

QP Code : 78991

(2½ Hours)

[Total Marks : 75

- N.B. :** (1) All questions are compulsory.
(2) Figures to the right indicate full marks.
(3) Use of calculators is allowed.

1. (a) (i) Define the following terms : 3
(1) Random Experiment
(2) Mutually Exclusive Events
(3) Equally Likely Events
(ii) Two fair dice are tossed 4
(1) Find the probability of getting 'an even number on the first die or a total of 8'.
(2) Find the probability that the sum is neither 7 nor 11.
(b) (i) Give Mathematical definition of probability of an event. 2
(ii) State addition theorem for two events A and B. Derive the result for three events A, B and C. Write down the modified formula if A, B, C are mutually exclusive and exhaustive events. 6

OR

1. (p) (i) Define the following terms : 3
(1) Outcome
(2) Sample Space
(3) Complementary Events
(ii) What is a run? In an arrangement of ' r_1 ' α 's and ' r_2 ' β 's, find the probability of having ' n_1 ' α runs and ' n_1 ' β runs. 4
(q) (i) Obtain an expression for number of distinguishable distributions of putting ' r ' indistinguishable balls in ' n ' cells. 4
(ii) A car is parked among N cars in a row, not at either end. On his return the owner finds that exactly r of the N places are still occupied. What is the probability that both neighbouring places are empty? 4
2. (a) State and prove the expression for the probability of simultaneous occurrence of exactly ' m ' events out of N events A_1, A_2, \dots, A_N where ' m ' is an integer such that $1 \leq m \leq N$. 7

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- (b) The chance that doctor A will diagnose a disease X correctly is 60%. The probability that a patient will die by his treatment after correct diagnosis is 0.4 and the probability of death by wrong diagnosis is 0.7. A patient of doctor A, who had disease X, died. What is the probability that his disease was diagnosed correctly? 3
- (c) State and prove Bayes' Theorem. 5

OR

2. (p) State and prove the expression for the probability of occurrence of at least one event out of N events A_1, A_2, \dots, A_N . 7
- (q) A letter is known to have come either from TATANAGAR or from KOLKATA. On the envelope just two consecutive letters TA are visible. What is the probability that the letter came from KOLKATA? 3
- (r) Let A and B be two events such that $P(A) = \frac{3}{4}$ and $P(B) = \frac{5}{8}$. Show 5
that (i) $P(A \cup B) \geq \frac{3}{4}$ and (ii) $\frac{3}{8} \leq P(A \cap B) \leq \frac{5}{8}$.

3. (a) Define joint moment generating function of two random variables. X and Y. Explain, how it can be used to generate the moments. 5
- (b) The joint p.d.f. of X and Y is : 10

$$f(x, y) = \frac{e^{-(a+b)} a^x b^{y-x}}{x!(y-x)!} ; \quad x = 0, 1, 2, \dots$$

$$= 0 \text{ Otherwise} ; \quad y = x, x+1, x+2, \dots$$

Find :

- (i) The joint moment generating function of (X, Y).
(ii) The marginal distribution of X and of Y.
(iii) The p.d.f. of Y-X.

OR

3. (p) Define trinomial distribution. If the joint distribution of (X, Y) is trinomial, Obtain : 8
- (i) marginal distribution of X .
 - (ii) Conditional distribution of Y given $X = x$.
 - (iii) The p.d.f. of $X+Y$.
- (q) Three fair dice are cast n times. Let X denote the number of times all three faces are alike and let Y denote the number of times only two faces are alike. Find the joint p.d.f. of (X, Y) : 7
- Also state,
- (i) Marginal p.d.f. of X and of Y
 - (ii) The p.d.f. of $X+Y$.
4. (a) Based on a random sample of size n drawn from a population of the continuous type : 8
- (i) Define order statistics
 - (ii) Derive the joint probability density function of the r^{th} and s^{th} order statistics where $1 \leq r \leq s \leq n$.
- (b) A random sample of size n is drawn from a population whose distribution is exponential with mean 1. Obtain density function of sample range. Hence or otherwise find the probability that the sample range is more than 1. 7
- OR**
4. (p) Define r^{th} order statistic in a sample of size n from a continuous population. Derive its distribution function. 7
- (q) A random sample of size n is drawn from uniform distribution on $(0, 1)$. Find the correlation coefficient between Y_1 and Y_n order statistics. 8
- Where Y_1 : 1st order statistic,
 Y_n : n^{th} order statistic.
5. (a) The odds that person X speaks the truth are 3:2 and the odds that person Y speaks the truth are 5:3. In what percentage of cases are they likely to contradict each other on an identical point. 6
- (b) Define multinomial distribution. State its moment generating function. Hence find correlation coefficient between X_i and X_j ($i \neq j$). 9

TURN OVER

OR

5. (p) State and prove multiplication theorem for three events. 5
(q) Five observations are randomly drawn from a population with p.d.f. 10

$$f(x) = 2(1-x) \quad ; \quad 0 < x < 1$$
$$= 0 \quad ; \quad \text{Otherwise}$$

Find the probability that :

- (i) The sample median is more than $\frac{1}{2}$
 - (ii) Fourth order statistics is less than $\frac{1}{4}$
 - (iii) Fifth order statistics is greater than $\frac{3}{4}$.
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