

[Max Marks:75]

Revised Course

Duration: 21/2 Hours

N.B. 1. All questions are compulsory.**2.** From Question 1,2 and 3, Attempt any one from part(a) and any two from part(b).**3.** From Question 4, Attempt any THREE**4.** Figures to the right indicate marks for the respective parts.

- 1 a i Define triple integral of a bounded function $f: Q \rightarrow \mathbb{R}$ where $Q = [a_1, b_1] \times [a_2, b_2] \times [a_3, b_3]$ is a rectangular box in \mathbb{R}^3 . Further show with usual notations $m(b_1-a_1)(b_2-a_2)(b_3-a_3) \leq \iiint_Q f \leq M(b_1-a_1)(b_2-a_2)(b_3-a_3)$ 8
- ii State and prove Fubini's theorem for a rectangular domain in \mathbb{R}^2
- b i State the change of variables formula for triple integral, stating clearly the condition under which it is valid. Use it to express the triple integral $\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} f(x, y, z) dz dy dx$ in spherical co-ordinates (ρ, θ, ϕ) . 12
- ii Evaluate $\int_0^1 \int_{\sqrt{x}}^3 e^{y^3} dy dx$ by reversing the order of integration. Sketch the region of integration.
- iii Evaluate $\iiint_S y dV$ where S is the solid enclosed by the planes $z = 0, z = y$ and the parabolic cylinder $y = 1 - x^2$
- iv Evaluate $\iint_R \frac{y-4x}{y+4x} dA$ where R is the region bounded by the lines $y = 4x, y = 4x + 2, y = 2 - 4x, y = 5 - 4x$.
- 2 a i Let f be a continuously differentiable scalar field defined on an open set U in \mathbb{R}^n . Suppose C be a closed curve in U, with parameterization $r(t), t \in [a, b]$. then prove that $\oint_C \nabla f \cdot \overline{dr} = 0$ 8
- ii State and prove Green's Theorem for a rectangle. Further find the area of the region between two concentric circles of radii r_1 and r_2 where $r_1 < r_2$, using Green's theorem.
- b i $F = (P, Q)$ is a continuously differentiable function defined on a simply connected region D in \mathbb{R}^2 . Show that $\oint_C P dx + Q dy = 0$ around every closed curve C in D if and only if $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} \forall (x, y) \in D$ 12
- ii Evaluate $\int_C F$, where $F(x, y, z) = (x^2 - xy, 1)$ and C is the circle of radius 1, with centre at the origin and lying in the yz, plane, traversed counterclockwise as viewed from the positive x axis.
- iii Show that the line integral $\int_{(-1,2)}^{(3,1)} (y^2 + 2xy) dx + (x^2 + 2xy) dy$ is path independent. Further evaluate the line integral.
- iv Use Green's theorem to find the area of the region $D = \{(x, y) \in \mathbb{R}^2 : x^{\frac{2}{3}} + y^{\frac{2}{3}} \leq 4\}$
- 3 a i State and prove Stoke's Theorem for an oriented smooth, simple parameterized surface in \mathbb{R}^3 bounded by a simple, closed curve traversed counter clockwise assuming general form of Green's Theorem. 8

- ii For the surface $\vec{r}(u, v)$ described by the vector equation $\vec{r}(u, v) = X(u, v)\hat{i} + Y(u, v)\hat{j} + Z(u, v)\hat{k}$, $(u, v) \in T$ where X, Y, Z are differentiable on T , define the fundamental vector product $\frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v}$. If C is a smooth curve lying on the surface, $C = \vec{r}(\alpha(t))$, $\alpha: [a, b] \rightarrow T$, then show that $\frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v}$ is normal to C at each point. Further assume S and C satisfies the hypotheses of Stokes' Theorem and f, g have continuous second order partial derivatives. Prove with usual notations that $\int_C (f \nabla f) \cdot d\vec{r} = 0$.
- b i Assuming S and V satisfy the conditions of the Divergence Theorem and scalar fields f and g , components of \vec{F} have continuous partial derivatives, \hat{n} is unit outward normal. Prove 12
- p) $|V| = \frac{1}{3} \iint_S \vec{r} \cdot \hat{n} dS$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $|V| =$ volume of V .
- q) $\iint_S \text{curl} \vec{F} \cdot \hat{n} dS = 0$.
- ii Using Stoke's theorem evaluate $\iint_S \text{curl} \vec{F} \cdot \hat{n} dS$ where $F(x, y, z) = x\hat{i} + z^2\hat{j} + y^2\hat{k}$ and S is the plane surface $x + y + z = 1$ lying in the first octant.
- iii Evaluate $\iint_S y ds$, where S is cylinder $x^2 + y^2 = 1, 0 \leq z \leq 1$
- iv Using Gauss Divergence Theorem to evaluate $\iint_S \vec{F} \cdot \hat{n} dS$ where $F(x, y, z) = (y - x, z - y, y - x)$ and S is the cube bounded by the planes $x = \pm 1, y = \pm 1, z = \pm 1$.
- 4 i Using Spherical coordinates find the volume of the solid region bounded by the surface $\rho = \cos \phi$. 15
- ii Find the area of the region R bounded by the curves $y = \sin x$ and $y = \cos x$ for $0 \leq x \leq \frac{\pi}{4}$.
- iii A particle starts at the point $(-2, 0)$ moves along the X -axis to $(2, 0)$ and then along the semi circle $y = \sqrt{4 - x^2}$ to the starting point. use Green's theorem to find the work done on this particle by the force field $F(x, y) = x\hat{i} + (x^3 + 3xy^2)\hat{j}$
- iv Evaluate the line integral of $f(x, y, z) = e^{\sqrt{z}}$, along the path parametrised by $\gamma(t) = (1, 2, t^2), 0 \leq t \leq 1$
- v Find the surface area of S which is parametrically given by $r(\theta, z) = (a \cos \theta, a \sin \theta, z)$ and $(\theta, z) \in [0, 2\pi] \times [-1, 1]$, $a > 0$ is constant.
- vi Using Stoke's theorem evaluate $\oint_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x, y, z) = (x^3 + y^3)\hat{i} + (x - y)\hat{j}$, C is the boundary of the rectangular lamina in the xy -plane. Bounded by the lines $x = 0, x = 2, y = 2$ and $y = 5$ oriented counter clockwise as viewed from above.