External (Revised) (3 Hours) [Total marks: 80]

Instructions:

1) Attempt any two questions from each section.
2) All questions carry equal marks.
3) Answer to Section I and section II should be written in the same answer book.

SECTION-I (Attempt any two questions)

Q.1] A) Let p be an odd price and $\gcd(a, p) = 1$. Prove that " 'a' is a quadratic residue of p if and only if $a^{(p-1)/2} \equiv 1(mod\ p)$". [10]

B) Use Cardanos method to find the roots of the cubic equation $64x^3 - 48x^2 + 12$. [10]

Q.2] A) i) Determine the number of ways to put k indistinguishable balls into n indistinguishable boxes. [05]

ii) How many non-negative integer solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 67$? [05]

B) In a survey of students it was found that 80 students knew Marathi, 60 knew English, 50 knew Hindi , 30 knew Marathi and English, 20 knew English and Hindi, 15 knew Marathi and Hindi and 10 knew all the three languages. How many students knew a) At least one language , b) Only Marathi and c) English but not both English and Hindi. [10]

Q.3] A) Let m and n be relative prime positive integer then prove that the system $x \equiv a(mod\ m)$ and $x \equiv (mod\ n)$ has a solution. [10]

B) A chess master who has 11 weeks to prepare for a tournament decides to play at least one game every day but, order not to tire himself, he decides not to play more than 12 games during any calendar weeks. Show that there exist a succession of consecutive days during which the master will have played exactly 21 games. [10]

Q.4] A) State and prove De-Morgans law for Boolean expressions in two variables. [10]

B) Show that a simple graph G is a tree if and only if any two distinct vertices are connected by a unique path. [10]

SECTION-II (Attempt any two questions)

Q.5] A) Verify the conditions of the existence and uniqueness theorem to conclude that the initial value problem $\frac{dy}{dx} = 3y + 1,\ y(0) = 2$ has a solution. Find the domain where the solution exists and use Picard iteration scheme to compute the first four approximations of the solution. [10]

B) Prove that " If $f: I \times \Omega \to \mathbb{R}^n$ has the locally Lipschitiz property then the initial value problem $\frac{dy}{dx} = f(t,x),\ x(t_0) = x_0$ has a solution $X: (t_0 - \delta, t_1 + \delta) \to \Omega$". [10]

Q.6] A) Solve the initial value problem :

$$\frac{dx}{dt} = 3x + 4y, \qquad x(1) = 2$$

$$\frac{dy}{dt} = -4x + 3y, \qquad y(1) = 3$$

$$\frac{dz}{dt} = yz + 3, \qquad z(1) = 4.$$

[10]

B) Show that $\{\emptyset_1, u_2\emptyset_1, u_3\emptyset_1, \dots \dots, u_n\emptyset_1\}$ is a basis for solution of $L_n(y) = 0$ on I where $\emptyset_1(x) \neq 0$ on I and $V_k = u'_k$ (k=1,2,3,……,n) are linearly independent solution of $\emptyset_1 V^{n-1} + \cdots \dots + (n{\emptyset_1}^{n-1} + a_1{\emptyset_1}^{n-2} + \cdots \dots + a_{n-1}\emptyset_1)V = 0.$ [10]

Q.7] A) Solve $y'' - 2xy' + y = 0,\ y(0) = 0,\ y'(0) = 1$ using power series. [10]

B) Solve Bessel's equation $x^2y'' + xy' + (x^2 - p^2)y = 0, \quad p \geq 0.$ [10]

Turn Over

Q.8] A) For the partial differential equation $u_y = {u_x}^3$ find the solution satisfying

$u(x,0) = 2x^{3/2}$. [10]

B) Find the integral surface of the equation $(2xy-1)p + (z-2x^2)q = 2(x-yz)$,

which passes through the line $x_0(t) = 1$, $y_0(t) = 0$, $z_0(t) = t$. [10]

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