QP Code : 78959

(21/2 Hours) [Total Marks: 75

N.B.: (1) All questions are compulsory.
(2) Figures to the right indicate marks for respective subquestions.

1. (a) Attempt any *one* of the following: [8]

(i) Show that the rate of convergence of the secant method is $\frac{1}{2}(1 \pm \sqrt{5})$.

(ii) Show that the Newton – Raphson iterative formula applied to the function $f(x)= x^2 - a$, $a > 0$ leads to the iterative formula

$$x_{k+1} = \frac{1}{2}(x_k + \frac{a}{x_k}),\ x_0 > 0$$

for evaluating $\sqrt{a}$. Also for the function $f(x)= x^p - a$, show that the sequence given by,

$x_{k+1} = \frac{1}{p}((p-1)x_k + \frac{a}{x_k^{p-1}})$, $x_0 > 0$ can be used to evaluate $a^{\frac{1}{p}}$.

(b) Attempt any *three* of the following: [12]

(i) Find the number of terms n to be taken in the expansion of e^x correct to 7 places of decimals, when x = 1.

(ii) Apply Muller's method, with $x_0= 1.9$, $x_1 = 2$ and $x_3= 2.1$ to find the root of $x^3 - 2x - 5 = 0$, perform one iteration.

(iii) Taking $x_0 = 0$ and $x_1 = 1$, solve by Regula- Falsi method the equation $x - \cos x = 0$. Perform two iterations.

(iv) Using the Fixed point iterative method, is it possible to find a root of $x^3 - 10x - 5 = 0$? If yes determine it. Perform one iteration.

2. (a) Attempt any *one* of the following: [8]

(i) If p_k is an approximation of the root p of the polynomial equation $P_n(x) = a_0x^n + a_1x^{n-1} + \ldots + a_{n-1}x + a_n = 0$, then show that the next approximation to the root using Birge-Vieta method is $p_{k+1} = p_k - \frac{b_n}{c_{n-1}}$, k = 0, 1, 2... where b_k satisfies the recurrence relation $b_k = a_k + p\,b_{k-1}$ with $b_0 = a_0$ and c_k satisfies the recurrence relation $c_k = b_k + pc_{k-1}$ and $c_0 = b_0$.

(ii) If $x^{(k)}$ is the approximation to the solution x of the system of linear equations Ax = b then show that the next approximation $x^{(k+1)}$ of x using Jacob iterative method is obtained by
$x^{(k+1)} = x^{(k)} + v^{(k)}$, where $v^{(k)} = D^{-1}r(k)$, $r^{(k)} = b - Ax^{(k)}$ and D the diagonal part of A.

(b) Attempt any *three* of the following: [12]

(i) Using synthetic division find the value of P(2), P ′(2) for the polynomial
$x^5 - 2x^4 + 4x^3 - x^2 - 7x + 5 = 0$.

(ii) Apply Graeffe's root squaring method to find root of $x^3 - 2x + 2 = 0$. Perform upto second squaring.

(iii) Solve the following system of equations using Triangularization method by assuming $u_{ii} = 1$:
$3x + 4y = 10,\ 4x + 3y = 11.$

(iv) Solve the following system of equations using Jacobi iterative method. Do one iteration.
$10x_1 + 2x_2 + x_3 = 9,\ x_1 + 10x_2 - x_3 = -22,\ -2x_1 + 3x_2 + 10x_3 = 22.$

3. (a) Attempt any *one* of the following: [8]

(i) Let A be a real symmetric matrix. Using Jacobi method reduce A to a diagonal matrix by a series of orthogonal transformations $S_1, S_2, \ldots$ in 2 × 2 subspaces. Let $|a_{ik}|$ be the numerically largest off diagonal element of the matrix S_1^* such that $S_1^*A\,S^*$ is diagonalized and hence show that

$$\tan 2\theta = \frac{2a_{ik}}{a_{ii} - a_{kk}},\ -\frac{\pi}{4} \le \theta \le \frac{\pi}{4}.$$

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(ii) Define condition number of a matrix A. Let $A(\alpha) = \begin{bmatrix} 0.1\alpha & 0.1\alpha \\ 1.00 & 1.5 \end{bmatrix}$. Determine α such that cond(A(α)) is minimized. Use maximum absolute row sum norm.

(b) Attempt any ***three*** of the following: **[12]**

(i) Let A be a symmetric matrix given below. Apply Jacobi method to find orthogonal transformations S_1 and S_2. Do one iteration ; $A = \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$

(ii) Determine eigen value for $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ using Rutishauser method. Do three iterations.

(iii) Find the largest eigen value and eigen vector in magnitude of the following matrix using Power method at the end of second iteration ; $\begin{bmatrix} 1 & 1 \\ 5 & 3 \end{bmatrix}$

(iv) Find the smallest eigen value and eigen vector in magnitude of the matrix A using inverse power method at the end of second iteration ; $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{bmatrix}$.
Take initial approximation as $[1\ 1\ 1]^T$ near 2.

4. Attempt any ***three*** of the following: **[15]**

(i) Solve the equation $x^3 - 45x = 0$ using Newton- Raphson method with initial approximation $x_0 = 3$.Do three iterations.

(ii) Define spectral radius for square matrix and hence determine spectral radius for

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

(iii) Find the inverse of the following matrix by Cholesky method ,

$$\begin{pmatrix} 1 & -1 & 2 \\ -1 & 4 & 6 \\ 2 & 6 & 29 \end{pmatrix}$$

(iv) Using Strum's sequence obtain the exact number of real root and complex roots of the polynomial $x^3 + x - 1 = 0$.

(v) Find a root by False–Position method correct upto 4 decimal places for the equation $e^{-x} = \sin x$.

(vi) Obtain first iteration matrix (A_2) and the eigenvalues of the matrix $A = \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}$ using Rutishauser method.

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