Q. P. Code: 11362

(3 hours) **[Total marks: 60]**

N.B.: (1) Attempt any two questions from Question Nos. 1,2 and 3, and any two from Question Nos. 4,5 and 6.
(2) Figures to the right indicate full marks.
(3) Simple non-programmable calculator is allowed.

1. (a) An urn contains $N_1, N_2, \ldots, N_k$ items of type 1,2,…,k respectively. In a random sample of size n drawn without replacement, let X_i denote the number of items of type i, where i = 1,2,…,k. Obtain: (05)
i) joint probability distribution of $X_1, X_2, \ldots, X_k$.
ii) $Cov(X_1, X_2)$

(b) From an urn containing b black and r red balls, balls are drawn one by one with replacement until k black balls are obtained. Let Y denote the number of red balls drawn. Obtain probability distribution of Y and its cumulant generating function. Hence deduce its mean, variance and β_1. (07)

(c) An unbiased coin is tossed indefinitely. Let Y_1 denote the length of the first run. Obtain $E(Y_1)$. (03)

2. (a) The failure time of an equipment is exponential with mean λ. (10)
i) State and prove the forgetfulness property of this distribution.
ii) Show that the number of failures in a given time t follows Poisson distribution.
iii)Obtain pdf of the sample range based on a random sample of size n.

(b) Show that iid random variables $X_1, X_2, \ldots, X_n$ are geometric if and only if Minimum$(X_1, X_2, \ldots, X_n)$ is geometric. (05)

3. (a) i) Obtain mean deviation of a random variable following N(0,1). (05)
ii) Let X_1, X_2 be a random sample of size 2 from N(0,1). Obtain expectation of Maximum(X_1, X_2).

(b) i) If X and Y are iid N(0,1) random variables, obtain pdf of U = X/Y. (05)
ii) State the characteristic function of Cauchy distribution with location parameter μ and scale parameter σ.
iii) Hence deduce the distribution of average of n independent and identical Cauchy random variables.

(c) Explain how to simulate random observations from discrete uniform distribution over the set {1,…,n}. (05)

4. (a) State and prove Bhattachrya's bound . (07)

P.T.O.

(b) Let $X_1, X_2, \dots, X_n$ be a random sample of size n drawn from the normal distribution with mean μ and variance σ^2, both unknown. $S^2 = \sum_{i=1}^{n}(x_i - \bar{x})^2$. Three estimators of σ^2 are defined as $T_1 = \frac{S^2}{n}$, $T_2 = \frac{S^2}{n-1}$, $T_3 = \frac{S^2}{n+1}$. Find the M.S.E of each of them, compare their rate of convergence . (08)

5. (a) Prove that sample quantiles are consistent estimators of population quantiles. (07)

(b) Let $X_1, X_2, \dots, X_n$ be a random sample from Bernoulli with parameter p. Obtain Jack-knife estimator of p^2. (08)

6. (a) Let $X_1, X_2, \dots, X_n$ be a random sample from exponential distribution with mean . Obtain Cramer Rao lower bound for variance of unbiased estimator of $\frac{e^\theta}{\theta}$. (07)

(b) State the Pitman estimator for scale parameter. Further obtain Pitman estimator for σ^r, if $X_1, X_2, \dots, X_n$ is a random sample from $f(x|\sigma) = \frac{2}{\sigma}\left(1 - \frac{x}{\sigma}\right), 0 < x \leq \sigma.$ (08)

****