QP Code : 75677

Revised **(3 Hours)** **(Total marks: 80)**

Instructions:-

i. **Attempt any two questions from each section.**
ii. **All questions carry equal marks.**
iii. **Answer to section I and section II should be written in the same answer book.**

SECTION I (Attempt any Two Questions)

1. (a) If U(F) and V(F) are two vector spaces and T be a linear transformation, then, dim U= dim Ker T + dim Image T.

   (b) Consider the basis $S = \{v_1, v_2, v_3\} for\ R^3, where\ v_1 = (1,1,1), v_2 = (1,1,0) and \quad v_3 = (1,0,0)$. Let $T: R^3 \rightarrow R^2$ be the linear transformation such that $T(v_1) = (1,0), T(v_2)\ (2,1), T(v_3) = (4,3)$. Find a formula for $T(x_1, x_2, x_3)$; then use this formula to compute T(2,-3, 5).

2. (a) Let $C_1, C_2, \ldots\ldots, C_n$ be column vectors of dimension n. They are linearly dependent if and only if det $(C_1, C_2, \text{-----}, C_n) = 0$.

   (b) Find the rank of the following matrices:

   i) $\begin{bmatrix} 3 & 1 & 2 & 5 \\ 1 & 2 & -1 & 2 \\ 1 & 1 & 0 & 1 \end{bmatrix}$

   ii) $\begin{bmatrix} 3 & 1 & 1 & -1 \\ -2 & 4 & 3 & 2 \\ -1 & 9 & 7 & 3 \\ 7 & 4 & 2 & 1 \end{bmatrix}$

3. (a) Find Eigen values and the Eigen vectors of the following matrix

$$A = \begin{bmatrix} 4 & 4 & 4 \\ -2 & -3 & -6 \\ 1 & 3 & 6 \end{bmatrix}$$

   (b) Find Minimal Polynomial of the following matrix

$$\begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

   Also find Eigen values of A.

4. (a) Prove that an orthogonal set of non-zero vectors is linearly independent.

   (b) Find real orthogonal matrix P such that $P^T A P$ is diagonal for the following matrix A.

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

SECTION II (Attempt any Two Questions)

5. (a) Any finite cyclic group of order n is isomorphic to Z_n, the group of integer residue classes modulo under addition.

   (b) State and prove First Isomorphism Theorem i.e., let f: G to $\overline{G}$ be a homomorphism of groups. If f is onto, $\frac{G}{kerf} \approx \overline{G}$ or $(\frac{G}{Kerf} \approx Imf)$

6. (a) Let H be a subgroup of a group G. Then the following statements are equivalent:
   i) $H \Delta G$ $(i.e. aHa^{-1} \subseteq H \; \forall \; a \in G)$
   ii) $aHa^{-1} = H$ *for each* $a \in G$
   iii) $aH = Ha,$ *for each* $a \in G$
   iv) $H_a H_b = H_{ab}$ *for each* $a, b \in G$

   (b) A group G of order p^n where p is prime and $n \geq 1$ has non-trivial centre.

7. (a) M is maximal ideal if and only if R/M is field. (Prove it)

   (b) Let R = $\left\{\begin{bmatrix} a & b \\ b & a \end{bmatrix} : a, b \in Z\right\}$. *Let* $\emptyset: R \to Z$ *be defined by*

$$\emptyset\left\{\begin{bmatrix} a & b \\ b & a \end{bmatrix}\right\} = a - b \text{ then } \emptyset \text{ is ring homomorphism.}$$

8. (a) In unique factorization domain, irreducible polynomials are prime.

   (b) Let F be a field. If $f(x) \in F[x]$ *and* $deg\, f(x) = 2$ *or* 3 then $f(x)$ is reducible over F if and only if $f(x)$ has zero in F.

— End —

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