{(-a-j\omega)t}\, dt = \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$= \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\right]_0^{\infty} = \frac{1}{a+j\omega}$$

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c(ii) $x(t) = \sin\omega_0(t)\,u(t)$. 5M.

→ $F[\sin\omega_0 t\, u(t)] = = F\left[\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}\, u(t)\right]$

$$= \frac{1}{2j}\left\{\left[\pi\delta(\omega-\omega_0) + \frac{1}{j(\omega-\omega_0)}\right] - \left[\pi\delta(\omega+\omega_0) + \frac{1}{j(\omega+\omega_0)}\right]\right\}$$

$$= \frac{\omega_0}{\omega_0^2 + (j\omega)^2} + \frac{\pi}{j2}\left[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)\right]$$

a5b) $x(n) = 4\cos\frac{\pi n}{2}$ 5M.

→ Test for periodicity

$x(n+N) = 4\cos\left(\frac{\pi n}{2} + \frac{\pi N}{2}\right)$.

$\frac{\pi N}{2} = 2\pi \times M$, $N = 4M$.

N is integer for $M = 1, 2, 3, \ldots$

Let $M = 1$, $N = 4$.

$x(n)$ → periodic with $N = 4$, $\omega_0 = \frac{2\pi}{N} = \frac{2\pi}{4} = \frac{\pi}{2}$

Fourier series

$$C_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n)\, e^{\frac{-j2\pi kn}{N}}$$

for $k = 0, 1, 2, 3, \ldots N-1$.

$N = 4$, $x(n) = 4\cos\frac{\pi n}{2}$

$$C_K = \frac{1}{4}\sum_{n=0}^{3} 4\cos\frac{\pi n}{2}\, e^{-j\frac{2\pi k n}{4}}$$

$= 1 - \cos\pi k + j\sin\pi k$

$k=0,\ C_k = C_0 = 1-\cos 0 - j\sin 0 = 1-1+j0 = 0.$

$k=1,\ C_k = C_1 = 1-\cos\pi + j\sin\pi = 1+1+j0 = 2$

$k=2,\ C_k = C_2 = 1-\cos 2\pi + j\sin 2\pi = 1-1+j0 = 0.$

$k=3,\ C_k = C_3 = 1-\cos 3\pi + j\sin 3\pi = 2$

Fourier series representation is given by

$$x(n) = \sum_{k=0}^{N-1} c_k e^{\frac{j2\pi k n}{N}}$$

$$= C_0 + C_1 e^{\frac{j\pi n}{2}} + C_2 e^{j\pi n} + C_3 e^{\frac{j3\pi n}{2}}$$

$= 2e^{j\omega_0 n} + 2e^{j3\omega_0 n}$ where $\omega_0 = \pi/2$

Q5() $x(n) = \{1, \underset{\uparrow}{1}, 2, 2\}$ & $y(n) = \{1, \underset{\uparrow}{3}, 1\}$

→ ~~Ans.~~ For $x(n)$, $n_1 = -1$, $N_1 = 4$

For $y(n)$, $n_2 = -1$, $N_2 = 3$

$r_{xy}(m)$ will have $N_1 + N_2 - 1 = 4+3-1 = 6$ samples.

Initial value of $m = n_1 - (n_2 + N_2 - 1)$

$= -1 - (-1+3-1) = -1-(1) = -2$

Final value of $m = m_i + (N_1 + N_2 - 2)$

$= -2 + (4+3-2) = 3$

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By ~~tab~~ Matrix Method or any equivalent method

$$\begin{array}{c|ccc} & 1 & 3 & 1 \\ \hline 1 & 1 & 3 & 1 \\ 1 & 1 & 3 & 1 \\ 2 & 2 & 6 & 2 \\ 2 & 2 & 6 & 2 \end{array}$$

$r_{xy}(-2) = 1$, $r_{xy}(-1) = 1+3 = 4$, $r_{xy}(0) = 6$

$r_{xy}(1) = 2+6+1 = 9$, $r_{xy}(2) = 6+2 = 8$,

$r_{xy}(3) = 2$, $r_{xy}(m) = \{1, 4, \underset{\uparrow}{6}, 9, 8, 2\}$

Q6a) → $x(t) = e^{-3t}u(t)$, $h(t) = u(t-1)$

Using convolution integral,

$$x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} e^{-3\tau} u(\tau) \cdot u(t-1-\tau)\, d\tau$$

Here, $u(\tau) = 0$ for $\tau < 0$

& $u(t-1-\tau) = 0$ for $\tau > \cancel{t+3}(t-1)$

$u(\tau)\, u(t-1-\tau) = 1$ only for $0 < \tau < t-1$

for all other value of τ, $u(\tau)\, u(\tau \cancel{+} 1 - \tau) = 0$.

$$x_1(t) * x_2(t) = \int_0^{t-1} e^{-3t}\, d\tau$$

$$= \left[\frac{e^{-3t}}{-3}\right]_0^{t-1} = \frac{e^{-3(t-1)} - 1}{-3} = \frac{1 - e^{-3(t-1)}}{3}$$

$y(c) = 0$ for $(t < -3)$

$y(c) = \dfrac{1 - e^{-3(\cancel{t+3})}}{3}$ (for $(t > -3)$.

$$y(t) = \frac{1-e^{-3(t-1)}}{3}$$

$y(t) = 0 \quad$ (for $t < 1$)

$$y(t) = \frac{1-e^{-3(t-1)}}{3} \quad (\text{for } t > 1).$$

Q 6b) (i) $x(n) = n^2 u(n)$ 5M

$\rightarrow \quad Z[u(n)] = \frac{z}{z-1}$ Using multiplication property

$$Z[n\,u(n)] = -z\frac{d}{dz}\left(\frac{z}{z-1}\right) = \frac{z}{(z-1)^2}.$$

$$Z[n^2 u(n)] = -z\frac{d}{dz}\{Z[n\,u(n)]\}$$

$$= -z\frac{d}{dz}\left[\frac{z}{(z-1)^2}\right] = \frac{z(z+1)}{(z-1)^3}$$

(ii) $x(n) = a^n \cos\omega_0 n\, u(n)$

$$\rightarrow \quad x(n) = (a)^n \left[\frac{e^{j\omega_0 n} + e^{-j\omega_0 n}}{2}\right] u(n).$$

$$= \frac{1}{2}\left[(ae^{j\omega_0})^n + (ae^{-j\omega_0})^n\right] u(n).$$

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} \frac{1}{2}(ae^{j\omega_0})^n z^{-n} + \sum_{n=0}^{\infty} \frac{1}{2}(ae^{-j\omega_0})^n z^{-n}$$

$$= \frac{1}{2}\,\frac{1}{1-ae^{j\omega_0}z^{-1}} + \frac{1}{2}\,\frac{1}{1-ae^{-j\omega_0}z^{-1}}$$