External (Scheme A) (3 Hours) [Total Marks:100
Internal (Scheme B) (2 Hours) [Total Marks:40
Note:

(1) External (Scheme A) students answer any five questions.

(2) Internal (Scheme B) students answer any three questions.

(3) All questions carry equal marks. Scientific calculator can be used.

(4) Write on top of your answer book the scheme under which you are appearing.

Que. 1 (a) Define: Absolute error and Percentage error.
Evaluate the sum $S = \sqrt{5}+\sqrt{7}+\sqrt{11}$ upto 4 significant digits and find its absolute and relative errors.

(b) Convert the decimal fraction $(391.6875)_{10}$ to the binary form and then convert to the octal form.

Que. 2 (a) Define the term rate of convergence of iterative method and also find the rate of convergence of the Iteration method.

(b) Perform two iterations of the Birge-Vieta method to find a root (correct upto four decimal places) of the equation $x^3+2x^2+10x-20=0$. Use initial approximation $p_0 = 1$.

Que. 3 (a) Describe Crout's method to solve the following system of linear equations:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3. \end{aligned}$$

(b) Find the Singular Value Decomposition of the matrix

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

Que. 4 (a) Estimate the error in Newton's backward difference interpolation formula.

(b) From the following data obtain the first and second derivatives of $y = \log_e x$ at $x = 550$.

x:	510	520	530	540	550
y:	6.2344	6.2538	6.2729	6.2916	6.3099.

Que. 5 (a) Derive Newton-Cotes quadrature formula and use it to derive Simpson's three eighth rule for numerical integration.

(b) Use Romberg's method to evaluate $\int_0^1 \frac{1}{1+x^2}\,dx$. Take $h = 0.5, 0.25, 0.125$.

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Que. 6 (a) Using the least-squares method, obtain the normal equations to find the values of a, b and c when the curve $y = c + bx + ax^2$ is to be fitted for the data points $(x_i, y_i), i = 1, 2, 3....., n$.

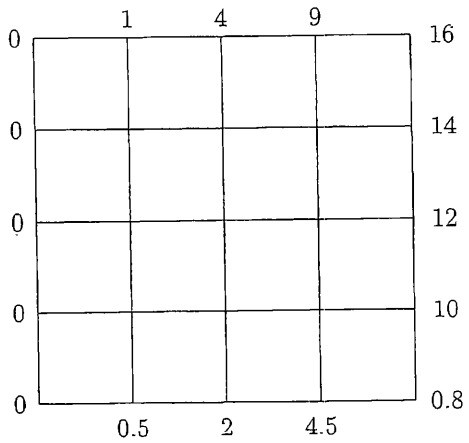
(b) Using Chebyshev polynomials, obtain the least squares approximation of second degree for $f(x) = x^4 + x^3 - x - 9$ on $[-1, 1]$ with respect to the weight function $w(x) = \frac{1}{\sqrt{1-x^2}}$.

Que. 7 (a) Derive the Adams-Bashforth corrector formula to solve the differential equation $\frac{dy}{dx} = f(x, y)$ with $y(x_0) = y_0$.

(b) Use Milne's method to compute $y(0.8)$ correct upto four decimal places, given that $\frac{dy}{dx} = 1 + y^2$ with $y(0) = 0,\ y(0.2) = 0.2027,\ y(0.4) = 0.4228,\ y(0.6) = 0.6841$.

Que. 8 (a) Derive a Crank-Nicolson's numerical method to obtain the numerical solution of one dimensional heat equation with initial and boundary conditions.

(b) Use Liebmann's method to solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ at the interior mesh points of the square region with boundary values given in the following figure.



[Take 2 iterations and obtain result correct upto three decimal places.]