Q.P.Code: 11391

( 2 ½ hours) Total Marks : 60

**N.B (1)** All Questions are compulsory, and carry equal marks.

(2) In first four questions, attempt any two subquestions from each question.

(3) In question five attempt any four subquestions.

(3) All subquestions of each question carry equal marks.

(4) The norm of vector x is usually denoted by | x | and in the case of Function spaces the norm of a mapping f, is usually denoted by || f ||.

Q (1 )

(a) Define a Baire space. Show that a compact Hausdroff space is a Baire space.

(b) Define equicontinuous family of mappings. Show that if a family $\mathbb{F}$ of complex valued continuous functions defined on a compact metric space is compact in the space C(X) of continuous complex valued functions on a compact metric space X, then $\mathbb{F}$ is uniformly bounded.

( c) i) Is Cantor set a Baire space ? Is a Discrete space a Baire space ?

ii) Show that a locally compact Hausdroff space is a Baire space.

[Please Turn over

Q (2)

(a) Define a normed linear space and a Banach space. Show that $\mathbb{C}$ is a Banach space over $\mathbb{R}$.

(b) Let Y be a closed proper linear subspace of a normed linear space X. Then given any r, $0 < r < 1$, show that there exists a unit vector $x_r \in X$, such that $r < d(x_r, Y)$.

(c) Show that any two norms are equivalent on $\mathbb{R}^n$.

Q (3)

(a) Let Y be a closed linear subspace of normed linear space X. Show that he quotient map is bounded linear map, and if Y and X/Y are both complete then X is complete.

(b) Let B(X, Y) denote the space of bounded linear maps from a Banach space X to a Banach space Y. Show B(X,Y) is a Banach space.

(c) i) Suppose X and Y are normed spaces. Show that projections of the product space $X \times Y$ on X, are bounded linear mappings.

ii) Give an example of an unbounded linear transformation.

Q (4)

(a) State and Prove Hahn Banach theorem for real vector spaces.

(b) Define graph of a linear transformation. State and Prove closed graph theorem.

(c) State and Prove the uniform bounded ness principle. [Please turn over

Q (5)

(a) Discuss for equicontinuity. $\mathbb{F} = \{ f_n ; f_n(x) = \sin nx \}, x \in X = [0, 2\pi]$.

(b) i) Let X be a normed linear space. Show that the norm is continuous function.

ii) Show that a linear transformation is continuous if and only if it is continuous at the origin.

(c) Let X be a normed linear space and $x_0 \neq 0$ be an arbitrary vector in X. Then there exists a bounded linear functional f in X*, such that $\| f \| = 1$, and $f(x_0) = |x_0|$.

(d) X = C[0, 1] under sup norm. T:X → K is defined as, $T(f) = \int_0^1 f(x)\, dx$ .

Show that Y is bounded . Find the norm of T.

e) Show that $l^\infty$ is a normed linear space.

f) State open mapping theorem.

Show that the graph of a bounded linear transformation from a Banach space X to a Banach space Y is a closed subspace of X × Y.

****************************************************************************