External (Scheme A)	(3 Hours)	Total Marks: 100
Internal (Scheme B)	(2 Hours)	Total Marks: 40

N.B.:Scheme A students should attempt any five questions.
Scheme B students should attempt any three questions.
Write the scheme under which you are appearing, on the top of the answer book.

Q.1.	a)	Prove that the set of all real numbers is uncountable.	10
	b)	Let f : X→ Y . Prove that f is bijective iff f(X\ A) = Y \ f(A) for A⊏ X.	10
Q.2	a)	Define subspace topology. If $\mathcal{B}$ is a basis for the topology of X then prove that the collection $B_Y = \{B\cap Y / B\epsilon\mathcal{B}\}$ is a basis for the subspace topology on Y .	10
	b)	Let X be a topological space and A ⊂_ X. Prove that the following statements are equivalent: (i) A is open (ii) A =$A°$.	10
Q.3	a)	Define a connected topological space. Let X be a topological space and A be a subset of X. Show that if A is connected then its closure A is also connected.	10
	b)	Define a path connected topological space. Prove or disprove: Every connected space is path connected. Justify your answer.	10
Q.4.	a)	Define connected topological space. Prove that the cartesian product of two connected spaces is connected.	10
	b)	Give an example of a continuous bijection from one topological space to the other, which is not a homeomorphism.	10
Q.5	a)	Show that a limit point compact metric space is sequentially compact.	10
	b)	Prove that the continuous image of a compact metric space is compact.	10
Q.6	a)	Define a dense set. Let X be a topological space with countable basis. Show that: (i) Every open covering of X has a countable subcollection covering X. (ii) There exists a countable subset of X that is dense in X.	10
	b)	Show that every open subset of ℝ is the union of disjoint sequence of open intervals.	10
Q.7	a)	Define a quotient map. Prove that if p : X → Y is a continuous, surjective and open map then it is a quotient map.	10
	b)	Prove that every compact subset of a Hausdroff space is closed.	10
Q.8.	a)	State and prove path lifting lemma	10
	b)	Let f and g be two paths in a topological space with same initial point and same end point. Define the path homotopy relation $\sim_p$ and show that it is an equivalence relation.	10
