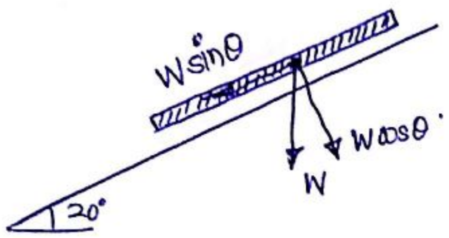
FLUID MECHANICS - Nov-Dec 2017

QP Code: 13610

Q1.a



Area = $A = 0.9 \times 0.9 = 0.81 m^2$.

$W = 392.4 N$

$\therefore$ shear stress $\tau = \frac{F}{A} = \frac{W\sin\theta}{A}$

$\therefore \boxed{\tau = \frac{392.4 \times \sin 20}{0.81} = 165.68 N/m^2}$ — 2 marks

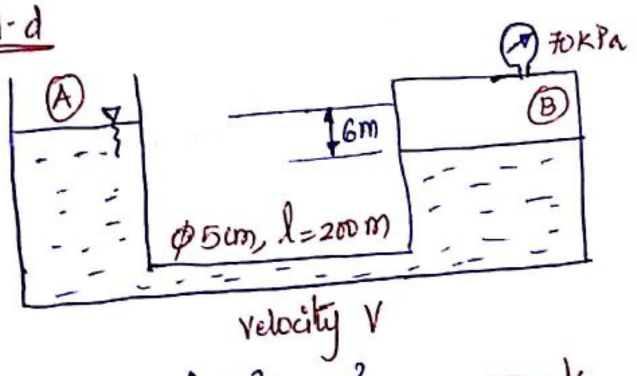
Applying Newton's law of viscosity

$\boxed{\tau = \mu \cdot \frac{dv}{dy}}$ — 1 mark

velocity gradient $= \frac{dv}{dy} = \frac{0.2}{1.5 \times 10^{-3}} = 133.33 \frac{1}{s}$

$\therefore$ Dynamic viscosity $\mu = \frac{\tau}{dv/dy} = \frac{165.68}{133.33} = \boxed{1.242 \frac{N\text{-}s}{m^2}}$ — 2 marks.

Q1.d



The entire head H = 6m must be lost in overcoming the pipe friction and in creating the exit velocity.

f = friction factor = 0.02

$\therefore H = \frac{fLV^2}{2gd} + \frac{V^2}{2g}$ — 1 mark

$6 = \left[\frac{0.02 \times 200}{2 \times 0.05} + 1\right]\frac{V^2}{2 \times 9.81}$

$\therefore \cancel{6} = \frac{41V^2}{19.62}$

$\boxed{V = 1.6944 m/s}$ — 2 marks

Discharge

$Q = AV$

$= \frac{\pi}{4} \times 0.05^2 \times 1.6944$

$\boxed{Q = 3.32 \times 10^{-3} \frac{m^3}{s}}$ — 2 marks

Q1-e
$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right)^2 + \left(\frac{y}{\delta}\right)^3 - 2\left(\frac{y}{\delta}\right)^4$$

$$\therefore \frac{du}{dy} = \frac{d}{dy}\left[\frac{2y^2}{\delta^2} + \frac{y^3}{\delta^3} - \frac{2y^4}{\delta^4}\right]U$$
$$= \left[\frac{4y}{\delta^2} + \frac{3y^2}{\delta^3} - \frac{8y^3}{\delta^4}\right]U$$

Now $\left(\frac{du}{dy}\right)_{y=0} = 0$. This implies that the flow is NEITHER attached NOR completely seperated from the boundary. It is on the VERGE of seperation.

Q2a Note that the tank is closed. Hence the air will exert some pressure. P_{air}. Manometric fluid is to be assumed as mercury.

$$\therefore \begin{pmatrix}\text{pr. due to}\\ \text{1m of air}\end{pmatrix} + \begin{pmatrix}\text{pr due to}\\ \text{2m of oil}\end{pmatrix} + \underbrace{\begin{pmatrix}\text{pr due to}\\ \text{3m of water}\end{pmatrix} + \begin{pmatrix}\text{pr due}\\ \text{to 100 cm}\\ \text{of water}\end{pmatrix}}_{\text{water}} = \begin{pmatrix}\text{pr due to}\\ \text{'y m of Hg}\end{pmatrix}$$

(4m)

$$\therefore p_a + (0.82\times1000\times9.81\times2) + (1\times1000\times9.81\times4) = 13.6\times1000\times9.81\times y$$

$$\therefore p_a + 16088.4 + 39240 = 133416\ y$$

$$\therefore \boxed{p_a + 55328.4 = 133416\,y}$$ —— 4m

The air pressure must be assumed, so as to get a +ve value of 'y' in meters.

Q3a $x = x_0 e^{-tk} + y_0(1-e^{-2tk})$ is $x = f(t)$

$\therefore$ Horizontal velocity component $V_x = \frac{dx}{dt}$

$$\therefore V_x = \frac{dx}{dt} = \left[-x_0 k \cdot e^{-tk} + 2ky_0 e^{-2tk}\right] \quad \text{—— (1)}$$

And $y = y_0 e^{tk}$

$$\therefore V_y = \frac{dy}{dt} = y_0 k e^{tk} \quad \text{—— (2)}$$

5marks

Now, eqn of the path can be found by eliminating the time variable 't' among the two position variables x & y

$$\therefore \quad e^{tk} = \frac{y}{y_0}$$

$$\therefore x = \frac{x_0}{e^{tk}} + y_0\left[1 - \frac{1}{(e^{tk})^2}\right]$$

$$\boxed{x = \frac{x_0 y_0}{y} + y_0\left[1 - \left(\frac{y_0}{y}\right)^2\right]}$$ — This is the relatn between x & y.

(5 marks)

Q4 b $K = \gamma = 1.66$ - for Helium

ρ_1 = density upstream
ρ_2 = density downstream
$\rho_2 = 3\rho_1 \quad \therefore \frac{\rho_2}{\rho_1} = \frac{3}{1}$

(1.) Using the continuity equation: $m = \rho_1 A_1 V_1 = \rho_2 A_2 V_2$

let $A_1 = A_2 \quad \therefore \rho_1 V_1 = \rho_2 V_2$

$$\Rightarrow \boxed{\frac{V_2}{V_1} = \frac{\rho_1}{\rho_2} = \frac{1}{3}}$$ — velocity ratio.

1 mark

(2) Pressure Ratio

$$\frac{p_2}{p_1} = \frac{\left[\left(\frac{\gamma+1}{\gamma-1}\right)\frac{\rho_2}{\rho_1} - 1\right]}{\left[\left(\frac{\gamma+1}{\gamma-1}\right) - \frac{\rho_2}{\rho_1}\right]} = \frac{\left[\left(\frac{2.66}{0.66}\right)\left(\frac{3}{1}\right) - 1\right]}{\left(\frac{2.66}{0.66} - \frac{3}{1}\right)} = \frac{11.0909}{1.0303} = 10.746.$$

$$\Rightarrow \boxed{p_2 = 10.746\, p_1}$$ — 3 marks

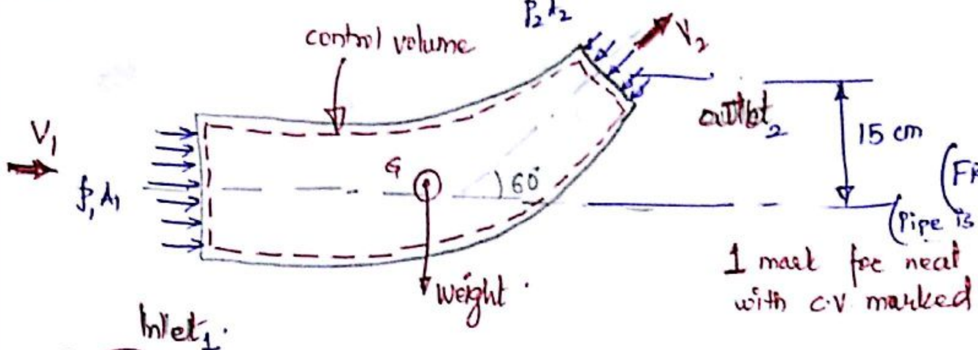
(3) $$\frac{p_2}{p_1} = \left[\frac{2\gamma M_1^2 - (\gamma-1)}{(\gamma+1)}\right]$$ — substituting we get $\boxed{M_1 = 2.902771}$ — 3 marks

ie supersonic

(4) $$M_2^2 = \frac{[(\gamma_1 - 1)M_1^2 + 2]}{[2\gamma M_1^2 - (\gamma-1)]}$$ — substituting $\boxed{M_2 = 0.5261} < 1$ — 3 marks.

$\therefore$ subsonic.

Q5 a



(FRONT VIEW)
(Pipe is in VERTICAL plane)

1 mark for neat sketch with c.v marked

(2 marks) – alloted if the Reynold Transport Theorem is actually written

NOTE:

① suffix 1 is used for INLET | $d_1 = 0.25\,m$ | Volume $= 0.015\,m^3$

suffix 2 is used for OUTLET. | $d_2 = 0.15\,m$ | $Z = 15\,cm$.

$V_1 = 1.5\,m/s$ & $p_1 = 300\,kN/m^2$.

Discharge $Q = A_1 V_1 = \frac{\pi}{4} \times (0.25)^2 \times 1.5 = 0.073631\,\frac{m^3}{s}$.

And $V_2 = \frac{Q}{A_2} = 4.166\,m/s$ or $V_2 = \left(\frac{A_1}{A_2}\right) V_1 = \left(\frac{d_1}{d_2}\right)^2 V_1$

② Σ Forces along the HORIZONTAL

$\Sigma F_x = \rho Q \left[\overline{V_{1x}} - \overline{V_{2x}} \right] + \overline{p_{1x} A_1} + \overline{p_{2x} A_2}$ (all vectors +ve →) — 1 mark

$\overline{V_{1x}}$ = initial horizontal velo $= +1.5\,m/s$

$\overline{V_{2x}}$ = final horizontal velo $= V_2 \cos 60 = 4.166 \cos 60 = 2.0833\,\frac{m}{s}$ →

$\overline{p_{1x}}$ = pressure at inlet along horizontal $= 300 \times 10^3\,N/m^2$

Apply Bernoulli's Eq^n at inlet & outlet

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{Z_1} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{Z_2}$$

0   0.15 m

$$\left(\frac{p_1 - p_2}{\rho}\right) + \frac{V_1^2}{2} = \frac{V_2^2}{2}$$

ie $\frac{(V_1^2 - V_2^2)}{2} + \frac{p_1}{\rho} = \frac{p_2}{\rho}$

$\therefore \boxed{p_2 = 292.447\,\frac{kN}{m^2}}$ — 1 mark.

$\overline{p_{2x}}$ = outlet pressure along horizontal $= p_2 \cos 60\ (\leftarrow) = 146.223\,\frac{kN}{m^2}$

$\therefore \Sigma F_x = 1000 \times 0.073631 \left[(+1.5) - (+2.0833)\right] + 300 \times 10^3 \times \frac{\pi}{4}(0.25)^2 - 146223 \times 10^3 \times \frac{\pi}{4}(0.15)^2$

$\boxed{\Sigma F_x = 12099.28\,N}$ — 1 mark.

Q5-a (continued)

(3) ΣF_y = forces along the vertical (Include the self weight of the control volume)

$\bar{V}_{1y} = 0$

$\bar{V}_{2y} = V_2 \sin 60 = 4.166 \sin 60 = 3.60786$ m/s ↑

$\bar{p}_{1y} = 0 \qquad \bar{p}_{2y} = -p_2 \sin 60 \downarrow = -292.447 \sin 60$
$= -249.658 \ N/m^2$

Self weight W = mass × gravity
= mass density × volume × gravity
= $1000 \times 0.015 \times 9.81$
= 147.15 N (↓)

$\therefore \Sigma F_y = \rho Q[\bar{V}_{1y} - \bar{V}_{2y}] + \bar{p}_{1y} A_1 + \bar{p}_{2y} A_2 + \bar{W}$ — 1 mark

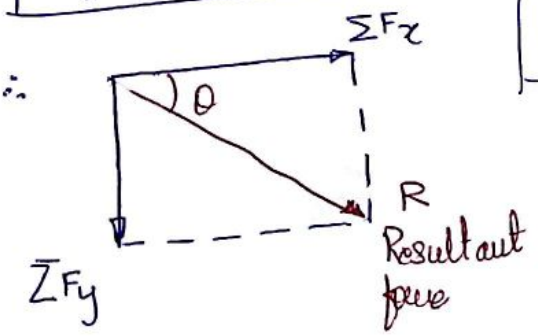
$= 1000 \times 0.073631[0 - (3.60786)] + 0 - 249.658 \times \frac{\pi}{4}(0.15)^2 - 147.15$

$= -265.6503 - 2895.327 - 147.15$

$\boxed{\Sigma F_y = -3308.12 \ N}$ — 1 mark

$\boxed{\begin{aligned} R &= 12543.37 N \\ \theta &= 15.29^\circ \end{aligned}}$ — 1 mark

∴



NOTE: These are forces from the free body diagram of the C.Volume hence they are forces OF the surroundings ON the C.V. (ie of the Bend on the fluid)

The force of the water back on the bend would be EQUAL & OPPOSITE to that of the Resultant force calculated — 1 mark

Q5-b Since the pipes are in parallel $\therefore Q = Q_1 + Q_2 + Q_3$ — (1) — 1mark

and Loss of head in each pipe = h. $Q = 0.3\,m^3/s$.

(1)

$l_1 = 1500\,m$	$d_1 = 0.2\,m$	$\therefore A_1 = \frac{\pi}{4}(0.2)^2 = 0.031415\,m^2$	$f_1 = 0.02$
$l_2 = 2000\,m$	$d_2 = 0.3\,m$	$\therefore A_2 = \frac{\pi}{4}(0.3)^2 = 0.070685\,m^2$	$f_2 = 0.015$
$l_3 = 1000\,m$	$d_3 = 0.15\,m$	$\therefore A_3 = \frac{\pi}{4}(0.15)^2 = 0.01767\,m^2$	$f_3 = 0.02$

(2) V_1, V_2 & V_3 are velocities in three pipes which are unknown

(3) $h_{f_1} = h_{f_2}$ $\therefore \frac{f_1 L_1 V_1^2}{2gd_1} = \frac{f_2 L_2 V_2^2}{2gd_2}$ $\therefore \frac{0.02 V_1^2}{0.2} = \frac{0.015 V_2^2}{0.3}$

$\therefore \frac{V_2^2}{V_1^2} = 2$ $\boxed{V_2 = \sqrt{2}\,V_1}$ — (2) 1mark $\frac{V_1^2}{10} = \frac{V_2^2}{20}$

and $h_{f_1} = h_{f_3}$ $\therefore \frac{0.02 V_1^2}{0.2} = \frac{0.02 V_3^2}{0.15}$

$\boxed{V_3 = \sqrt{0.75}\,V_1}$ — (3) 1mark

Now using Equation (1)

$Q = Q_1 + Q_2 + Q_3$

$\therefore 0.3 = 0.031415\,V_1 + 0.070685 \times \sqrt{2}\,V_1 + 0.01767(\sqrt{0.75})V_1$

$0.3 = \cancel{0.14668}V_1$ $\Rightarrow \boxed{V_1 = \cancel{2.0452}\,m/s}$ 3.83 m/s 1mark

$= 0.07832\,V_1$ $\Rightarrow \boxed{V_2 = \cancel{2.8943}\,m/s}$ 1.712 m/s 1mark

$\boxed{V_3 = \cancel{1.7712}\,m/s}$ 3.3168 m/s 1 mark

$\therefore Q_1 = A_1V_1 = 0.031415 \times \cancel{2.0452}\,3.83 = \cancel{0.0642}\,m^3/s$ = 0.1203 m^3/s

$Q_2 = A_2V_2 = 0.070685 \times \cancel{2.8942}\,1.712 = \cancel{0.2042}$ 0.1210 m^3/s

$Q_3 = A_3V_3 = 0.01767 \times \cancel{1.7712}\,3.3168 = \cancel{0.0313}\,m^3/s$ 0.0586 m^3/s

=

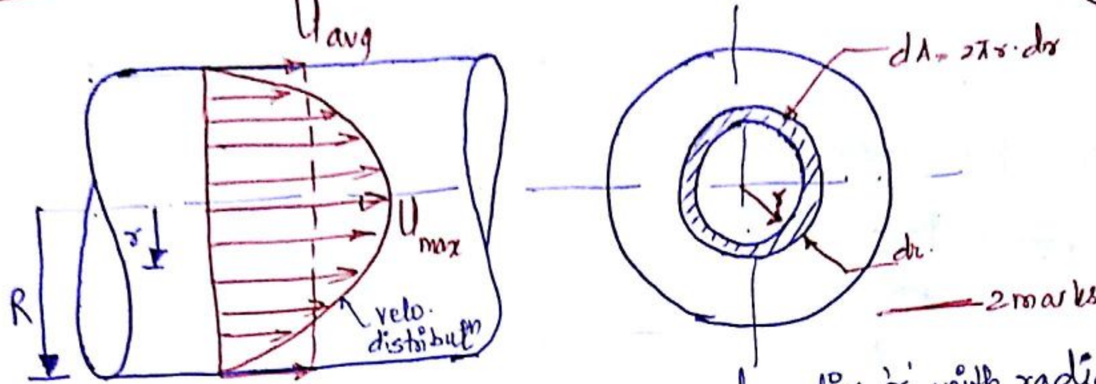
(3marks)

[redacted] Loss of head $= h = \frac{f_1 L_1 V_1^2}{2gd_1} = \frac{f_2 L_2 V_2^2}{2gd_2} = \frac{f_3 L_3 V_3^2}{2gd_3}$

$\therefore h = \frac{0.02 \times 1500 \times (0.1203)^2}{2 \times 9.81 \times 0.2}$

$\boxed{h = 0.1106\,m}$ — 1mark

Q6a



—— 2 marks

Consider an elementary ring at a general radius 'r' with radial thickness 'dr'

$\therefore$ Elementary area = $dA = (2\pi r)\, dr$

velocity distribution = $u = \left[1 - \left(\frac{r}{R}\right)^2\right] U_{max}$

$\therefore$ Elementary discharge = $dQ = u \cdot dA = \left(1 - \frac{r^2}{R^2}\right) U_{max} \cdot dA$

$= \left(1 - \frac{r^2}{R^2}\right) U_{max} \times 2\pi r\, dr$. —— 1 mark

$\therefore$ Total discharge $Q = \int dQ = \int_0^R 2\pi \left(r - \frac{r^3}{R^2}\right) U_{max}\, dr$ —— (1) —— 2 marks

If average velocity U_{Avg} is considered throughout the c/s A then discharge $Q = A U_{avg}$ —— (2) —— 1 mark

Equating (1) = (2)

$$A \cdot U_{avg} = 2\pi \int_0^R \left(r - \frac{r^3}{R^2}\right) U_{max}\, dr$$

$$\pi R^2 \cdot U_{avg} = 2\pi \cdot U_{max} \left(\frac{r^2}{2} - \frac{r^4}{4R^2}\right)_0^R$$

$$= 2\pi U_{max} \left(\frac{R^2}{2} - \frac{R^4}{4R^2}\right)_{UL}$$

$$\therefore \pi R^2 \cdot U_{avg} = 2\pi U_{max} \left(\frac{R^2}{2} - \frac{R^2}{4}\right)$$

$$\cancel{\pi} R^2 \cdot U_{avg} = 2\cancel{\pi} U_{max} \times \frac{3}{4} R^2$$

—— 4 marks.

$$\therefore \boxed{\frac{U_{avg}}{U_{max}} = \frac{3}{2}}$$ —— Ratio.

Pg 8

Q 6 b(ii)

$R = 260\ J/kg\,k$ $\quad T = 25°C$

$k = 1.4$ $\quad = 25 + 273$

$= 298\,k$ —— 1 mark

$\therefore$ velocity $= \sqrt{k \cdot RT}$ —— 3 marks

$= \sqrt{1.4 \times 260 \times 298}$

$\boxed{Velo = 329.35\ m/s.}$ —— 1 mark

velocity of sound in Oxygen