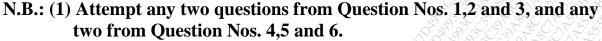
Q. P. Code: 11362

## (3 hours)

[Total marks: 60]



- (2) Figures to the right indicate full marks.
- (3) Simple non-programmable calculator is allowed.
- (a) An urn contains N<sub>1</sub>, N<sub>2</sub>,...,N<sub>k</sub> items of type 1,2,...,k respectively. In a random sample of size n drawn without replacement, let X<sub>i</sub> denote the number of items of type i, where i = 1,2,...,k. Obtain:

   joint probability distribution of X<sub>1</sub>, X<sub>2</sub>,..., X<sub>k</sub>.
   Cov(X<sub>1</sub>, X<sub>2</sub>)
  - (b) From an urn containing b black and r red balls, balls are drawn one by one with replacement until k black balls are obtained. Let Y denote the number of red balls drawn. Obtain probability distribution of Y and its cumulant generating function. Hence deduce its mean, variance and β1.
  - (c) An unbiased coin is tossed indefinitely. Let  $Y_1$  denote the length of the first run. Obtain  $E(Y_1)$ .
- (a) The failure time of an equipment is exponential with mean λ.
  i) State and prove the forgetfulness property of this distribution.
  ii) Show that the number of failures in a given time t follows Poisson distribution.
  iii) Obtain pdf of the sample range based on a random sample of size n.
  - (b) Show that iid random variables  $X_1, X_2, ..., X_n$  are geometric if and only if Minimum( $X_1, X_2, ..., X_n$ ) is geometric. (05)
- 3. (a) i) Obtain mean deviation of a random variable following N(0,1). (05) ii) Let  $X_1$ ,  $X_2$  be a random sample of size 2 from N(0,1). Obtain expectation of Maximum( $X_1$ ,  $X_2$ ).
  - (b) i) If X and Y are iid N(0,1) random variables, obtain pdf of U = X/Y.
    ii) State the characteristic function of Cauchy distribution with location parameter μ and scale parameter σ.
    iii) Hence deduce the distribution of average of n independent and identical Cauchy random variables.
  - (c) Explain how to simulate random observations from discrete uniform distribution over the set  $\{1,...,n\}$ .
- 4. (a) State and prove Bhattachrya's bound . (07) **P.T.O.**

Q. P. Code: 11362

- (b) Let  $X_1, X_2, ..., X_n$  be a random sample of size n drawn from the normal distribution with mean  $\mu$  and variance  $\sigma^2$ , both unknown.  $S^2 = \sum_{i=1}^n (x_i \bar{x})^2$ . Three estimators of  $\sigma^2$  are defined as  $T_1 = \frac{s^2}{n}, T_2 = \frac{s^2}{n-1}, T_3 = \frac{s^2}{n+1}$ . Find the M.S.E of each of them, compare their rate of convergence.
- 5. (a) Prove that sample quantiles are consistent estimators of population quantiles. (07)
  - (b) Let  $X_1, X_2, ..., X_n$  be a random sample from Bernoulli with parameter p. (08) Obtain Jack-knife estimator of  $p^2$ .
- 6. (a) Let  $X_1, X_2, ..., X_n$  be a random sample from exponential distribution with mean . Obtain Cramer Rao lower bound for variance of unbiased estimator of  $\frac{e^{\theta}}{\theta}$ .
  - (b) State the Pitman estimator for scale parameter. Further obtain Pitman estimator for  $\sigma^r$ , if  $X_1, X_2, ..., X_n$  is a random sample from  $f(x|\sigma) = \frac{2}{\sigma} \left(1 \frac{x}{\sigma}\right), 0 < x \le \sigma$ .

\*\*\*\*\*\*\*\*\*\*\*