(3 Hours) [Total Marks: 60]

N.B.

i. Attempt any **TWO** questions among question numbers 1, 2, 3, and any **TWO** questions among questions 4, 5, 6.
ii. Figures to the right indicate full marks.
iii. Simple non-programmable calculator is allowed

1. (a) Derive the UMPU test for testing $H_0 : p = p_0$ against $H_1 : p \neq p_0$ when $X_1, X_2, \ldots, X_n$ are iid Bernoulli (p) random variables. (07)

   (b) A coin, with probability p of falling heads, is tossed independently 100 times and 60 heads are observed. Use the UMPU test for testing the hypothesis $H_0 : p = 1/2$ against the alternative $H_1 : p \neq 1/2$, at level of significance $\alpha = 0.1$. (04)

   (c) Define an α-similar test. When do you say an α – similar test has Neyman structure? (04)

2. (a) Let $X_1, X_2, \ldots, X_m$ and $Y_1, Y_2, \ldots, Y_n$ be independent samples from $N(\mu, 1)$ and $N(\eta, 1)$ respectively. Find a UMPU test of size α for testing $H_0 : \mu \leq \eta$ against $H_1 : \mu > \eta$. (07)

   (b) Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$. Show that the UMPU test for testing $H_0 : \mu \leq 0$ against $H_1 : \mu > 0$ is UMP invariant. (The MLR property of non-central Student's t-distribution may be assumed without proof). (08)

3. (a) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n having probability density function $f(x, \theta)$. Construct a locally most powerful unbiased test of size α for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$. (07)

(b) Let $X_1, X_2, ..., X_m$ and $Y_1, Y_2, ..., Y_n$ be iid. Observations from Poisson (μ) and Poisson (λ) respectively. Find a Neyman structure test for $H_0 : \mu = \lambda$ against $H_1 : \mu > \lambda$. (08)

4. (a) Show that for bivariate normal distribution kendall's coefficient τ is given by $\tau = \frac{2}{\pi} \sin^{-1} \rho$. Find an unbiased estimator of τ and express it in the form resembling ordinary correlation coefficient. (09)

(b) Obtain expression for Spearman's rank correlation coefficient in terms of differences of ranks. Show that it satisfies criteria of good measure of association. (06)

5. (a) Generate by enumeration the exact null probability distribution of k-sample median test for $k = 3, n_1 = 1, n_2 = 2, n_3 = 1$. If the rejection region consists of those arrangements which are least likely under the null hypothesis, find the rejection region R and exact α . (09)

(b) How is Kruskal-Wallis test different from k sample median test? Describe Kruskal-Wallis test. (06)

6. (a) Show that SPRT terminates with probability one. (07)

(b) Let $X_1, X_2, ...$ be iid $B(1, p)$ random variables. Write SPRT procedure to test $H_0 : p = p_0$ against $H_1 : p = p_1 > p_0$. Derive the OC and ASN functions. Prepare table showing values of the functions at five standard points. (08)

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