

[Time: 3 Hours]

[Marks:70]

Please check whether you have got the right question paper.

- N.B:**
1. All questions are **compulsory**.
 2. **Use of simple calculator is allowed**.
 3. **Figures to the right indicate full marks**.

Q.1 (A) Attempt any 7 [2 marks each]:

14

- 1) The value of $\int_2^5 (3x^2 + 2x + 1)dx$ is:
 - a) 411
 - b) 141
 - c) 142
 - d) 35
- 2) If $\begin{vmatrix} 2 & x \\ 1 & 4 \end{vmatrix} = \begin{vmatrix} x & 6 \\ 2 & 4 \end{vmatrix}$ then the value of x is:
 - a) 1
 - b) -2
 - c) 4
 - d) 2
- 3) The N^{th} derivative of $f(x)=\log(2x+1)$ is:
 - a) $y_n = \frac{1}{2(2x+1)}$
 - b) $y_n = \frac{(1)^{n-1}(n-1)!2^n}{(2x+1)^n}$
 - c) $y_n = \frac{(1)^n(n)!2^n}{(2x+1)^n}$
 - d) $y_n = \frac{(1)^n(n-1)!2^n}{(2x+1)^n}$
- 4) For $f(x,y)=x^2+xy+y^2$, the value of $\frac{\partial^2 f}{\partial x^2}$ is:
 - a) $2x+y$
 - b) 1
 - c) 2
 - d) $x+2y$
- 5) If $y=2x$, then Δy by taking $h=1$ is:
 - a) 4
 - b) 2
 - c) 3
 - d) 1
- 6) If $A = \begin{bmatrix} k & k & 4 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ is a singular matrix, then value of k is:
 - a) $5/4$
 - b) $5/2$

- c) 15/4
- d) 40/8
- 7) The solution of the differential equation $x dx + y dy = 0$ is:
 - a) $x^2 + y^2 = c$
 - b) $x^2 - y^2 = c$
 - c) $x + y = c$
 - d) $x - y = c$
- 8) The value of $\int \log x dx$ is:
 - a) $x \log x - 1 + c$
 - b) $x \log x + 1 - c$
 - c) $x(\log x + 1) + c$
 - d) $x(\log x - 1) + c$
- 9) General solution for the differential equation $(D^3 - 6D^2 + 9D)y = 0$ is:
 - a) $(c_1x + c_2)e^{3x} + c_3$
 - b) $c_1e^{3x} + c_2e^{3x} + c_3e^{0x}$
 - c) $(c_1x + c_2x)e^{3x} + c_3$
 - d) $(c_1x + c_2)e^{3x} + c_3e^{3x}$

(B) Attempt any 1:

1

- 10) The value of $\int_{-2}^2 x^5 dx$ is:
 - a) 16/3
 - b) 8/3
 - c) 0
 - d) 3/16
- 11) Which of the following is not a homogeneous differential equation?
 - a) $f(x,y) = 2x - 9y$
 - b) $f(x,y) = 3x^2 - 7y^2$
 - c) $f(x,y) = x^2 + 3y^2 - 1$
 - d) a and b

Q.2 (A) Attempt any two (4 marks each)

8

- 1) Find the N^{th} derivative of $y = \frac{2x+1}{x^2+3x+2}$
- 2) Using Maclaurin's series, give the expansion of $f(x) = \cos x$
- 3) Examine the function $f(x,y) = xy(3-x-y)$ for maxima and minima.

(B) Attempt any one (3 marks)

3

- 1) State Roil's Mean Value Theorem. Use it to verify for $f(x) = x^2 - 5x + 6$ in $[2, 3]$
- 2) Differentiate the equation: $(1+x^2)y_2 - xy_1 + y = 0$, n times w.r.t.x.

Q.3 (A) Attempt any two (4 marks each)

8

- 1) Obtain the reduction formula for $\int_0^{\frac{\pi}{2}} \sin^n x dx$, hence evaluate $\int_0^{\frac{\pi}{2}} \sin^{10} x dx$

- 2) Find the whole area of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$
- 3) Evaluate : $\int_0^{\frac{\pi}{2}} \sin^4 x \, dx$

(B) Attempt any one (3 marks)

- 1) The portion of the curve $x^2=9y$ between $x=0$ and $x=3$ is revolved about x -axis. Find the volume of solid of revolution.
- 2) By using the properties of Definite integral, Prove $\int_0^{\pi/2} \left(\frac{\cos x}{\sin x + \cos x} \right) dx = \frac{\pi}{4}$

Q.4 (A) Attempt any two (4 marks each)

- 1) By using the Adjoint method, find the inverse of the matrix $A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$
- 2) Find the Eigen values and one of the Eigen vectors of the matrix: $\begin{bmatrix} 2 & -1 & 1 \\ 2 & 2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$
- 3) Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(B) Attempt any one (3 marks)

- 1) Solve the following equations by Cramer's rule:
 $x+y=3, y+z=5, x+z=4$

- 2) Find the Rank of the matrix $A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$

Q.5 (A) Attempt any two (4 marks each)

- 1) Solve $(x^3+y^3)dy=x^2y \, dx$
- 2) Solve $(1-x) dy-(1+y) dx=0$. Also find the particular solution, if $y=2$ when $x=1$
- 3) Find the particular solution of $(D^3-D^2-D+1)y=0$, when $x=0, y=1$ and $\frac{dy}{dx} = 0$

(B) Attempt any one (3 marks)

- 1) Form the differential equation for $y=ae^x+be^{-x}$, where a, b are arbitrary constants.
- 2) Solve the following homogeneous differential equations: $\frac{dy}{dx} = \frac{xy+y^2}{x^2+xy}$

Q.6 (A) Attempt any two (4 marks each)

- 1) Using Lagrange's interpolation formula obtain a polynomial which passes via the points $(0,5), (1,-1)$ and $(3,13)$.
- 2) Given :

x	1	3	5	7	9	11	13
f(x)	5	21	53	101	165	245	341

Find $\int_1^{13} f(x)dx$, using Simpson's $1/3^{\text{rd}}$ rule.

3) Estimate the missing term by using E and Δ from the following:

x	0	1	2	3	4
Y	1	3	9	-	81

(B) Attempt any one (3 marks)

1) Estimate the value of $f(x)$ at $x=2.65$ from the following data by Newton's forward difference formula.

x	-1	0	1	2	3
f(x)	-21	6	15	12	3

2) Solve $\left(\frac{\Delta^2}{E}\right) x^3$ by taking $h=1$

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