Revised **(3 Hours)** **Total Marks : 80**

Instructions :

- **Attempt any two questions from each section**
- **All questions carry equal marks. Scientific calculator can be used.**
- **Answers to section I and section II should be written in the same answer book**

Section I(Attempt any two questions)

Q1 a) Define: Absolute error, Relative error and Percentage error. Find the Relative and Absolute error in calculation of $Z = 3x - 3$ by taking approximate value of x as 3.45, and true value of x as 3.457.

b) i) Convert decimal number $(0.859375)_{10}$ to corresponding binary number.
ii) Convert binary number $(10110101.110011100)_2$ to Octal number.

Q2 a) Prove that Newton Raphson method has quadratic rate of convergence. Hence find correct root using Newton-Raphson Method for $f(x) = x^4 - x - 10$ with initial approximation $x_0 = 1$ upto two decimal places.

b) Derive the Muller's formula to find a root of the algebraic or transcendental equation $f(x) = 0$. Perform one iteration with muller method for

$$f(x) = x^2 + x - 1 \ \& x_0 = 0, x_1 = 0.5, x_2 = 1$$

Q3 a) Solve the system by using cholesky method

$12x + 4y - z = 15$
$4x + 7y + z = 12$
$-x + y + 6z = 6$

b) Determine the largest eigenvalues and the corresponding eigenvector of the matrix $\begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix}$ correct to three decimal places using power method. Take the initial approximate vector as $v^{(0)} = [1 \quad 1]^t$.

Q4 a) Obtain the Newton's forward interpolating polynomial, for the following tabular data and interpolate the value of the function at $x = 0.0045$.

x	0	0.001	0.002	0.003	0.004	0.005
y	1.121	1.123	1.1255	1.127	1.128	1.1285

b) From the following table, find x for which y is minimum and find this value of y.

x	3	4	5	6	7
y	2.7	6.4	12.5	21.6	34.3

Section II (Attempt any two questions)

Q5 a) Derive two point Guassian quadrature formula to evaluate the integral $\int_{-1}^{1} f(x)\,dx$.

b) Evaluate $\int_0^{\pi} \frac{\sin^2 x}{5 + 4\cos x} dx$ by taking 5 ordinates by Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule.

Q6 a) Obtain the least squares approximation of second degree for $f(x) = sinx$ on $[0, \frac{\pi}{2}]$ with respect to the weight function $w(x) = 1$.

b) Explain the term Discrete Fourier Transform(D.F.T) and compute the (4-point) D.F.T of the sequence x = (1,2,3,4)

Q7 a) Derive the Milne's Method to solve the differential equation $\frac{dy}{dx} = f(x, y)$with $y(x_0) = y_0$.

b) Solve

$$\frac{dx}{dt} = y - t, \qquad \frac{dy}{dt} = x + t$$

With $x(0) = 1, y(0) = 1$ for $x(0.1)$and $y(0.1)$ by Runge –Kutta Method.

Q8 a) Derive the Bender-Schmidt method to obtain the numerical solution of one dimensional heat equation with initial and boundary conditions.

b) Solve $u_t = u_{xx}$ subject to the initial condition $u(x, 0) = sin\pi x \; \forall \; x \in [0,1]$ and $u(0, t) = 0, u(0, t) = 1 \; \forall \; t > 0$ by the Gauss-Seidel Method.
