Solution for Discrete Structures (old)
Comp. sem III

Q.P. code $\boxed{00024629}$

Q.1 a) Prove by that n^2+n is even number

Let $p(n) = n^2+n$ is an even nu.

Basis of Induction. $\longrightarrow$ 2 M

for $n=1$

$(1)^2+1 = 2$ is an even nu.

$\therefore p(1)$ is true $\longrightarrow$ 1 M.

Induction step

Assume $p(k)$ is true — 1 M

i.e. k^2+k is true.

To prove that $p(k+1)$ is true.

for $n = k+1$

$$(k+1)^2+k+1 = k^2+2k+1+k+1$$
$$= k^2+k+2k+2$$
$$= k^2+k+2(k+1) \quad \text{— 2 M}$$

k^2+k is ~~divisi~~ even nu. by (1)

$2(k+1)$ is an even nu. as multiplied by 2

$\therefore p(k+1)$ is true $\therefore$ Hence proved.

b) Find G.F. — 2½ M each.

i) 2, 2, 2, 2, 2, 2

$$\rightarrow \sum_{n=0}^{5} (2)^n x^n = 2^0x^0 + 2^1x^1 + 2^2x^2 + 2^3x^3 + 2^4x^4 + 2^5x^5$$

ii) 1, 1, 1, 1, 1, 1

$$\longrightarrow \sum_{n=0}^{5} (1)^n x^n = 1+x+x^2+x^3+x^4+x^5 = \frac{1}{1-x}$$

c) $R^2 = \{(a,a), (a,b), (a,c), (b,e), (b,d), (b,e)\}$

aR^2b if there is a path of length 2 from a to b in R.

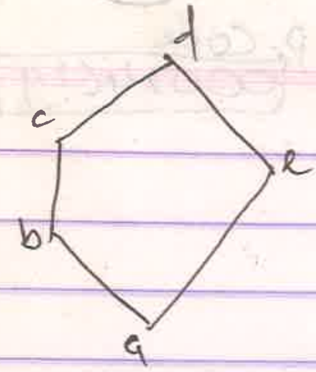
$aR^\infty b$ if ——— " ——— any length ——— " ———

$R^\infty = \{(a,a), (a,b), (a,c), (a,d), (a,e), (b,c), (b,d), (b,e), (c,d), (c,e), (d,e)\}$

d) Defination — 1 M

It is a lattice as every pair of elements has LUB & GLB

1)



LUB	a	b	c	d	e
a	a	b	c	d	e
b	b	b	c	e	e
c	c	c	c	e	e
d	d	e	e	d	e
e	e	e	e	e	e

GLB	a	b	c	d	e
a	a	a	a	a	a
b	a	b	b	a	b
c	a	b	c	a	c
d	a	a	a	d	d
e	a	b	c	d	e

1 1/2 for each table.

Q.2 a) Def^n of Isomorphism — 2M

No. of Nodes in Graph 1 = 5
— " — Graph 2 = 5
No. of Edges in Graph 1 = 6
— " — Graph 2 = 6
} 2M.

Degrees of vertices in G_1	
a	2
b	2
c	3
d	2
e	3

Degrees of vertices in G_2	
a	4
b	2
c	3
d	2
e	1

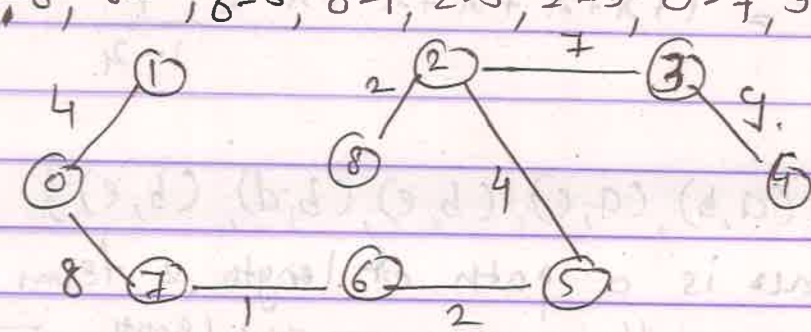
— 2M

As degrees of vertices do not match, the incidence relationship is not preserved

∴ Given Graphs are not isomorphic — 2M

b) MST Edges are added as follows

7-6, 8-2, 6-5, 0-1, 2-5, 2-3, 0-7, 3-4



c)

p	q	p∨q ①	¬q	p∧q ②	①∧②∨q
T	T	T	F	F	F ∨ T = T
T	F	T	T	T	T ∨ F = T
F	T	T	F	F	F ∨ T = T
F	F	F	T	F	F ∨ F = T

(A)

③

Q.3 a) Addition Table

$+_6$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

i) $\forall a, b \quad a +_6 b \in G \quad \therefore$ closed

ii) Associative

$a +_6 (b +_6 c) = (a +_6 b) +_6 c$

Let $a = 1, b = 2, c = 3$

$1 +_6 (2 +_6 3) = (1 +_6 2) +_6 3$

$1 +_6 5 = 3 +_6 3$

$0 = 0$

iii) If for every element a

$0 +_6 a = a +_6 0 = 0 \quad \therefore 0$ is an identity

iv) An element b is an inverse of a if $b * a = e$ where e is an identity element.

Let inverse of 0, 1, 2, 3, 4, 5 are 0, 5, 4, 3, 2, 1 resp.

v) Commutative if $a * b = b * a$ for all $a, b \in G$

From the table it is seen that corresponding row and column are identical

$\therefore$ operation is commutative.

vi) The number of elements in $G = 6$

$\therefore (G, +_6)$ is a Abelian group of order 6.

b) $M_R = W_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\quad W_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$W_2 = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\quad W_3 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$W_4 = W_3$

$R^\infty = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (2,1)\ (2,2)\ (2,3)\ (2,4)\ (3,4)\ (4,4)\}$

c) $f: \mathbb{Z} \to \mathbb{Z} \quad f(x) = x^2 + x + 1$

$f(2) = f(-3) = 7 \quad \therefore$ Not one-to-one

Negative elements of $\mathbb{Z}$ are not images of any element

$\therefore$ Not onto

Q.4 a) $d(000000, 011110) = 4$ $\quad d(011100, 101010) = 3$

$d(000000, 101010) = 3$ $\quad d(011100, 111000) = 3$

$d(000000, 111000) = 3$ $\quad d(101010, 111000) = 2$ — 6M

i) ∴ Minimum distance = 2 — 1M

ii) The code will detect k or fewer errors iff min distance is at least k+1

∴ No. of errors that can be detected = 1 or fewer — 1M.

b) First compute encoding function $e_H : B^2 \rightarrow B^5$

$e(00) = 00000$ $\quad e(10) = 10110$

$e(01) = 01011$ $\quad e(11) = 11101$ (02 M)

Decoding table.

00000	01011	10110	11101
00001	01010	10111	11100
00010	01001	10100	11111
00100	01111	10010	~~10101~~ 11001
01000	00011	11110	~~01101~~ 10101
10000	11011	00110	~~10001~~ 01101
01100	00111	11010	10001
11000	10011	01110	00101

(03 M)

i) 01111 → 01

ii) 01110 → 10

iii) 11001 → 11 (03 M)

c) Let n = no. of friends = pigeons

Let no. of months be pigeonholes = m = 12 —(1M)

By extended pigeonhole principle,

$$\left\lfloor \frac{(n-1)}{m} \right\rfloor + 1 = 5 \quad \text{5 people selected.}$$

$$\frac{n-1}{12} = 4$$

$$n = 49$$

∴ 49 friends are required.

Q.5 a) $a*b = a+b-ab$ 2 M each step.

i) $\forall a,b \in G \quad a*b \in G \quad \therefore$ closed

ii) Associative

show $(a*b)*c = a*(b*c)$

$(a+b-ab)*c = a*(b+c-bc)$

iii) Identity $a*0 = a+0-a\times 0$

$= a$

$\therefore$ '0' is an identity element.

iv) if $a*b$ = identity b is inverse of a

$a*1 = a+1-a\times 1 = a$

$\therefore$ '1' is inverse of both a & b

$\therefore (G,*)$ is a group.

Q.5 b) $a_r - a_{r-1} - 6a_{r-2} = -30$ with $a_0 = 20$, $a_1 = 5$

Homogenous recurrence relⁿ $a_r - a_{r-1} - 6a_{r-2} = 0$

Characteristic eqⁿ. $a^2 - a - 6 = 0$

$a = -2, 3$ — 2M

$a_r^{(h)} = A_1(-2)^r + A_2(3)^r$ — 1M

R.H.S. is a constant $\therefore a_r^{(p)}$ is also a constant p.

$a_r = p$ for all r

$a_{r-1} = a_{r-2} = p$ — 1M

$p - p - 6p = -30$

$p = 5 \quad \therefore a_r^{(p)} = 5$ — 2M

Total Solⁿ. $= a_r^{(h)} + a_r^{(p)}$

$a_r = A_1(-2)^r + A_2(3)^r + 5$

Using initial conditions

$A_1 + A_2 + 5 = 20 \qquad -2A_1 + 3A_2 = -10$

$A_1 = 11, \quad A_2 = 4$ — 2M

$\therefore \boxed{a_r = 11(-2)^r + 4(3)^r + 5}$

c) $\{A_1, A_2\}$ is not a partition as $A_1 \cap A_2 \neq \phi$

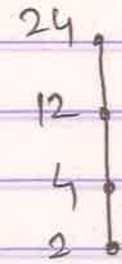
$\{A_3, A_4, A_5\}$ is a partition as

$A_3 \cup A_4 \cup A_5 = A$ and $A_3 \cap A_4 = A_4 \cap A_5 = A_3 \cap A_5 = \phi$

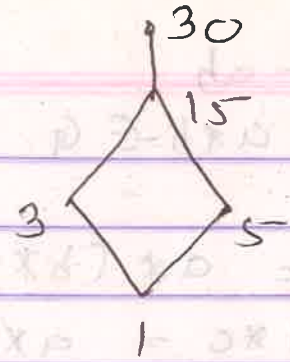
— 2M each.

4 M each
2 M diagram
2 M Justification

Q.6 a) Hasse Diagrams



Chain



Not a chain
as 3 ∤ 5

b) $g \circ f = g(f(x)) = g(2x+3) = 6x+13$
$f \circ g = f(g(x)) = f(3x+4) = 6x+11$
$f \circ h = f(h(x)) = f(4x) = 8x+13$
$g \circ f \circ h = g(8x+3) = 24x+13$

c) Def^n of Euler path and cycle – 2 M
Euler path BBADCDEBC or CDCBBADEB
Euler ckt CDCBBADEBC or CDEBBADC.