Q. P. Code: 11354

(3 hours) **[Total marks: 60]**

N.B.: (1) Attempt any two questions from Question Nos. 1, 2 and 3, and any two from Question Nos. 4,5 and 6.
(2) Figures to the right indicate full marks.
(3) Simple non-programmable calculator is allowed.

1. (a) Define $E(X)$, the expectation of an arbitrary random variable X. Let $F(x)$, the distribution function of X be $0, 0.2 + x$ and 1 according as $x < 0, 0 \le x < 0.4$ and $0.4 \le x$. Find Variance of X. (05)

(b) Define convergence in probability and almost sure convergence. Let $X_1, \dots, X_n, \dots$ be iid $N(0,1)$. Prove that $(X_1 + \cdots. + X_n)/n$ converges in probability to 0. Does it converge almost surely? (05)

(c) Let $\{X_n\}$ be a sequence of independent random variables with $P\{X_n = 0\} = 1 - \frac{1}{n}$ and $P\{X_n = 1\} = \frac{1}{n}$. Discuss i) convergence in distribution ii) convergence in probability and iii) almost sure convergence of $\{X_n\}$. (05)

2. (a) State and prove Holder's inequality and hence derive Cauchy-Schwartz inequality. (05)

(b) State and prove Jensen's inequality. For a positive random variable X, prove or disprove $E(\log X) \le \log E(X)$. (05)

(c) State and prove central limit theorem for sequence of iid random variables. (05)

3. (a) Describe Markovian property of a sequence of random variables. Let $\{X_n : n = 0,1,2, \dots\}$ be a sequence of iid discrete uniform random variables taking values -1,0 and 1. Prove that $S_n = X_1 + \cdots + X_n$ has Markovian property. (05)

(b) Let $\{X_n : n = 0,1,2, \dots\}$ be a Markov Chain with states space $\{1,2\}$ and transition probability matrix (tpm) $\begin{bmatrix} 0.6 & 0.4 \\ 0.4 & 0.6 \end{bmatrix}$ and the initial distribution is uniform. Find (i) $P\{X_{n+k} = 2 | X_n = 2\}$ (ii) $f_{12}^{(n)}$, probability of being in 1 reaching 2 for the first time after n steps (iii) the stationary distribution (iv) Periodicity of states (v) the marginal distribution of X_2. (10)

P.T.O.

4. (a) In circular systematic sampling, in usual notations, prove that mean of circular systematic sample is an unbiased estimator of population mean with variance given by (07)

$$V(\bar{y}) = \frac{\sigma^2}{n}\left[1+\rho(n-1)\right]$$

Derive the condition when circular systematic sampling is more precise than simple random sampling without replacement.

(b) In cluster sampling with unequal cluster sizes, define i) estimator based on cluster means ii) ratio estimator iii) probability proportional to size estimator of population mean per element. In each case obtain its variance and estimator of the variance. (08)

5. (a) In two stage sampling when units are drawn from the population using simple random sampling with replacement at both stages, show that, sample mean per element is an unbiased estimator of population mean per element. Derive its variance. Also obtain estimator of variance. (10)

(b) In double sampling (in usual notations) if cost function is of the form $C = nC_n + n' C_{n'}$ obtain the optimum values of n and n' so as to minimise variance of the estimator of population mean for given cost. Also obtain minimum variance. (05)

6. (a) Describe Lahiri's method of drawing probability proportional to size (PPS) sample. Show that probability of drawing a specified unit at any draw is proportional to its size. What is the drawback of this method? (05)

(b) In a random sample of size n, n_1 units respond and n_2 units do not respond at the first attempt. r_2 units respond out of n_2 in the second attempt where $n_2=kr_2$. If the cost function is of the form $C= nC_0 + n_1C_1 + r_2C_2$, obtain the optimum values of n and k such that cost is minimum for given variance of the estimator of population mean. (06)

(c) If the population exhibits linear trend, show that weighted mean of systematic sample using end corrections is an unbiased estimator of population mean. (04)

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