QP Code : 77059

(2 ½ Hours) [Total Marks :75

N.B. : (1) **All** questions are **compulsory**
(2) Figures to the right indicate full marks.
(3) Use of calculator is allowed.

1. (a) Let $x_1, x_2 \ldots, x_n$ be a random sample drawn from N $(0,\sigma^2)$. Examine whether **8**

$$T = \frac{\sum x_i^2}{n}$$

is (i) Unbiased, (ii) Consistent, (iii) Sufficient estimator of σ^2.

(b) Explain the term consistency. **7**
Let $x_1, x_2 \ldots, x_n$ be a random sample drawn from population with mean μ and variance σ^2.

Check whether $T = 2\sum_{k=1}^{n} \frac{kx_k}{n(n+1)}$ is unbiased and consistent estimator of μ.

OR

1. (p) In each of the following cases, find sufficient estimator of θ : **8**

(i) $f(x, \theta) = \frac{1}{\theta} e^{-x/\theta}$; $x > 0$, $\theta > 0$
$= 0$; otherwise

(ii) $f(x, \theta) = e^{-(x-\theta)}$; $x > \theta$
$= 0$; otherwise

(q) Prove that an estimator T of parameter θ is consistent estimator of θ if its bias $\to 0$ and V (T) $\to 0$ as $n \to \infty$, in usual notations. **7**

2. (a) Prove that if MVUE exists then it is unique. **8**

[TURN OVER]

b) Let $x_1, x_2, \ldots x_n$ be a random sample taken from population with p.m.f. 7

$$f(x, \theta) = \theta^x (1-\theta)^{1-x} \quad ; x = 0,1 \; ; 0 < \theta < 1$$
$$= 0 \quad ; \text{otherwise}$$

Find MVUE of θ and its standard error.

OR

2. (p) State and prove Cramer-Rao Inequality. 8

(q) If $X \sim N(\mu, \sigma^2)$ where σ^2 is known, find CRLB for the variance of an unbiased estimator of μ. 7

3. (a) With reference to point estimation, explain 8

(i) method of minimum chi-square,

(ii) method of modified minimum chi-square.

(b) Obtain an estimator of θ based on a random sample of size n drawn from a population with p.d.f. 7

(i) $f(x, \theta) = (1+\theta)\, x^{\theta} \quad ; 0 < x < 1$

$= 0 \quad ; \text{otherwise}$

(ii) $f(x, \theta) = \dfrac{1}{\theta} \quad ; 0 < x < \theta$

$= 0 \quad ; \text{otherwise},$

using method of moments.

OR

3. (p) State the properties of Maximum Likelihood Estimator (MLE). 8

(q) Let $x_1, x_2, \ldots x_n$ be a random sample of size n from the population with p.d.f. 7

$$f(x,\alpha,\beta) = \frac{1}{\beta} e^{-(x-\alpha)/\beta} \quad ; \; \alpha \le x < \infty, \; \beta > 0$$
$$= 0 \quad ; \text{otherwise}$$

Find estimators of parameters α and β using method of maximum likelihood.

4. (a) Let $x_1, x_2, \ldots x_n$ be a random sample from exponential distribution with mean $1/\theta$ where θ is itself distributed as gamma with parameter α and β Assuming squared error loss function, find Bayes' estimator for θ. 8

(b) Two independent samples of sizes n_1 and n_2 are drawn from two populations 7

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having normal distributions with means μ_1 and μ_2 and variances σ_1^2 and σ_2^2 respectively. Construct 100 (1-α) % confidence interval for σ_1^2 / σ_2^2 where μ_1 and μ_2 are unknown.

OR

4. (p) Let $x_1, x_2, ...x_n$ be a random sample from Poisson distribution with parameter θ. The prior distribution of θ is exponential with mean $\frac{1}{\alpha}$. Assuming squared error loss function, find Bayes' estimator of θ. **8**

(q) Obtain 100 (1 -α) % confidence interval for θ, parameter of Poisson distribution using asymptotic distribution of M.L.E. **7**

5. (a) Let x_1, x_2, x_3 be a random sample of size 3 drawn from N (μ, σ^2) distribution. Show that sample mean $\overline{X}$ and $T = \frac{x_1 + 2x_2 + 3x_3}{6}$ are both unbiased estimators of μ. Which estimator would you prefer? Justify. **5**

(b) Let $x_1, x_2, ...x_n$ be a random sample drawn from population with p.m.f **5**

$$f(x,\theta) = \frac{x}{\theta^2} e^{-x/\theta} \quad ; x > 0, \theta > 0$$
$$= 0 \quad ; \text{otherwise}$$

Find MLE of θ

(c) Explain the following terms as used in Bayesian estimation : **5**

(i) Prior distribution,

(ii) Posterior distribution.

OR

5. (p) Define : **5**

(i) relative efficiency of an estimator,

(ii) exponential family of distributions.

(q) Let $x_1, x_2, ...x_n$ be a random sample from a distribution with p.m.f **10**

$$f(x, \theta) = \theta (1-\theta)^{x-1} \quad ; x = 1, 2, ... \quad 0 < \theta < 1$$
$$= 0 \quad ; \text{otherwise}$$

Show that variance of sample mean $\overline{X}$ attains CRLB.
