QP Code : 75685

Scheme A (External)]	(3 Hours)	**[Total Marks:100**
Scheme B (Internal)]	(2 Hours)	**[Total Marks: 40**

Instructions:
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- **Mention** on the top **of the answer book the scheme** under which you are appearing
- **Scheme A** students should **attempt any five** questions
- **Scheme B** students should **attempt any three** questions
- All **questions** carry **equal marks**

1. (a) State and prove Nested Intervals theorem.

   (b) Define the supremum and infimum of a non-empty subset S of $\mathbb{R}$ and state Supremum property (axiom) of $\mathbb{R}$. SHow that a real number M is the supremum of S iff $M \geq x, \forall x \in S$ and for any $\epsilon > 0, \exists y \in S$ such that $M - \epsilon < y \leq M$, where $\phi \subseteq S \subseteq \mathbb{R}$.

2. (a) If S is a nonempty, open subset of $\mathbb{R}^n$ and $f : S \to \mathbb{R}$, define continuity of f at $a \in S$. Show that if $f, g : S \to \mathbb{R}$ are both continuous at a and α, β are real numbers then $(\alpha f + \beta g)$ is continuous at a.

   (b) Examine the continuity and differentiablity of f at $(0, 0)$ given that $f(x, y) = \frac{x^3 y}{x^6+y^2}$ for $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$

3. (a) Define a real valued Cauchy sequence in $\mathbb{R}^n$ and show that a convergent (real valued) sequence in $\mathbb{R}^n$ is Cauchy.

   (b) Examine the pointwise and uniform convergence of the sequence $\{f_n(x)\}$ defined by $f_n(x) = x^n$ on $[0, 1]$. Justify your answers.

4. (a) State and prove Weirstrass test for uniform convergence of a series $\sum f_n(x)$ defined on a non-empty subset S of $\mathbb{R}$.

   (b) State Root test for convergence of a positive term series $\sum a_n$. Hence or otherwise discuss the convergence of $\sum \frac{3^n}{n^n . x^n}$, where $x \in \mathbb{R}^+$.

5. (a) Let S be a non-empty open subset of $\mathbb{R}^n$ and $a \in S$. Suppose $f : S \to \mathbb{R}$. Define the total derivative of f at a. Find the total derivative of $f(x, y, z) = xy + yz + zx$ at $(1, -2, 3)$. State the result used.

   (b) If $f : \mathbb{R}^2 \to \mathbb{R}^3$ is given by $f(x; y) = (x - y, 2y^2, x + y)$ and $g : \mathbb{R}^3 \to \mathbb{R}^2$ is given by $g(u, v, w) = (u - rv, w^2 u)$ then find the jacobians of f and g respectively at $(2, -3)$ and at $f(2, -3)$. Also find the jacobian of $g \circ f$ at $(2, 3)$.

6. (a) If $x = se^{\sin t}, y = te^{\cos t}$ and $s = r\cos\theta, t = r\sin\theta$, use chain rule to find $\frac{\partial x}{\partial r}, \frac{\partial x}{\partial \theta}, \frac{\partial y}{\partial r}, \frac{\partial y}{\partial \theta}$ in terms of functions of r, θ.

   (b) State Taylor's theorem and use it to expand the function $f(x, y) = e^x \cos y$ near $(0, \pi/4)$ upto and including degree two terms.

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7. (a) State and prove Fubini's theorem for the double integral of a bounded real-valued function $f(x, y)$ over a rectangle D in xy-plane.

   (b) Evaluate the double integral of $f(x, y) = x^2 + y^2$ over the disc $x^2 + y^2 \leq 4$ in the xy-plane

8. (a) Define the convergence of an improper integral $\int_a^\infty f(x)dx$ and show that an improper integral $\int_a^\infty \frac{dx}{x^p}, p > 0$, converges iff $p > 1$. Hence show that $\int_1^\infty \frac{dx}{5x^5}$ converges.

   (b) Discuss the convergence of (i) $\int_0^1 \frac{dx}{x^3\sqrt{1-x^2}}$, (ii) $\int_2^3 \frac{dx}{(x-2)^2 x^3}$

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