($2\frac{1}{2}$ Hours) **[Total Marks : 60**

N.B. : (1) **All** questions are **compulsory**.

(2) **Figures** to the **right** indicate **full** marks.

(3) Draw **neat** diagrams wherever **necessary**.

(4) Symbols have usual meanings unless otherwise stated.

1. (a) Attempt any **one:---** **08**

(i) Obtain the expression

$$\frac{d}{dt}(\overline{P}_{mech}+\overline{P}_{field})=\frac{1}{4\pi}\int_V[(\overline{\nabla}\cdot\overline{E})\overline{E}+(\overline{\nabla}\cdot\overline{B})\overline{B}+(\overline{\nabla}\times\overline{E})\times\overline{E}+(\overline{\nabla}\times\overline{B})\times\overline{B}]dV$$

For momentum conservation.

(ii) Obtain the expression for the energy conservation law of electrodynamics. Give the physical interpretation of poynting vector $\overline{S}$.

(b) Attempt any **one:---** **04**

(i) Derive the continuity equation from Maxwell's equations.

(ii) Explain why the tails of the comets are always directed away from the Sun.

2. (a) Attempt any **one:---** **08**

(i) From Maxwell's equations in vacuum (for ρ=0 and j=0).Obtain the wave equation .For Partial solution $\overline{E}=\overline{E}_0e^{i(k.r-\omega t)}$ and $\overline{B}=\overline{B}_0e^{i(k.r-\omega t)}$, Show that $\overline{k},\overline{E}$ and $\overline{B}$ in that order form an orthogonal right handed system. Also show that $E_0=B_0$

(ii) For the simplest atomic model (where we imagine electron elastically bound to the atomic nucleus and performing damped harmonic oscillations of frequency ω_0). Obtain the expression for atomic polarizability $\alpha(\omega)$. Hence comment on the frequency dependence of polarizability.

(b) Attempt any **one:---** **04**

(i) For plane harmonic waves in matter obtain the following relation

$$\overline{k}\times\overline{E}=\frac{\omega}{c}\mu\overline{H}$$

P.T.O.

(ii) Distinguish TE,TM and TEM waves on the basis of boundary conditions.

3. (a) Attempt any **one:---** **08**

(i) The LienardWiechert electric field of a point charge is given by:

$$\vec{E} = e\left[\frac{(\hat{n} - \hat{\beta})(1-\beta^2)}{k^3R^2} + \frac{\hat{n} \times \{(\hat{n} - \vec{\beta}) \times \vec{a}\}}{c^2k^3R}\right]$$

Where the symbols have their usual meaning.

Using this relation show that the power radiated, P, by a non relativistic charged particle is given by

$$P = \frac{2}{3}\frac{e^2a^2}{c^3}$$

(ii) Define retarded potential and show that it leads to the generalized Coulomb's law:

$$\vec{E}(\vec{r},t) = \int\left[\frac{\rho}{R^2}\hat{e}_R + \frac{1}{cR}\frac{\partial\rho}{\partial t}\hat{e}_R - \frac{1}{c^2R}\frac{\partial\vec{J}}{\partial t}\hat{e}_R\right]d\tau'$$

Where $\vec{R} = \vec{r} - \vec{r}'$. When does it reduce to the fundamental form of Coulomb's law?

(b) Attempt any **one:---** **04**

(i) Obtain Lorentz force law in terms of scalar potential ϕ and vector potential $\vec{A}$.

(ii) In the use of multipole expansion in the context of radiation, we make three assumptions. Explain them.

4. (a) Attempt any **one:---** **08**

(i) Construct Energy – Momentum Tensor (asymmetric form) $T^{\alpha\beta}$. Derive an expression for the conservation of energy using it.

(ii) Construct field strength tensor $F^{\alpha\beta}$ and Lorentz transformation matrix Λ for frames with common $x - x'$ axes and relative velocity v. and show how fields transform.(You can use covariant or contra variant notations)

(b) Attempt any **one:---** **04**

(i) State and explain the postulates of special theory of relativity.

(ii) State the properties of Lorentz transformation matrix.

5. Attempt any **four:---** **12**

(i) Write Maxwell's equations in differential form.

P.T.O.

(ii) A wire is has resistance **R**, a potential difference of **V** volts is applied across its length **L** If the radius of the wire is **a**, find the Poynting vector at the surface of the wire.

(iii) The components of instantaneous electric fields in a plane wave propagated in Z direction are $E_x=E_1 Cos\ (kz-\omega t)$ and $E_y=E_2 Sin\ (kz-\omega t)$ .Comment on the state of polarization of the wave.

iv) Explain: 'Phase velocity and group velocity are equal in non dispersive medium.'

(v) What is the gauge transformation? State the Coulomb gauge and Lorentz gauge.

(vi) The LienardWiechert electric field of a point charge is given by:

$$\vec{E} = e\left[\frac{(\hat{n}-\hat{\beta})(1-\beta^2)}{k^3R^2} + \frac{\hat{n}\times\{(\hat{n}-\vec{\beta})\times\vec{a}\}}{c^2k^3R}\right]$$

Write the term responsible for radiation. Explain.

(vii) Using $\Lambda^{\alpha}_{\mu}\Lambda^{\beta}_{\nu}\, g^{\mu\nu} = g^{\alpha\,\beta}$ show that

$$(ds)^2 = dx^{\sigma}\, dx^{\epsilon} g_{\sigma\epsilon}$$

is invariant under Lorentz transformation.

(viii) Write down the components of ∂^{α}
