Answer Key

Exam: SE | Sem III | IT | CBCGS (R 2016)

Subject: Applied Maths III

Exam Date: 23/11/17

Q. P Code: 24935

Space for Marks	Question No.	Solution of SE / sem III / IT ①

Q1(a) Every integer between 1 to 26 can be written as.

$1 = 2^0 \cdot 1$, $2 = 2^1 \cdot 1$, $3 = 2^0 \cdot 3$, $4 = 2^2 \cdot 1$

$5 = 2^0 \cdot 5$, . . . till 26

Hence there are thirteen no for representation of 1 to 26 no

{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25}.

Every number choosen from 1 to 26 will be one of the above forms.

If Fourteen are taken then two will be definitenitely multiple of each other.

Q1(b)

$f : R \to R, \quad g : R \to R.$

$f(x) = 2x+3, \quad g(x) = 3x-4$

(i) $f \circ g = f[g(x)] = f[3x-4]$

$= 2(3x-4)+3$

$= 6x - 8 + 3 = 6x-5$

(ii) $g \circ f \circ g = g[f \circ g]$

$= g[6x-5]$

$= 3(6x-5)-4$

$= 18x - 15 - 4$

$= 18x - 19$

Q1(c) $L\left\{ \frac{d}{d} \frac{\sin 3t}{t} \right\}$

$L\{\sin 3t\} = \frac{3}{s^2+9} = \phi(s)$

$L\left\{\frac{\sin 3t}{t}\right\} = \int_s^\infty \frac{3}{s^2+9}\, ds = \left[\tan^{-1}\frac{s}{3}\right]_s^\infty$

$= \tan^{-1}\infty - \tan^{-1}\frac{s}{3} = \cot^{-1}\frac{s}{3}$

$L\{$ Let $f(t) = \frac{\sin 3t}{t}$

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$f'(t) = \frac{d}{dt}\left(\frac{\sin 3t}{t}\right)$

$L\{f'(t)\} = s\,\phi(s) - f(0)$ —— (1)

$f(0) = \frac{\sin 3 0}{0} \quad \left(\frac{0}{0}\right)$

$f(0) = \lim_{t \to 0} \frac{3\cos 3t}{1} = \frac{3\cos 0}{1} = 3$

using above (1) thm

$L\left\{\frac{d}{dt}\frac{\sin 3t}{t}\right\} = s\cot^{-1}\frac{s}{3} - 3$

Q(d) Let $u = 3x^2 - 2x^2y + y^2$

$\frac{\partial u}{\partial x} = 6x - 4xy$

$\frac{\partial^2 u}{\partial x^2} = 6 - 4y$

$\frac{\partial u}{\partial y} = -2x^2 + 2y$

$\frac{\partial^2 u}{\partial y^2} = 2$

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 6 - 4y + 2 = 8 - 4y \neq 0$

For any Analytic function $f(z) = u + iv$

u & v should satisfy Laplace eq^n.

u is not satisfying Laplace eq^n Hence

u is not part of analytic function.

Q2a $\int_0^\infty e^{-t}\left(\frac{\cos 3t - \cos 2t}{t}\right)dt$

Sol^n. $L\left(\cos 3t - \cos 2t\right) = \frac{s}{s^2+9} - \frac{s}{s^2+4}$

$L\left\{\frac{\cos 3t - \cos 2t}{t}\right\} = \int_s^\infty \left(\frac{s}{s^2+9} - \frac{s}{s^2+4}\right)ds$

$= \frac{1}{2}\left[\log(s^2+9) - \log(s^2+4)\right]_s^\infty$

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$$=\frac{1}{2}\left[\lim_{s\to\infty}\log\left(\frac{s^2+9}{s^2+4}\right)-\lim_{s\to s}\log\left(\frac{s^2+9}{s^2+4}\right)\right]$$

$$= 0 + \frac{1}{2}\log\left(\frac{s^2+4}{s^2+9}\right)$$

Using def of LT

$$\int_0^\infty e^{-st}\left(\frac{\cos 3t-\cos 2t}{t}\right)dt = \frac{1}{2}\log\left(\frac{s^2+4}{s^2+9}\right)$$

putting $s=1$.

$$\int_0^\infty e^{-t}\left(\frac{\cos 3t-\cos 2t}{t}\right)dt = \frac{1}{2}\log\frac{5}{10}$$

$$= \frac{1}{2}\log\frac{1}{2}.$$

b) $L^{-1}\left\{\frac{s}{(s^2+1)(s^2+4)(s^2+9)}\right\}$

Let consider

$$\frac{1}{(s^2+1)(s^2+4)(s^2+9)} \quad \& \quad s^2=x$$

$$\frac{1}{(x+1)(x+4)(x+9)} = \frac{A}{x+1}+\frac{B}{x+4}+\frac{C}{x+9}$$

$$1 = A(x+4)(x+9)+B(x+1)(x+9) + C(x+1)(x+4)$$

putting $x=-1,-4,-9$, we k

$$1 = A(3\times 8) \Rightarrow A=\frac{1}{24}$$

$$1 = B(-3)(5) \Rightarrow B=-\frac{1}{15}$$

$$1 = C\cdot(-8)(-5) \Rightarrow C=\frac{1}{40}$$

Hence

$$\frac{s}{(s^2+1)(s^2+4)(s^2+9)} = \frac{s}{24(s^2+1)}-\frac{s}{15(s^2+4)}+\frac{s}{40(s^2+9)}$$

$$L^{-1}\{\ \ \} = \frac{1}{24}\cos t-\frac{1}{15}\cos 2t+\frac{1}{40}\cos 3t$$

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(4)

Q2(c) The cross ratio property of Bilinear transformation

$$\frac{(w-w_1)(w_2-w_3)}{(w_1-w_2)(w_3-w)} = \frac{(z-z_1)(z_2-z_3)}{(z_1-z_2)(z_3-z)}$$

$w = i, 0, -i$, & $z = 1, i, -1$

$$\frac{(w-i)(0+i)}{(i-0)(-i-w)} = \frac{(z-1)(i+1)}{(1-i)(-1-z)}$$

$$\frac{(w-i)\cancel{i}}{-\cancel{i}(w+i)} = \frac{(z-1)(i+1)}{-(1-i)(1+z)}$$

$$\frac{w-i}{w+i} = \frac{(z-1)(i+1)}{(1-i)(1+z)}$$

using componendo & dividendo

$$\frac{w-\cancel{i}+w+\cancel{i}}{\cancel{w}-i-\cancel{w}-i} = \frac{(z-1)(i+1)+(1-i)(1+z)}{(z-1)(i+1)-(1-i)(1+z)}$$

$$\frac{2w}{-\cancel{2}i} = \frac{\cancel{iz}+z-i-\cancel{1}+\cancel{1}+z-i-\cancel{iz}}{iz+\cancel{z}-\cancel{i}-1-1-\cancel{z}+\cancel{i}+iz}$$

$$\frac{w}{-i} = \frac{\cancel{2}(z-i)}{\cancel{2}(iz-1)}$$

$$w = \frac{-iz-1}{iz-1} = \frac{\cancel{i}(-z+i)}{\cancel{i}(z+i)}$$

$$w = \frac{i-z}{i+z}.$$

for fixed points $z = \dfrac{i-z}{i+z}$

$$iz+z^2 = i-z.$$

$$z^2+(1+i)z-i = 0$$

$$z = \frac{-(1+i) \pm \sqrt{(1+i)^2+4i}}{2} = \frac{-(1+i)\pm\sqrt{6i}}{2}$$

For image $|z|<1$ $\quad w(i+z) = i-z$

$$wi+wz = i-z-$$

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$$wz + z = i - iw$$

$$z(w+1) = i - iw$$

$$z = \frac{i(1-w)}{1+w}$$

$|z| < 1$ ie $\left|\frac{i(1-w)}{1+w}\right| < 1.$

$$|i(1-w)| < |1+w|$$

$$|i(1-u-iv)| < |1+u+iv|$$

$$\sqrt{(1-u)^2+v^2} < \sqrt{(1+u)^2+v^2}$$

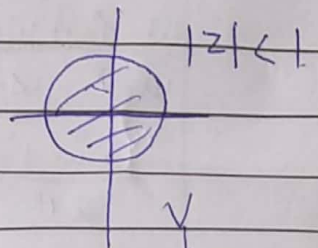
$$(1-u)^2+v^2 < (1+u)^2+v^2$$

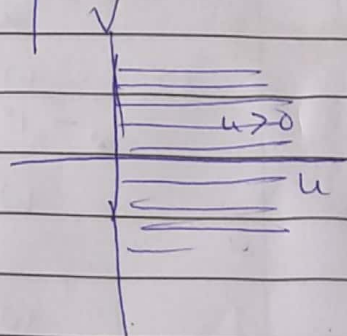
$$\cancel{1} - 2u + \cancel{u^2} + \cancel{v^2} < \cancel{1} + \cancel{u^2} + 2u + \cancel{v^2}$$

$$-4u < 0$$

$$u > 0.$$

$|z| < 1$





Q3 (A) ~~If~~ A

Let $(x, y) \in A \times (B \cup C)$

$\{x, y \mid x \in A$ and $y \in (B \cup C)\}$

$\{x, y \mid x \in A$ and $y \in B$ or $y \in C\}$

$\{(x, y) \mid (x \in A$ and $y \in B)$ or $(x \in A$ and $y \in C)\}$

$(x, y) \in (A \times B) \cup (A \times C)$

$A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$ — (1)

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The ~~Yeo~~

(ii) let $(x,y) \in (A \times B) \cup (A \times C)$

$\therefore (x,y) \in (A \times B)$ or $(x,y) \in (A \times C)$

$\therefore (x \in A$ and $y \in B)$ or $(x \in A$ and $y \in C)$

$\therefore x \in A$ and $y \in B$ or $y \in C$

$x \in A$ and $y \in B \cup C$

$(A \times B) \cup (A \times C) \subseteq A \times (B \cup C)$ — (ii)

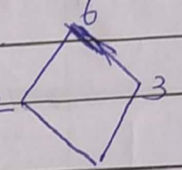
Hence By 1 & 2

$A \times (B \cup C) = (A \times B) \cup (A \times C)$

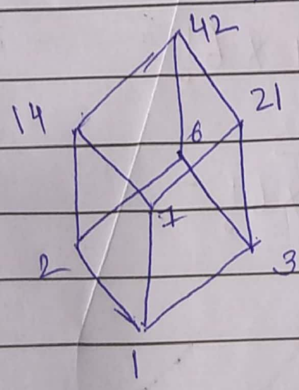
Q.2 b) $A = \{1, 2, 3, 6\}$

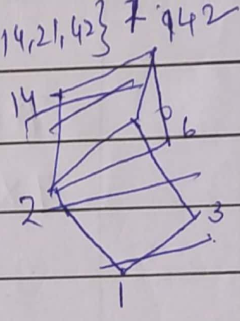
$R = \{(1,1)\ (1,2)\ (1,3)\ (1,6)\ (2,6)\ (3,6)$
$(2,2)\ (3,3)\ (6,6)\}$

~~Hasse diagram~~



(ii) ~~Let we~~ $B = \{1, 2, 3, 6, 7, 14, 21, 42\}$ ~~7~~ 942





$R = [\{1,1\}\ (1,2)\ (1,3)\ (1,6)\ (1,7)\ (1,14)\ (1,21)\ (1,42),$
$(2,6)\ (2,14)\ (2,42)\ (3,21)\ (3,42)\ (6,42)\ (7,42)$
$(2,2)\ (3,3)\ (6,6)\ (7,7)\ (14,14)\ (21,21)\ (42,42)\}$

For both set

(i) $a|a$ R is reflexive.

(ii) $a|b$ and $b|a$ Then $a = b$ R is reflexive.

(iii) if $a|b$ and $b|c$ Then $a|c$. Hence R is transitive.

There. (A, R) & (B, R) ~~are~~ Poset

Q3 (i) $L\{e^{-2t}\sqrt{1-\sin t}\}$

$$L\{\sqrt{1-\sin t}\} = L\{\sqrt{\sin^2\tfrac{t}{2}+\cos^2\tfrac{t}{2}-2\sin\tfrac{t}{2}\cos\tfrac{t}{2}}\}$$

$$= L\{\sin\tfrac{t}{2}-\cos\tfrac{t}{2}\}$$

$$= \frac{\frac{1}{2}}{s^2+\frac{1}{4}} - \frac{s}{s^2+\frac{1}{4}}$$

$$L\{e^{-2t}\sqrt{1-\sin t}\} = \frac{\frac{1}{2}}{(s+2)^2+\frac{1}{4}} - \frac{s+2}{(s+2)^2+\frac{1}{4}}$$

Q3(ii) $L\{t\,e^{-2t}H(t-1)\} = e^{-s}\,L\{(t+1)e^{-2(t+1)}\}$

$$= e^{-s}e^{-2}\,L\{e^{-2t}(t+1)\}$$

$$= e^{-(2+s)}\left[\frac{1}{(s+2)^2}+\frac{1}{s+2}\right]$$

Q4(a) For ladies cant sit together there 7 possible ways. 2 corner and 5 between the gentlemen. The four ladies can occupy those 7 seats $^7P_4$ ways.

6 genentemen can occuply these 6 seats in $^6P_6$ ways.

Hence no of arragement for ladies and gentlemen $^7P_4 \times {}^6P_6 = 840\times 720 = 60,4800$

(b) Find analytic function whose real part is.

Let $u = \dfrac{\sin 2x}{\cosh 2y + \cos 2x}$

$$\frac{\partial u}{\partial x} = \frac{2\cos 2x(\cosh 2y+\cos 2x) - \sin 2x(-2\sin 2x)}{(\cosh 2y+\cos 2x)^2}$$

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$$\frac{\partial u}{\partial x} = \frac{2\cos 2x \cosh 2y + 2\cos^2 2x + 2\sin^2 x}{(\cosh 2y + \cos 2x)^2}$$

$$\phi_1(z,0) = \left(\frac{\partial u}{\partial x}\right)_{(z,0)} = \frac{2\cos 2z + 2}{(1+\cos 2z)^2} = \frac{2(1+\cos 2z)}{(1+\cos 2z)^2}$$

$$\phi_1(z,0) = \frac{2}{2\cos^2 z} = \sec^2 z$$

$$\frac{\partial u}{\partial y} = \frac{-\sin 2x\,(2\sinh 2y)}{(\cosh 2y + \cos 2x)^2}$$

$$\phi_2(z,0) = \left(\frac{\partial u}{\partial y}\right)_{(z,0)} = 0$$

Analytic $f(z)$ is given by

$$f(z) = \int [\phi_1(z,0) - i\,\phi_2(z,0)]\,dz + c.$$

$$f(z) = \int \sec^2 z\,dz + c$$

$$f(z) = \tan z + c$$

Q4(1) $L^{-1}\left\{\frac{1}{(s-3)(s+4)^2}\right\}$

Let $\phi_1(s) = \frac{1}{s-3}$ $\quad L^{-1}\left\{\frac{1}{s-3}\right\} = e^{3t} = f_1(t)$

$\phi_2(s) = \frac{1}{(s+4)^2}$, $\quad L^{-1}\left\{\frac{1}{(s+4)^2}\right\} = e^{-4t} L^{-1}\left\{\frac{1}{s^2}\right\}$

$= e^{-4t}\,t = f_2(t)$.

By convolution thm

$$L^{-1}\left\{\frac{1}{(s-3)(s+4)^2}\right\} = \int_0^t e^{3(t-x)} e^{-4x} x\,dx$$

$$= e^{3t}\int_0^t x e^{-7x}\,dx$$

$$= e^{3t}\left[x\left(\frac{e^{-7x}}{-7}\right) - \frac{e^{-7x}}{49}\right]_0^t$$

$$= e^{3t}\left[-\frac{te^{-7t}}{7} - \frac{e^{-7t}}{49} + \frac{1}{49}\right] = \frac{1}{49}\left[e^{3t} - e^{-4t} - 7te^{-4t}\right]$$

4 Q(ii) $L^{-1}\left\{\log\left(1+\frac{a^2}{s^2}\right)\right\} = L^{-1}\left\{\log\left(\frac{s^2+a^2}{s^2}\right)\right\}$

$$L^{-1}\left\{\frac{d}{ds}\log\left(\frac{s^2+a^2}{s^2}\right)\right\} = -t\,f(t)$$

$$L^{-1}\left\{\frac{d}{ds}\left[\log(s^2+a^2) - \log s^2\right]\right\} = -t\,f(t)$$

$$L^{-1}\left\{\frac{2s}{s^2+a^2} - \frac{2s}{s^2}\right\} = -t\,f(t)$$

$$2\left[\cos at - 1\right] = -t\,f(t)$$

$$f(t) = \frac{2\left[1-\cos at\right]}{t}$$

Q5(a) Solve $\frac{dy}{dt} + 2y + \int_0^t y\,dt = \sin t$ given that $y(0)=1$

$$y'(t) + 2y(t) + \int_0^t y(t)\,dt = \sin t$$

Taking LT from bothside

$$\left[s\bar{y}(s) - y(0)\right] + 2\bar{y}(s) + \frac{1}{s}\bar{y}(s) = \frac{1}{s^2+1}$$

$$\left(s+2+\frac{1}{s}\right)\bar{y}(s) = 1 + \frac{1}{s^2+1}$$

$$\left(\frac{s^2+2s+1}{s}\right)\bar{y}(s) = 1 + \frac{1}{s^2+1}$$

$$\bar{y}(s) = \frac{s}{s^2+2s+1} + \frac{s}{(s^2+1)(s^2+1+2s)}$$

$$\bar{y}(s) = \frac{s}{s^2+2s+1} + \frac{1}{2}\left[\frac{1}{s^2+1} - \frac{1}{s^2+1+2s}\right]$$

Taking ILT

$$y(t) = L^{-1}\left\{\frac{s}{(s+1)^2}\right\} + \frac{1}{2}L^{-1}\left\{\frac{1}{s^2+1}\right\} - \frac{1}{2}L^{-1}\left\{\frac{1}{(s+1)^2}\right\}$$

$$L^{-1}\left\{\frac{s+1-1}{(s+1)^2}\right\} + \frac{1}{2}L^{-1}\left\{\frac{1}{s^2+1}\right\} - \frac{1}{2}L^{-1}\left\{\frac{1}{(s+1)^2}\right\}$$

$$L^{-1}\left\{\frac{s+1}{(s+1)^2}\right\} - \frac{3}{2}L^{-1}\left\{\frac{1}{(s+1)^2}\right\} + \frac{1}{2}L^{-1}\left\{\frac{1}{s^2+1}\right\}$$

$$e^{-t} - \frac{3}{2}e^{-t}t + \frac{1}{2}\sin t$$

(b) Find P such that the function $\frac{1}{2}\log(x^2+y^2)+i\tan^{-1}\frac{px}{y}$ is analytic

(10)

Let $f(z) = u+iv = \frac{1}{2}\log(x^2+y^2)+i\tan^{-1}\frac{px}{y}$

$u = \frac{1}{2}\log(x^2+y^2) \quad v = \tan^{-1}\frac{px}{y}$

$\frac{\partial u}{\partial x} = \frac{2x}{2(x^2+y^2)} \qquad \frac{\partial v}{\partial x} = \frac{1}{1+\frac{p^2x^2}{y^2}}\left(\frac{p}{y}\right), \frac{\partial v}{\partial y} = \frac{1}{1+\frac{p^2x^2}{y^2}}\left(\frac{-px}{y^2}\right)$

$\frac{\partial u}{\partial y} = \frac{2y}{2(x^2+y^2)} \qquad \frac{\partial v}{\partial x} = \frac{py}{p^2x^2+y^2}, \quad \frac{\partial v}{\partial y} = \frac{-px}{y^2+p^2x^2}$

using C-R equation

$$\left.\begin{aligned}\frac{\partial u}{\partial x} &= \frac{\partial v}{\partial y} \Rightarrow \frac{x}{x^2+y^2} = \frac{-px}{p^2x^2+y^2} \\ \frac{\partial u}{\partial y} &= \frac{-\partial v}{\partial x} \Rightarrow \frac{y}{x^2+y^2} = \frac{-py}{p^2x^2+y^2}\end{aligned}\right\}$$

By observation $p = -1$

(c) For $x, y \in Z$ xRy if and only if $2x+5y$ is divisible by 7

Sol Putting $y = x$.

(I) xRx i e $2x+5x = 7x$ divisible by 7.

Therefor R is Reflexive

(II) xRy i e $2x+5y$ is divisible by 7

Let $\frac{2x+5y}{7} = m \Rightarrow 2x+5y = 7m$.

$yRx \Rightarrow 2y+5x = 7y-5y+7x-2x$

$= 7(y+x)-(2x+5y)$

$= 7(y+x-m)$ divisible by 7

Hence xRy Then yRx Hence R is symmetric

(III) xRy and yRz

$2x+5y = 7m, \quad 2y+5z = 7n \quad m,n \in I$

$2x+5y+2y+5z = 7m+7n$

$2x+5z = 7m+7n-7y = 7(m+n-y)$

$2x+5z$ is divisible by 7 Hence R is transitive

therefor R is equivalent Relation

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Q 6(a) $ax^2+bx+c=0$.

a, b, c are determined by throwing a die, a, b, c Can take 1 to 6 only

Total number of possible outcomes $= 6\times6\times6 = 216$

The no of favourable where $b^2-4ac \geq 0$ i.e $b^2 \geq 4ac$

ac	a	c	4ac	b^2	No of case
1	1	1	4	2, 3, 4, 5, 6	5
2	1	2	8	3, 4, 5, 6	8.
	2	1	8	3, 4, 5, 6	
3	1	3	12	4, 5, 6	6
	3	1	12	4, 5, 6	
4	1	4	16	4, 5, 6	6.
	4	1	16	4, 5, 6	
	2	2	16	4, 5, 6	
5	1	5	20	5, 6	4
	5	1	20	5, 6	
6	1	6	24	5, 6	8.
	6	1	24	5, 6	
	2	3	24	5, 6	
	3	2	24	5, 6	
8	2	4	32	6	2.
	4	2	32	6	
9	3	3	36	6	1
					Total 43.

So required probability $= \frac{43}{216}$

(b) We have p_1 = probability of person has cancer

$$= \frac{2000}{100000} = .02$$

p_2 = probability that a persons does not have cancer

$$1 - .02 = .98$$

p_1' = test is positive when a person has cancer $= \frac{95}{100} = .95$

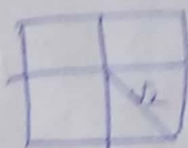
p_2' = test is positive when a person does not have a cancer $= \frac{5}{100} = .05$

Required probability $= \dfrac{p_1 p_1'}{p_1 p'_1 + p_2 p'_2}$

$$= \frac{\left(\frac{2}{100}\right)\left(\frac{95}{100}\right)}{\left(\frac{2}{100}\right)\left(\frac{95}{100}\right) + \left(\frac{98}{100}\right)\left(\frac{5}{100}\right)}$$

$$= \frac{190}{680} = .279$$

6(c)(i) Consider square of side 2 units. Divide the square in four smaller squares. Consider small square as pigeonholes. If five points are taken on square by pigeonhole principal at least two of them will be in same square. The maximum distance is diagonal in box i.e $\sqrt{2}$. Hence atleast two points are not more than $\sqrt{2}$ units apart.



(ii) Since there are twelve months, Considering even distribution if there are 48 friends Four will have birthday in same month. Hence, if there are 49 friends then five will have birthday in same month. By extended pigenhole principle

$$\left\lceil \frac{n-1}{12} \right\rceil + 1 = 5, \quad \frac{n-1}{12} = 4 \quad \therefore \quad n = 48+1 = 49.$$