

Note:

1. All questions are compulsory.
2. Attempt any two sub questions out of four from question no.1to question no. 3.
3. From question no.4, attempt any one out of (a) and (b) and any one out of (c) and (d).
4. Use of Non- Programmable Scientific Calculator is allowed.

1. (a) Define the rectangular distribution in the interval (a, b) . Obtain its moment generating function (MGF) and hence expression for the mean and variance. 10
- (b) Obtain the Moment Generating Function (M.G.F.) of the exponential distribution with parameter θ . Hence, show that the coefficient of skewness and kurtosis are independent of parameter. 10
- (c) Define Beta distribution of first kind. Obtain the expression for its r^{th} raw moment. Hence, find its mean and variance. 10
- (d) State the moment generating function (M.G.F.) of normal distribution. Obtain its cumulant generating functions (C.G.F.). Hence, obtain coefficient of skewness. 10
2. (a) Derive the expression of probability density function of Chi-square variate. 10
- (b) Define the Chi-square distribution with 'n' degrees of freedom. Obtain the expression for its r^{th} order cumulant generating function. Hence, obtain the coefficients of skewness and kurtosis. 10
- (c) Obtain the limiting form of a Chi-square distribution. 10
- (d) If the two independent random variables X_1 and X_2 follows Chi-square distribution with parameters n_1 and n_2 respectively, obtain the distribution of $Y_1 = X_1 + X_2$ and $Y_2 = \frac{X_1}{X_1 + X_2}$ and identify their distributions. 10
3. (a) If U is a standard normal variate and V is a Chi-square variate with 'n' degrees of freedom. Obtain the probability density of function (p.d.f.) of $T = \frac{U}{\sqrt{V/n}}$. 10
- (b) State the applications of t-distribution. Explain the test procedure for any two applications. 10

(c) Define 'F' distribution. Obtain its r^{th} raw moment and hence obtain expressions for mean and variance of F-distributions.

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(d) Obtain mean and mode of F distribution.

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4. (a) Obtain the recurrence relation for even ordered central moments in terms of σ^2 of a normal distribution with mean μ and variance σ^2 .

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(b) If X and Y are two independent Chi square variates with n_1 and n_2 degrees of freedom respectively, then show that $\frac{X}{Y}$ follows Beta distribution of second type.

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(c) Define the t- distribution and derive its p.d.f.

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(d) Obtain relationship between F and χ^2 .

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