QP : 24232

HINTS & SOLUTIONS. (Numericals only)

Q.1. A) Given : $Q_1 = 300\ \mu C$ at $P_1(1,-1,-3)$ m

$Q_2 = Q$ at $P_2(3,-3,-2)$ m.

$\vec{F}_{21} = 8\hat{a}_x - 8\hat{a}_y + 4\hat{a}_z$ (N).

To find : $Q_2 = Q$.

Sol^n : $\vec{F}_{21} = \frac{Q_1 Q_2}{4\pi\epsilon_0 (R_{21})^2}\hat{a}_{21}$; $\vec{R}_{21} = -2\hat{a}_x + 2\hat{a}_y - \hat{a}_z$

$\therefore R_{21} = 3$ m.

$\therefore \hat{a}_{21} = \frac{\vec{R}_{21}}{R_{21}} = \frac{1}{3}(-2\hat{a}_x + 2\hat{a}_y - \hat{a}_z)$

$\therefore$ $\boxed{Q_2 = Q = -40\ \mu C.}$

B) E & F regions are present during day & night time. Maximum attenuation of wave takes place in D-region.

C) With neat labeled diagram & appropriate theory, the explanation of Super refraction & Sub refraction should be done.

D) "Modified Ampere's law" eq^n. tells the propagation of EM wave in air.

Q.2. A) Following boundary cond^{ns}. need to be derived,

For E field : $\vec{D}_t = \vec{E}_t = 0$ & $\vec{D}_n = \epsilon\vec{E}_n = \rho_s$

For H-field : $\vec{H}_n = \vec{B}_n = 0$ (Inside conductor) & $\vec{H}_t = \frac{\vec{B}_t}{\mu} = \bar{a}_{N12} \times \bar{K}$ ← Surface current.

B) Derive following eq^n. & state significance of each term in it.

$$\int_v (\vec{E}\cdot\vec{J})\,dv = -\frac{\partial}{\partial t}\int_v \left(\frac{\mu}{2}H^2 + \frac{\epsilon}{2}E^2\right)dv - \oint_s (\vec{E}\times\vec{H})\cdot\vec{ds}$$

Q.3. A) i) ρ_v :

$$\rho_v = (\nabla \cdot \vec{D})\Big|_{at\ (-1,0,3)} = 8y\Big|_{at\ (-1,0,3)} = 0\ (C/m^3)$$

ii) $$\Psi = \oint_S \vec{D}\cdot d\vec{s} = \int_v \rho_v\, dv = \int_0^2 dx \int_0^3 (8y)\,dy \int_0^5 dz = 360\ C.$$

iii) $Q_{enclosed} = \Psi = 360\ C.$

B) $V_0 = V(0) = 0V$, V_1, V_2, V_3, $V_4 = V(4) = 20V$.

By FDM ; $V_1 = \frac{V_0 + V_2}{2} \Rightarrow 2V_1 - V_2 = 0$ — ①

$V_2 = \frac{V_1 + V_3}{2} \Rightarrow -V_1 + 2V_2 - V_3 = 0$ — ②

$V_3 = \frac{V_2 + V_4}{2} \Rightarrow -V_2 + 2V_3 = 20$ — ③.

$$\therefore \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 20 \end{bmatrix}$$

$\therefore V_1 = 5V$; $V_2 = 10V$; $V_3 = 15V$.

$\therefore$ $\boxed{V(1) = V_1 = 5V.}$

C) Use Fleming's left hand rule to describe the theory.

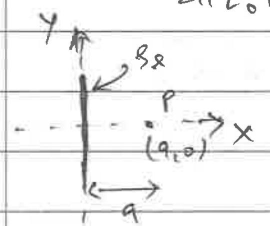
Q.4. A) Derivation of following expressions is required.

$$\Gamma_{R\perp} = \frac{|\vec{E}_r|}{|\vec{E}_i|} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\Gamma_{T\perp} = \frac{|\vec{E}_t|}{|\vec{E}_i|} = \frac{2\eta_2}{\eta_2 + \eta_1}$$

B) Derivation of following expression is required.

$$\vec{E} = \frac{\rho_l}{2\pi \epsilon_0 R} \hat{a}_R \ (V/m) \leftarrow \text{Cylindrical Co-ordinate system.}$$



$$\vec{E} = \frac{\rho_l}{2\pi \epsilon_0 a} \hat{a}_x \ (V/m) \leftarrow \text{Cartesian Co-ordinate System.}$$

Q.5 A) Obtain following expression for MUF in terms of d, H & f_c.

$$f_{MUF} = f_c \sqrt{1 + \left(\frac{d}{2H}\right)^2}$$

for $d = 2000$ Km ; $H = 200$ Km ; $f_c = 5$ MHz.

$$\boxed{f_{MUF} = 25.495 \text{ MHz.}}$$

B) Given : $h_t = 60$ m ; $h_r = 6$ m.
To find : d.
Soln : $d = 4.12 \left[\sqrt{h_t} + \sqrt{h_r}\right] = 42$ Km.

c) E-layer & Sporadic-E layer can be explained in differential form with following points.
i) Behaviour of layers
ii) Time of presence. (Day / night)
iii) Ionization density.
iv) Purpose in communication (wireless)

Q.6. A) Obtain following wave eqn. for E-field & M-field in lossy medium.

$$\nabla^2 \vec{E} = \mu(\sigma \dot{\vec{E}} + \epsilon \ddot{\vec{E}})$$

$$\nabla^2 \vec{H} = \mu(\sigma \dot{\vec{H}} + \epsilon \ddot{\vec{H}}).$$

B) i) γ :

$\omega = 2\pi f = 2\pi \times 10^9$ rad/sec

$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \sqrt{-438.65 + 0.7896j}$

$\therefore \boxed{\gamma = 0.019 + 20.944j} = \alpha + j\beta$.

ii) 'α' in dB :

$\alpha = 0.019$ (Np/m).

$\therefore \alpha_{dB} = 8.686\,\alpha = \underline{0.165 \text{ dB}}$.

iii) λ

$\lambda = \frac{2\pi}{\beta} = 0.299 \approx \underline{0.3 \text{ m}}$.

iv) (n)

$\boxed{n = \frac{c}{v} = 1}$ ($\because \epsilon_r = 1$ & $\mu_r = 1$)

v) Loss tangent :

$\tan\Delta = \frac{\sigma}{\omega\epsilon} = \underline{1.798 \times 10^{-3}}$.

$\because \tan\Delta << 1 \Rightarrow$ Medium is lossy dielectric.

c) Formation of duct propagation should be described by proper theory & neat diagram.

Cond^n. for duct propagation :

i) Transmitting antenna should be inside the duct.

ii) Radio wave should enter the duct at very low angle of incidence.