QP Code : 75674

Scheme A(External) **(3 Hours)** **Total marks: 100**

N.B: 1) Scheme A students answer any five questions.
2) All questions carry equal marks.

Q1. (a) Show that the intersection of two subgroups of a group G is a subgroup of G. Give an example to show that the union of two subgroups of a group G need not to be a subgroup of G.

(b) P rove that a finite semi-group G is a group if and only if G satisfies both the canceliation laws.

Q2. (a) Let H be a subgroup of a group G and a,b ϵ G. Show that either

$Ha \cap Hb = \emptyset$ or $Ha = Hb$.

(b) Prove that every group of prime order is cyclic.

Q3. (a) Show that every quotient group of a group is a homomorphic image of the group.

(b) Prove that a group of order 99 is not simple.

Q4. (a) Define Integral Domain and prove that every field is an integral domain.

(b) Prove that order of a finite field F is P^n, for some prime p and some positive integer n.

Q5. (a) Prove that every integral domain can be imbedded in a field.

(b) Show that Z [√ -5] is not a principal ideal domain.

Q6. (a) Prove that the union of two subspaces is a subspace if and only if one is contained in the other.

(b) Let U and V be the vector spaces over the field F and let T be a linear transformation from U into V. Suppose that U is finite dimensional then prove that

Rank (T) + Nullity (T) = dim (U)

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Q7. (a) Let V be a finite dimensional vector space over the field F, and let W be a subspace of V. Then prove that

$\dim W + \dim W^0 = \dim V$

Where W^0 is the annihilator of W.

(b) In $V_3(R)$, where R is the field of Real numbers, examine each of the following sets of vectors for linear dependence

i. { (1,2,1), (3,1,5), (3,-4.7) }
ii. { (2,1,2), (8,4,8) }

Q8. (a) Suppose that α and β are vectors in an inner product space. Then show that

$\| \alpha + \beta \|^2 + \| \alpha - \beta \|^2 = 2 \| \alpha \|^2 + 2 \| \beta \|^2$

(b) Show that every square matrix satisfies its characteristic equation.
