Q1 ⓐ $\because L[\operatorname{erf}\sqrt{t}] = \dfrac{1}{s\sqrt{s+1}}$

$\therefore L[\operatorname{erf} 2\sqrt{t}] = L[\operatorname{erf}\sqrt{4t}] = \dfrac{1}{4}\dfrac{1}{\frac{s}{4}\sqrt{\frac{s}{4}+1}} = \dfrac{2}{s\sqrt{s+4}}$

$\therefore L[t \operatorname{erf} 2\sqrt{t}] = -\dfrac{d}{ds}\left[2(s^3+4s^2)^{-\frac{1}{2}}\right]$

$\therefore L[t \operatorname{erf} 2\sqrt{t}] = \dfrac{3s+8}{s^2(s+4)^{3/2}}$

ⓑ characteristic eqn of A is $|A-\lambda I|=0$

ie $\lambda^2 - 2\lambda\cos\theta + 1 = 0$

show that $A^2 - 2A\cos\theta + I = 0$

ie A satisfies its own characteristic eqn.

$\therefore$ Cayley - Hamilton theorem is verified.

& premultiply by A^{-1} on both sides.

$\therefore A - 2\cos\theta I + A^{-1} = 0$

ie $A^{-1} = 2\cos\theta I - A$

$= \begin{bmatrix} 2\cos\theta & 0 \\ 0 & 2\cos\theta \end{bmatrix} - \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$\therefore A^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

ⓒ $f(z) = r^3\cos k\theta + i r^k \sin k\theta$

$\therefore u = r^3\cos k\theta$, $v = r^k \sin k\theta$

$\because f(z)$ is Analytic, it satisfies C-R in polar eqn.

ie $\dfrac{\partial u}{\partial r} = \dfrac{1}{r}\dfrac{\partial v}{\partial \theta}$, $\dfrac{\partial u}{\partial \theta} = -r\dfrac{\partial v}{\partial r}$

$\dfrac{\partial u}{\partial r} = 3r^2\cos k\theta$, $\dfrac{\partial u}{\partial \theta} = -kr^3\sin k\theta$, $\dfrac{\partial v}{\partial r} = kr^{k-1}\sin k\theta$, $\dfrac{\partial v}{\partial \theta} = kr^k\cos k\theta$.

$\because u_r = \dfrac{1}{r} v_\theta$

$3r^2\cos k\theta = \dfrac{1}{r} kr^k \cos k\theta$

ie $3r^2\cos k\theta = kr^{k-1}\cos k\theta$.

$\Rightarrow \boxed{k=3}$

(d) Probability of getting (3 or 5) in a single toss $= \frac{1}{6}+\frac{1}{6}=\frac{1}{3}$

This is a binomial distribution with $n=7$, $p=\frac{1}{3}$, $q=\frac{2}{3}$.

$\therefore P(X=x) = {}^{n}C_{x}\, p^{x}\, q^{n-x} = {}^{7}C_{x} \left(\frac{1}{3}\right)^{x} \left(\frac{2}{3}\right)^{7-x}$

$\therefore P(\text{at least 4 successes}) = P(X=4,5,6,7)$

$$= \sum_{x=4}^{7} {}^{7}C_{x} \left(\frac{1}{3}\right)^{x} \left(\frac{2}{3}\right)^{7-x}$$

$$= {}^{7}C_{4}\left(\frac{1}{3}\right)^{4}\left(\frac{2}{3}\right)^{3} + {}^{7}C_{5}\left(\frac{1}{3}\right)^{5}\left(\frac{2}{3}\right)^{2} + {}^{7}C_{6}\left(\frac{1}{3}\right)^{6}\left(\frac{2}{3}\right)^{1} + {}^{7}C_{7}\left(\frac{1}{3}\right)^{7}\left(\frac{2}{3}\right)^{0}$$

$$= \frac{379}{3^7}$$

$\therefore$ The expected number of times of getting (3 or 5) at least 4 times

$$= Np = 729 \times \frac{379}{3^7} = 126.3.$$

Q-2 (a) Differentiating the given relation w.r.to x & y

$$3\frac{\partial u}{\partial x} + 2\frac{\partial v}{\partial x} = -2x + 16y \quad — (1)$$

& $$3\frac{\partial u}{\partial y} + 2\frac{\partial v}{\partial y} = 2y + 16x \quad — (2)$$

But $u_x = v_y$ & $u_y = -v_x$. Hence, from (2), we get,

$$-3\frac{\partial v}{\partial x} + 2\frac{\partial u}{\partial x} = 2y + 16x \quad — (3)$$

$3 \times (1) + 2 \times (3) \Rightarrow$

$$13\frac{\partial u}{\partial x} = 26x + 52y$$

$$\therefore \frac{\partial u}{\partial x} = 2x + 4y = \phi_1(x,y)$$

& $(-2) \times (1) + 3 \times (3)$

$\Rightarrow$ $$-13\frac{\partial v}{\partial x} = 52x - 26y$$

$$\therefore \frac{\partial v}{\partial x} = -4x + 2y = \phi_2(x,y)$$

But $f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = \phi_1(x,y) + i\,\phi_2(x,y)$

$$= \phi_1(z,0) + i\,\phi_2(z,0)$$

$$= 2z - i\,4z$$

$$\therefore f(z) = z^2 - 2iz^2 + c = (1-2i)z^2 + c.$$

Q2 (b). $L^{-1}\left[\frac{16}{(s-2)(s+2)}\right] = L^{-1}\left[F(s)\,G(s)\right]$

$F(s) = \frac{16}{s-2}$, $f(t) = 16\,e^{2t}$

$G(s) = \frac{1}{s+2}$, $g(t) = e^{-2t}$

$$\therefore L^{-1}\left[\frac{16}{(s-2)(s+2)}\right] = f(t) * g(t)$$

$$= \int_0^t f(u)\,g(t-u)\,du$$

$$= \int_0^t 16 e^{2u}\, e^{-2(t-u)}\,du$$

$$= 16\,e^{-2t}\int_0^t e^{4u}\,du$$

$$= 16\,e^{-2t}\left[\frac{e^{4u}}{4}\right]_0^t$$

$$= 4e^{-2t}\left[e^{4t} - 1\right]$$

$$= 4\left[e^{2t} - e^{-2t}\right]$$

$$\therefore L^{-1}\left[\frac{16}{(s-2)(s+2)}\right] = 8\sinh(2t)$$

Q2 (c). The characteristic equation of A is

$|A - \lambda I| = 0$

$\therefore \lambda^2 - 7\lambda + 10 = 0$

$\therefore \lambda = 2, 5$

For $\lambda = 2$, $(A - \lambda I)X = 0$ gives

$$\begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\therefore 2x_1 + x_2 = 0$

put $x_2 = -2t$, $x_1 = t$

$$\therefore X_1 = \begin{bmatrix} t \\ -2t \end{bmatrix} = t\begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$\therefore X_1 = [1, -2]'$

For $\lambda = 5$, $(A - \lambda I)X = 0$ gives

$$\begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\therefore -x_1+x_2=0$

Put $x_2=t$, $x_1=t$

$$\therefore X_2=\begin{bmatrix} t \\ t \end{bmatrix}=t\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\therefore X_2=[1,1]'$$

Thus, the matrix $A=\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$ will be diagonalised to the diagonal matrix $D=\begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$ by the transforming matrix $M=\begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}$ & $M^{-1}AM=D$.

Q3 (a) $\displaystyle\int_0^\infty e^{-2t}\left(\int_0^t \frac{e^{-u}\sin u}{u}\,du\right)dt = L\left[\int_0^t \frac{e^{-u}\sin u}{u}\,du\right], \quad s=2.$

Now

$L[\sin u]=\dfrac{1}{s^2+1}$, $L[e^{-u}\sin u]=\dfrac{1}{(s+1)^2+1}$

$$\therefore L\left[\frac{1}{u}e^{-u}\sin u\right]=\int_s^\infty \frac{1}{(s+1)^2+1}\,ds$$

$$=\left[\tan^{-1}(s+1)\right]_s^\infty = \frac{\pi}{2}-\tan^{-1}(s+1)=\cot^{-1}(s+1)$$

$$\therefore L\left[\int_0^t \frac{e^{-u}\sin u}{u}\,du\right]=\frac{\cot^{-1}(s+1)}{s}$$

$$\therefore \int_0^\infty e^{-2t}\left(\int_0^t \frac{e^{-u}\sin u}{u}\,du\right)=\frac{\cot^{-1}(s+1)}{s}, \quad s=2$$

$$=\frac{1}{2}\cot^{-1}(3).$$

(b). we have $w=z+3+2i$

$\therefore u+iv=(x+iy)+(3+2i)$

$=(x+3)+i(y+2)$

$\therefore u=x+3$, $v=y+2$

But $|z|=2$ $\therefore \sqrt{x^2+y^2}=2$

$\therefore x^2+y^2=4$

$\therefore (u-3)^2+(v-2)^2=4$

Thus, the circle $x^2+y^2=4$ is transformed into the circle $(u-3)^2+(v-2)^2=4$. with same radius but with its centre shifted from origin to $(3,2)$.

Q3 (c). we write the given problem as

$f(x_1,x_2) = 10x_1 + 10x_2 - x_1^2 - x_2^2$

$h_1(x_1,x_2) = x_1 + x_2 - 8$

$h_2(x_1,x_2) = -x_1 + x_2 - 5$

The Kuhn - Tucker condition for maxima are

$$\frac{\partial f}{\partial x_1} - \lambda_1 \frac{\partial h_1}{\partial x_1} - \lambda_2 \frac{\partial h_2}{\partial x_1} = 0$$

$\therefore\ 10 - 2x_1 - \lambda_1 + \lambda_2 = 0 \quad — (1)$

$$\frac{\partial f}{\partial x_2} - \lambda_1 \frac{\partial h_1}{\partial x_2} - \lambda_2 \frac{\partial h_2}{\partial x_2} = 0$$

$\therefore\ 10 - 2x_2 - \lambda_1 - \lambda_2 = 0 \quad — (2)$

$\therefore\ \lambda_1 (x_1 + x_2 - 8) = 0 \quad — (3)$

$\lambda_2(-x_1 + x_2 - 5) = 0 \quad — (4)$

$x_1 + x_2 - 8 \leq 0 \quad — (5)$

$-x_1 + x_2 - 5 \leq 0 \quad — (6)$

$x_1, x_2 \geq 0 \quad — (7)$

$\lambda_1, \lambda_2 \geq 0 \quad — (8)$

Now depending upon λ_1 & λ_2 the following cases arise.

(Case - i) $\lambda_1 = 0,\ \lambda_2 = 0$

$\therefore$ from (1) & (2) $x_1 = 5, x_2 = 5$

But these values does not satisfy (5), Hence reject this pair.

Case - 2 $\lambda_1 = 0,\ \lambda_2 \neq 0$

To find x_1 & x_2 (1) & (2)

$10 - 2x_1 + \lambda_2 = 0 \quad , 10 - 2x_2 - \lambda_2 = 0$

Adding $\quad x_1 + x_2 = 10$

& from (4) $-x_1 + x_2 = 5$

$\therefore\ x_2 = 7.5 \ ,\ x_1 = 2.5$

But these values do not satisfy (5), Hence reject this pair.

Case - 3 : $\lambda_1 \neq 0,\ \lambda_2 = 0$ from (1) & (2)

$\therefore\ 10 - 2x_1 - \lambda_1 = 0 \quad , \quad 10 - 2x_2 - \lambda_1 = 0$

Subtracting, $x_1 = x_2$

from (3), $x_1 + x_2 = 8$

$\therefore\ x_1 = 4,\ x_2 = 4$

Now from (1) $10 - 2x_1 - \lambda_1 = 0 \Rightarrow \lambda_1 = 2$

Also $x_1 = 4, x_2 = 4$ satisfy (5) & (6)

Hence Z is maxima at $x_1 = 4, x_2 = 4$

$\therefore\ Z_{max} = 48$.

Case-4 $\lambda_1 \neq 0, \lambda_2 \neq 0$

from 3 & 4

$x_1 + x_2 = 8$, $-x_1 + x_2 = 5$

Adding the two, we get $x_1 = 1.5$, $x_2 = 6.5$

from 1 & 2 $\lambda_1 = 2$, $\lambda_2 = -5$

we reject this pair

$\therefore \boxed{Z_{max} = 48}$

Q4 (a). The circle $|z| = 3$ has centre at $(0,0)$ & radius 3. The points $z=1$ & $z=2$ lie inside the circle. Hence, $f(z)$ is not analytic in C.

$\therefore$ By partial fraction, we write $\dfrac{1}{(z-1)(z-2)} = \dfrac{1}{(z-2)} - \dfrac{1}{(z-1)}$

& $f(z) = e^{2z}$ which is analytic in C.

$$\therefore \int_C \frac{e^{2z}}{(z-1)(z-2)} dz = \int_C \frac{e^{2z}}{z-2} dz - \int_C \frac{e^{2z}}{z-1} dz$$

$$= 2\pi i (f(2)) - 2\pi i (f(1)) \qquad (\text{where } f(z) = e^{2z})$$

$$= 2\pi i e^{4} - 2\pi i e^{2}$$

$$= 2\pi i (e^{2} - 1) e^{2}$$

$$\therefore \int_C \frac{e^{2z}}{(z-1)(z-2)} dz = 2\pi i e^{2} (e^{2} - 1).$$

Q.4 (b) calculation of r.

X	X^2	Y	Y^2	XY
3	9	2	4	6
6	36	4	16	24
4	16	5	25	20
5	25	3	9	15
7	49	6	36	42
$\Sigma X = 25$	$\Sigma X^2 = 135$	$\Sigma Y = 20$	$\Sigma Y^2 = 90$	$\Sigma xy = 107$

$\boxed{\therefore r = 0.7}$

$\therefore \bar{x} = 5$, $\bar{y} = 4$

$$r = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{n}}{\sqrt{\left(\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right)\left(\Sigma y^2 - \frac{(\Sigma y)^2}{n}\right)}} = \frac{107 - \frac{(25)(20)}{5}}{\sqrt{135 - \frac{(25)^2}{5}}\sqrt{90 - \frac{(20)^2}{5}}} = \frac{107 - 100}{\sqrt{135-125}\sqrt{90-80}} = \frac{7}{\sqrt{10}\sqrt{10}}$$

$$= \frac{7}{10}$$

$= 0.7$.

Q5 (a) put $z = e^{i\theta}$, $dz = ie^{i\theta}\, d\theta$ $\therefore dz = iz\, d\theta$, $d\theta = \frac{dz}{iz}$

Now $\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2} = \frac{z+\frac{1}{z}}{2}$

$$\therefore \int_0^{2\pi} \frac{d\theta}{25-16\cos^2\theta} = \int_C \frac{1}{25-16\left(\frac{z+\frac{1}{z}}{2}\right)^2} \frac{dz}{iz}$$

$$= -\frac{1}{i}\int_C \frac{z}{4z^4-17z^2+4}\, dz$$ where C is the circle $|z|=1$,

Now, the poles of $4z^4-17z^2+4=0$ are $z = \frac{1}{2}, -\frac{1}{2}, 2, -2$

The poles $z=\frac{1}{2}, -\frac{1}{2}$ lie inside the unit circle & $z=2,-2$ lie outside it.

Residue of $f(z)$ (at $z=\frac{1}{2}$) $= \lim_{z\to\frac{1}{2}} \left(z-\frac{1}{2}\right) \frac{-z}{i(2z+1)(2z-1)(z^2-4)} = \frac{1}{30i}$

Residue of $f(z)$ (at $z=-\frac{1}{2}$) $= \lim_{z\to-\frac{1}{2}} \left(z+\frac{1}{2}\right) \frac{-z}{i(2z+1)(2z-1)(z^2-4)} = \frac{1}{30i}$

$\therefore \oint_C f(z)\, dz = 2\pi i\left[\text{Sum of the residues (at } z=\frac{1}{2} \text{ \& } z=-\frac{1}{2})\right] = 2\pi i\left(\frac{2}{30i}\right) = \frac{2\pi}{15}$.

$$\therefore \int_0^{2\pi} \frac{d\theta}{25-16\cos^2\theta} = \frac{2\pi}{15}.$$

(b) Let mean $\bar{x}$ of the sample is a S.N.V. we have $z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$

$\because \mu = 0.1, \sigma = 2.1, n = 900$ $\therefore z < \frac{-0.1}{2.1/\sqrt{900}} = -1.43$

$\therefore P(Z<-1.43) = P(z>1.43) = 0.5 - (\text{area from } z=0 \text{ to } z=1.43)$

$= 0.5 - 0.4236$

$= 0.0764$.

(c) The characteristic equation is

$|A-\lambda I| = 0$ $\therefore$ ~~...~~ $\lambda^3 - 9\lambda^2 + 24\lambda - 20 = 0$

$\therefore (\lambda-2)(\lambda^2-7\lambda+10) = 0$

$\therefore \lambda = 2, 2, 5$

For $\lambda = 2$, $(A-\lambda I)X = 0$ gives.

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The rank of the matrix is 1 and the number of variables is 3. Hence, there are $3-1=2$ linearly independent solutions.

Putting $x_2 = s$, $x_3 = -t$, $x_1 = s+t$

$$\therefore X = \begin{bmatrix} s+t \\ s \\ -t \end{bmatrix} = s\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$\therefore X_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad X_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$ are linearly independent.

Hence, the vectors X_1 and X_2 are basis for the eigen space corresponding to $\lambda = 2$.

for $\lambda = 5$, $(A-\lambda I)X = 0$ gives

$$\begin{bmatrix} -2 & -1 & 1 \\ -1 & -2 & -1 \\ 1 & -1 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-2x_1 - x_2 + x_3 = 0$, $x_1 + x_2 = 0$

Put $x_2 = -t$, $x_1 = t$, $x_3 = t$

$$\therefore X = \begin{bmatrix} t \\ -t \\ t \end{bmatrix} = t\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

Hence $X_3 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ is a basis for the eigenspace corresponding to $\lambda = 5$.

Q-6 (a). we have $f(x_1, x_2) = 6x_1 + 8x_2 - x_1^2 - x_2^2$

$h_1(x_1, x_2) = 4x_1 + 3x_2 - 16$; $h_2(x_1, x_2) = 3x_1 + 5x_2 - 15$

Now, the Lagrangian function is,

$$L(x_1, x_2, \lambda_1, \lambda_2) = f(x_1, x_2) - \lambda_1 h_1(x_1, x_2) - \lambda_2 h_2(x_1, x_2)$$
$$= 6x_1 + 8x_2 - x_1^2 - x_2^2 - \lambda_1(4x_1 + 3x_2 - 16) - \lambda_2(3x_1 + 5x_2 - 15)$$

We now obtain the following partial derivatives.

$$\frac{\partial L}{\partial x_1} = 6 - 2x_1 - 4\lambda_1 - 3\lambda_2, \quad \frac{\partial L}{\partial x_2} = 8 - 2x_2 - 3\lambda_1 - 5\lambda_2$$

$$\frac{\partial L}{\partial \lambda_1} = -(4x_1 + 3x_2 - 16), \quad \frac{\partial L}{\partial \lambda_2} = -(3x_1 + 5x_2 - 15)$$

we, then, solve the following equation.

$$\frac{\partial L}{\partial x_1} = 0, \quad \frac{\partial L}{\partial x_2} = 0, \quad \frac{\partial L}{\partial \lambda_1} = 0, \quad \frac{\partial L}{\partial \lambda_2} = 0$$

$\therefore 6 - 2x_1 - 4\lambda_1 - 3\lambda_2 = 0$ — (1)

$8 - 2x_2 - 3\lambda_1 - 5\lambda_2 = 0$ — (2)

$4x_1 + 3x_2 = 16$ — (3)

$3x_1 + 5x_2 = 15$ — (4)

multiply ① by 4, ② by 3 & add.

$\therefore 48 - 2(4x_1+3x_2) - 25\lambda_1 - 27\lambda_2 = 0$

$\therefore 25\lambda_1 + 27\lambda_2 = 16$ — ⑤

Multiply ① by 3, ② by 5 & add

$58 - 2(3x_1+5x_2) - 27\lambda_1 - 34\lambda_2 = 0$

$\therefore 27\lambda_1 + 34\lambda_2 = 28$ — ⑥

from ⑤ & ⑥

$\boxed{\lambda_1 = \frac{-212}{121}}$, $\boxed{\lambda_2 = \frac{268}{121}}$

from ① $x_1 = \frac{35}{11}$, from ② $x_2 = \frac{12}{11}$

$\because n=2, m=2, n-m=0$, the above method fails.

The criterion of bordered Hessian matrix can not be applied when $m=n$ or $m>n$ ie. when the number of constraints is equal to or greater than the number of variables.

Now $Z = 6x_1 + 8x_2 - x_1^2 - x_2^2$

$\therefore \frac{\partial z}{\partial x_1} = 6 - 2x_1$, $\frac{\partial^2 z}{\partial x_1 \partial x_2} = 0$, $\frac{\partial^2 z}{\partial x_2 \partial x_1} = 0$

$\frac{\partial^2 z}{\partial x_1^2} = -2$, $\frac{\partial z}{\partial x_2} = 8 - 2x_2$, $\frac{\partial^2 z}{\partial x_2^2} = -2$

$$\therefore \begin{vmatrix} \frac{\partial^2 z}{\partial x_1^2} & \frac{\partial^2 z}{\partial x_1 \partial x_2} \\ \frac{\partial^2 z}{\partial x_2 \partial x_1} & \frac{\partial^2 z}{\partial x_2^2} \end{vmatrix} = \begin{vmatrix} -2 & 0 \\ 0 & -2 \end{vmatrix} = 4$$

$\therefore A_1 = |a_{11}| = -2$ and $A_2 = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = 4$

$\because A_1$ is negative & A_2 is positive (x_0, y_0) is maximum, therefore, $X_0\left(\frac{35}{11}, \frac{12}{11}\right)$ is a maxima & $Z_{max} = 6\left(\frac{35}{11}\right) + 8\left(\frac{12}{11}\right) - \left(\frac{35}{11}\right)^2 - \left(\frac{12}{11}\right)^2 = 16.504$.

(b) The characteristic equation is

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ 1 & 2-\lambda & 1 \\ 2 & 2 & 3-\lambda \end{vmatrix} = 0$$

$\therefore (1-\lambda)(\lambda^2 - 5\lambda + 6) = 0$

$\therefore \lambda = 1, 2, 3$

Since, eigen values of A are all distinct & A satisfies its own characteristic eq^n by Cayley Hamilton theorem. A is derogatory.

Q.6 (c) (i). $L^{-1}\left[\frac{s+2}{s^2-4s+13}\right] = L^{-1}\left[\frac{s+2}{(s-2)^2+3^2}\right] = L^{-1}\left[\frac{(s-2)+4}{(s-2)^2+3^2}\right]$

$$= e^{2t} L^{-1}\left[\frac{s+4}{s^2+3^2}\right] = e^{2t} L^{-1}\left[\frac{s}{s^2+3^2}\right] + 4e^{2t} L^{-1}\left[\frac{1}{s^2+3^2}\right]$$

$$= e^{2t}\cos 3t + \frac{4}{3}e^{2t}\sin 3t.$$

(ii) $L^{-1}\left[2\tanh^{-1}s\right] = L^{-1}\left[2\cdot\frac{1}{2}\log\left(\frac{1+s}{1-s}\right)\right]$

$$= L^{-1}\left[\log\left(\frac{1+s}{1-s}\right)\right]$$

$$= -\frac{1}{t}L^{-1}\left[\frac{d}{ds}\log\left(\frac{1+s}{1-s}\right)\right]$$

$$= -\frac{1}{t}L^{-1}\left[\frac{1}{1+s}+\frac{1}{1-s}\right]$$

$$= -\frac{1}{t}\left[e^{-t}-e^{t}\right]$$

$$= \frac{2}{t}\left(\frac{e^{t}-e^{-t}}{2}\right)$$

$$= \frac{2}{t}\sinh t$$