May/June-2018 (1)

(CBCGS) | Code: 21376 |

Subject :- Fluid Mechanics Date: 17/05/2018

Q.1 c) Given velocity components

$u = 2x^2$

$v = 2xyz$

Calculate the unknown velocity component

ie $w = ?$

$\Rightarrow$ Continuity eqn for three dimensional flow field is given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\therefore \frac{\partial}{\partial x}(2x^2) + \frac{\partial}{\partial y}(2xyz) + \frac{\partial w}{\partial z} = 0$$

$$\therefore 4x + 2xz + \frac{\partial w}{\partial z} = 0$$

$$\therefore \frac{\partial w}{\partial z} = -4x - 2xz$$

$$\therefore w = -4xz - \frac{2xz^2}{2}$$

$$\boxed{\therefore w = -4xz - xz^2}$$

This is required third component of velocity

D) ~~AA~~ Given data

$\mu = 0.073$ poise $= 0.0073$ N.S/m^2

$S = 0.87$

Find i) ν in m^2/s & stokes

$\Longrightarrow$ We know that kinematic viscosity of a liquid is given by

$$\nu = \frac{\mu}{\rho} = \frac{0.0073}{0.87 \times 1000}$$

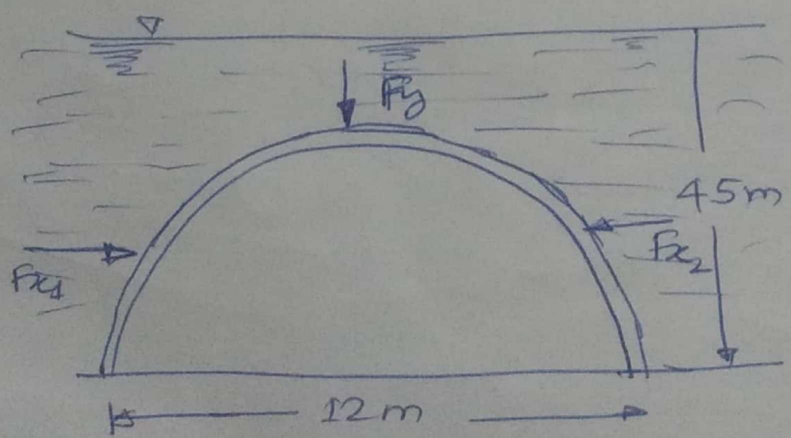
$$\boxed{\therefore \nu = 8.39 \times 10^{-6} \ m^2/s}$$

In stokes kinematic viscosity is given by

$$\nu = 8.39 \times 10^{-6} \times 10^{4} \ cm^2/s$$

$$\boxed{\therefore \nu = 8.39 \times 10^{-2} \ (cm^2/s) \text{. stokes}}$$

Q.2) A)



Given data:

$d = 12\,m$

$h = 45\,m$

$L = 240\,m$

Find :- i) F_R & θ

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i) Horizontal component of the force acting on : tunnel : F_x

$$F_{x1} = \rho g \bar{h} A$$

$$= 9810 \times 42 \times 240 \times 6$$

$$\boxed{F_{x1} = 593.31\ MN} \rightarrow$$

$$F_{x2} = \rho g \bar{h} A$$

$$= 9810 \times 42 \times 240 \times 6$$

$$\boxed{F_{x2} = 593.31\ MN \leftarrow}$$

Net horizontal force acting on tunnel is

$$\boxed{F_x = F_{x1} + F_{x2} = 0\ N}$$

ii) Vertical component of the force acting on tunnel : F_y

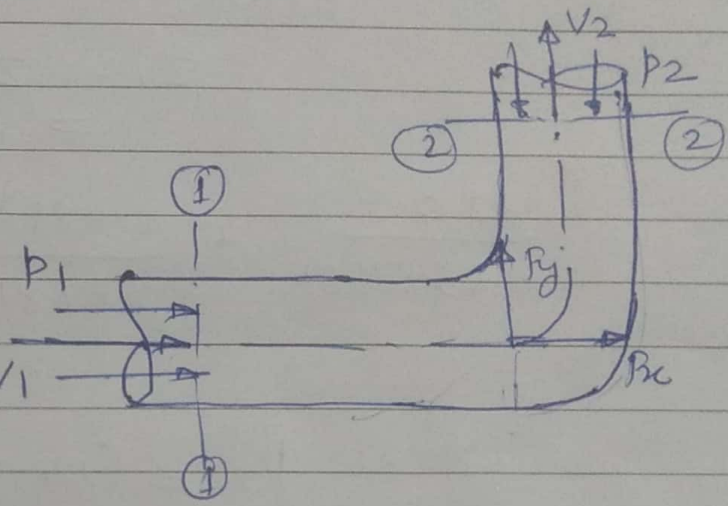
$\therefore F_y =$ Sp. wt of water × volume of water supported by tunnel upto free surface level

$$= 9810 \times \left[\left(12 \times 45 - \frac{\pi (6)^2}{2}\right) \times 240\right]$$

$$\boxed{F_y = 1138.24\ MN}$$

Vertical component of the force ie F_y will act at the top of the tunnel ie at $\bar{h} = 39$ m from the free surface level

Q.3 A) Given data



$d_1 = 300$ mm

$d_2 = 150$ mm

$Q = 0.6\ m^3/sec$

$S_{oil} = 0.85$

$P_1 = 120\ kN/m^2$

$\forall = 0.15\ m^3$

Find i) F_R & θ

$\Longrightarrow$

$$A_1 = \pi/4\, d_1^2 = \pi/4\,(0.3)^2 = 0.070686\ m^2$$

$$A_2 = \pi/4\, d_2^2 = \pi/4\,(0.15)^2 = 0.01767\ m^2$$

By continuity eqn

$$Q = A_1 V_1 = A_2 V_2 = 0.6$$

$$\therefore V_1 = Q/A_1 = \frac{0.6}{0.07068} = 8.488\ m/s$$

$$\therefore V_2 = Q/A_2 = \frac{0.6}{0.01767} = 33.955\ m/s$$

By appling Bernoullis principle at section

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + Z_2$$

As Bend is in horizontal plane

$$Z_1 = Z_2$$

$$\frac{120}{0.85 \times 9.81} + \frac{(8.488)^2}{2 \times 9.81} = \frac{p_2}{w} + \frac{(33.95)^2}{2 \times 9.81}$$

$$\therefore p_2 = (14.39 + 3.672 - 58.781) \times 0.85 \times 9.81$$

$$\boxed{\therefore p_2 = -339.54 \ KN/m^2}$$

a) Horizontal component of the force acting on the bend : F_x

$$\therefore F_x = \rho Q [V_1 - 0] + p_1 A_1 + \cancel{p_2 A_2}^{\,0}$$

$$\therefore F_x = 0.85 \times 0.6 [8.488 - 0] + 120 \times 0.07068$$

$$\boxed{\therefore F_x = 12.81 \ KN \leftarrow}$$

b) Vertical component of the force acting on the bend : F_y

$$\therefore F_y = \rho Q [0 - (V_2)] + \cancel{p_1 A_1}^{\,0} - p_2 A_2$$

$F_y = 0.85 \times 0.6 \left[-33.96\right] - (-339.54 \times 0.01767)$

$\therefore F_y = -17.32 + 5.99$

$\therefore F_y = -11.32$ KN

$\boxed{\therefore F_y = 11.32 \text{ KN} \downarrow}$

$\therefore$ Resultant force acting on the bend is

$F_R = \sqrt{F_x^2 + F_y^2}$

$\therefore F_R = \sqrt{(12.81)^2 + (11.32)^2}$

$\boxed{\therefore F_R = 17.1 \text{ KN}}$

Direction of resultant force is

$\theta = \tan^{-1}\left|\frac{F_y}{F_x}\right|$

$\boxed{\therefore \theta = 41.48^\circ \swarrow}$

Q4 a) Laminar boundary layer velocity distribution is given by

$$\frac{u}{U_\infty} = 2\frac{y}{\delta} - 2\frac{y^2}{\delta^2} + \frac{y^4}{\delta^4}$$

$\Rightarrow$ i) Boundary layer separation occurs when

$$\left(\frac{\partial u}{\partial y}\right)_{y=0} = 0$$

Lets check for the condition

$$\frac{\partial u}{\partial y} = \frac{2U_\infty}{\delta} - \frac{4yU_\infty}{\delta^2} + \frac{4y^3 U_\infty}{\delta^4}$$

A $y=0$, $\boxed{\left.\frac{\partial u}{\partial y}\right)_{y=0} = \frac{2U_\infty}{\delta}}$

As $\left.\frac{\partial u}{\partial y}\right)_{y=0}$ ~~is~~ is positive, hence flow will not separate or flow will remain attached with the surface.

ii) Boundary Layer thickness (in terms of Re)

$\Rightarrow$ By Von-Karman eq^n we can write

$$\frac{\tau_0}{\rho U_\infty^2} = \frac{\partial\theta}{\partial x} = \frac{\partial}{\partial x}\left[\int_0^\delta \left(\frac{2y}{\delta} - \frac{2y^2}{\delta^2} + \frac{y^4}{\delta^4}\right)\left(1 - \left(\frac{2y}{\delta} - \frac{2y^2}{\delta^2} + \frac{y^4}{\delta^4}\right)\right)\right] \quad \text{—(I)}$$

$$= \frac{\partial}{\partial x}\left[\int_0^\delta \left(\frac{2y}{\delta} - \frac{2y^2}{\delta^2} + \frac{y^4}{\delta^4}\right)\left(1 - \frac{2y}{\delta} + \frac{2y^2}{\delta^2} - \frac{y^4}{\delta^4}\right) dy\right]$$

$$= \frac{\partial}{\partial x}\left[\int_0^{\delta}\left(\frac{2y}{\delta} - \frac{2y^2}{\delta^2} + \frac{y^4}{\delta^4} - \frac{4y^2}{\delta^2} + \frac{4y^3}{\delta^3} - \frac{2y^5}{\delta^5} + \frac{4y^3}{\delta^3} - \frac{4y^4}{\delta^4} + \frac{2y^6}{\delta^6} - \frac{2y^5}{\delta^5} + \frac{2y^6}{\delta^6} - \frac{y^8}{\delta^8}\right) dy\right]$$

$$= \frac{\partial}{\partial x}\left[\int_0^{\delta}\left[\frac{2y}{\delta} - \frac{6y^2}{\delta^2} + \frac{8y^3}{\delta^3} - \frac{3y^4}{\delta^4} - \frac{4y^5}{\delta^5} + \frac{4y^6}{\delta^6} - \frac{y^8}{\delta^8}\right] dy\right]$$

$$= \frac{\partial}{\partial x}\left[\frac{2y^2}{\delta} - \frac{6y^3}{3\delta^2} + \frac{8y^4}{4\delta^3} - \frac{3y^5}{5\delta^4} - \frac{4y^6}{6\delta^5} + \frac{4y^7}{7\delta^6} - \frac{y^9}{9\delta^8}\right]_0^{\delta}$$

$$= \frac{\partial}{\partial x}\left(\delta - 2\delta + 2\delta - \frac{3}{5}\delta - \frac{2}{3}\delta + \frac{4}{7}\delta - \frac{1}{9}\delta\right)$$

$$= \frac{\partial}{\partial x}\left(\frac{61}{315}\delta\right)$$

$$\therefore \quad \frac{\tau_0}{\rho U_\infty^2} = \frac{61}{315}\frac{\partial \delta}{\partial x} \qquad \text{—— (II)}$$

Also shear stress is given by Newton's law of viscosity as

$$\tau_0 = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

where $u = \left(\frac{2y}{\delta} - \frac{2y^2}{\delta^2} + \frac{y^4}{\delta^4}\right) U_\infty$

$$\frac{\partial u}{\partial y} = U_\infty\left[\frac{2}{\delta} - \frac{4y}{\delta^2} + \frac{4y^3}{\delta^4}\right]$$

$$\left.\frac{\partial u}{\partial y}\right)_{y=0} = U_\infty \frac{2}{\delta}$$

$$\boxed{\therefore \quad \tau_0 = \mu \cdot \frac{2U_\infty}{\delta}} \qquad \text{—— (III)}$$

Substituting in eqn (II) we get

$$\frac{2\mu U_\infty}{\delta \cdot \rho U_\infty^2} = \frac{61}{315}\frac{\partial \delta}{\partial x}$$

$$\therefore \frac{2\mu \, \partial x}{\rho U_\infty} = \frac{61}{315}\delta \, \partial \delta$$

$$\frac{10.33 \, \mu \, \partial x}{\rho U_\infty} = \delta \, \partial \delta$$

Integrating above eqn, we get

$$\frac{10.33 \, \mu \, x}{\rho U_\infty} = \frac{\delta^2}{2} + C \quad \text{——— (IV)}$$

Substituting Boundary condition we can find const of integration C

$\therefore$ at $x=0$, $\delta = 0$ we get

$$\boxed{C = 0}$$

$\therefore$ Eqn (IV) becomes

$$\frac{10.33 \, \mu \, x}{\rho U_\infty} = \frac{\delta^2}{2}$$

$$\therefore \delta^2 = \frac{20.66 \, \mu x}{\rho U_\infty}$$

$$\delta^2 = \frac{20.66\, x^2\, \mu}{\rho\, U_\infty . x}$$

$$\therefore \delta^2 = \frac{20.66\, x^2}{Re_x} \qquad \left[Re_x = \frac{\rho U_\infty x}{\mu}\right]$$

$$\boxed{\therefore \delta = \frac{4.54\, x}{\sqrt{Re_x}}}$$

This is required bandary layer thickness in terms of Re.

Q.5) A). Given data

$V = 1000 \text{ Km/hr} = 277.78 \text{ m/s}$

$p_1 = -9.81 \text{ KN/m}^2$ (vacuum)

$T_1 = 47^\circ C = 320 K$

$P_{atm} = 98.1 \text{ KN/m}^2$

$R = 287 \text{ J/kg}^\circ K$

$\gamma = 1.4$

Find i) p_0 ii) T_0 iii) ρ_0

$\Rightarrow$

Pressure of air $p_1 = P_{atm} + P_{gauge}$

$p_1 = 98.1 + (-9.81) = 88.29 \text{ KN/m}^2$

velocity of sound $C_1 = \sqrt{\gamma RT}$

$C_1 = \sqrt{1.4 \times 287 \times 320}$

$\therefore C_1 = \underline{358.6} \text{ m/s}$

Mach number $M_1 = \frac{V_1}{C_1} = \frac{277.78}{358.6}$

$M_1 = \underline{\underline{0.775}}$

a) Stagnation pressure : p_0

$$\therefore p_0 = p_1 \left[1 + \left(\frac{\gamma - 1}{2} M_1^2\right)\right]^{\gamma/\gamma-1}$$

$$\therefore p_0 = 88.29 \left[1 + \frac{1.4-1}{2} \times (0.775)^2\right]^{\frac{1.4}{0.4}}$$

$$\boxed{\therefore p_0 = 131.27 \text{ KN/m}^2}$$

· Stagnation Temperature : T_0

$$T_0 = T_1 \left[1 + \left(\frac{\gamma - 1}{2}\right) M_1^2\right.$$

$$T_0 = 320 \left[1 + \left(\frac{1.4 - 1}{2}\right) \times 0.775^2\right]$$

$$\boxed{T_0 = 358.4 \text{ K} = 85.4^\circ C}$$

Stagnation Density : ρ_0

$$\rho_0 = \frac{P_0}{R T_0} = \frac{131.27}{287 \times 358.4} = 1.276 \text{ kg/m}^3$$

$$\boxed{\therefore \rho_0 = 1.276 \text{ kg/m}^3}$$

(B) Given velocity potential function is given by $\phi = x^3 - 3xy^2$

According to definition of potential function

$$u = \frac{-\partial \phi}{\partial x} = -\frac{\partial}{\partial x}(x^3 - 3xy^2) = -3x^2 + 3y^2$$

$$v = -\frac{\partial \phi}{\partial y} = -\frac{\partial}{\partial y}(x^3 - 3xy^2) = 6xy$$

To check condition for valid flow field let's apply continuity eq^n in two dimensional flow field

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\therefore \frac{\partial}{\partial x}(-3x^2 + 3y^2) + \frac{\partial}{\partial y}(6xy) = 0$$

$$-6x + 6x = 0$$

$$\boxed{\therefore 0 = 0}$$

~~the~~ Hence given velocity potential function satisfies the possible case of flow field.

According to definition of flow field

$$u = \frac{\partial \psi}{\partial y} = -3x^2 + 3y^2$$

$$v = -\frac{\partial \psi}{\partial x} = -6xy$$

$$\therefore \frac{\partial \psi}{\partial y} = -3x^2 + 3y^2$$

Integrals w.r.t y

$$\therefore \psi = -3x^2y + \frac{3y^3}{3} + c$$

$$\frac{\partial \psi}{\partial x} = -6xy$$

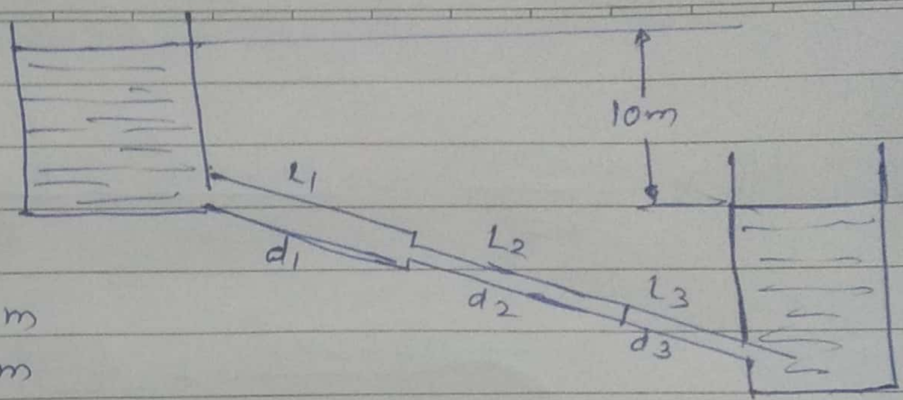
Integrals w.r.t x , we set

$$\psi = \frac{-6x^2y}{2} = -3x^2y + c$$

Stream function is given by

$$\therefore \boxed{\psi = -3x^2y + y^3}$$

Q.6 a) Given data



$L_1 = 500\ m$

$L_2 = 200\ m$

$L_3 = 100\ m$

$d_1 = 30\ cm = 0.3\ m$

$d_2 = 10\ cm = 0.1\ m$

$d_3 = 10\ cm = 0.1\ m$

$f_1 = 0.02$

$f_2 = 0.025$

$f_3 = 0.03$ Find : i) Q

→

Area of pipe 1; $A_1 = \pi/4\ d_1^2 = \pi/4\,(0.3)^2 = 0.07068\ m^2$

Area of pipe 2; $A_2 = \pi/4\ d_2^2 = \pi/4\,(0.1)^2 = 0.00785\ m^2$

Area of pipe 3; $A_3 = \pi/4\ d_3^2 = \pi/4\,(0.1)^2 = 0.00785\ m^2$

By Continuity eqn

$$Q = A_1V_1 = A_2V_2 = A_3V_3 \quad \text{——— (I)}$$

$$\therefore V_2 = A_1/A_2 \cdot V_1 = \frac{0.07068}{0.00785} = 9.0038V_1$$

$$\therefore V_3 = A_1/A_3\ V_1 = \frac{0.07068}{0.00785} = 9.0038V_1$$

Both reservoirs are connected by three pipes difference of head of 10 m. So by Bernoulli's principle total head loss between these two reservoirs is 10 m

$$\therefore H_L = 10\ m$$

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∴ Total losses takes place between two reservoirs is written as

$$H_L = \frac{0.5V_1^2}{2g} + \frac{f_1 L_1 V_1^2}{2gd_1} + \frac{K_1 V_2^2}{2g} + \frac{f_2 L_2 V_2^2}{2gd_2}$$

$$+ \frac{f_3 L_3 V_3^2}{2gd_3} + \frac{V_3^2}{2g} \quad \text{——— (II)}$$

$$10 = \frac{0.5V_1^2}{2g} + \frac{0.02 \times 500 V_1^2}{2g \times (0.3)} + \frac{0.5(9V_1)^2}{2g}$$

$$+ \frac{0.025 \times 200 \times (9V_1)^2}{2g \times (0.1)} + \frac{0.03 \times 100 (9V_1)^2}{2g \times (0.1)}$$

$$+ \frac{(9V_1)^2}{2g}$$

$$10 = \frac{V_1^2}{2g}\left[0.5 + \frac{0.02 \times 500}{0.3} + \frac{0.5 \times 81}{1} + \frac{0.025 \times 200 \times 81}{0.1} + \frac{0.03 \times 100 \times 81}{0.1} + 81\right]$$

$$\therefore 10 = 338.188\, V_1^2$$

$$\therefore \boxed{V_1 = 0.172 \text{ m/s}}$$

$$\boxed{V_2 = 1.55 \text{ m/s}}$$

substituting in eqn (I)

$$\therefore Q = A_1 V_1$$

$$\therefore Q = 0.07068 \times 0.172$$

$$\therefore Q = 0.0122 \text{ m}^3/\text{sec}$$

$$\boxed{\therefore Q = 12.2 \text{ lit/sec}}$$