M.Sc (Maths) [Part - I]
Algebra - I
(Paper- I) (Revised)
(May-2017)

QP Code : 75677

Revised **(3 Hours)** **(Total marks: 80)**

Instructions:-

i. **Attempt any two questions from each section.**
ii. **All questions carry equal marks.**
iii. **Answer to section I and section II should be written in the same answer book.**

SECTION I (Attempt any Two Questions)

1. (a) If U(F) and V(F) are two vector spaces and T be a linear transformation, then, dim U= dim Ker T + dim Image T.

   (b) Consider the basis
   $S = \{v_1, v_2, v_3\} for\ R^3, where\ v_1 = (1,1,1), v_2 = (1,1,0) and \quad v_3 = (1,0,0).$
   Let $T: R^3 \to R^2$ be the linear transformation such that
   $T(v_1) = (1,0), T(v_2)\ \ (2,1), T(v_3) = (4,3)$. Find a formula for $T(x_1, x_2, x_3)$;
   then use this formula to compute T(2,-3, 5).

2. (a) Let $C_1, C_2, \ldots\ldots, C_n$ be column vectors of dimension n. They are linearly dependent if and only if det $(C_1, C_2, \text{-----}, C_n) = 0$.

   (b) Find the rank of the following matrices:

   i) $\begin{bmatrix} 3 & 1 & 2 & 5 \\ 1 & 2 & -1 & 2 \\ 1 & 1 & 0 & 1 \end{bmatrix}$

   ii) $\begin{bmatrix} 3 & 1 & 1 & -1 \\ -2 & 4 & 3 & 2 \\ -1 & 9 & 7 & 3 \\ 7 & 4 & 2 & 1 \end{bmatrix}$

3. (a) Find Eigen values and the Eigen vectors of the following matrix

$$A = \begin{bmatrix} 4 & 4 & 4 \\ -2 & -3 & -6 \\ 1 & 3 & 6 \end{bmatrix}$$

   (b) Find Minimal Polynomial of the following matrix

$$\begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

   Also find Eigen values of A.

4. (a) Prove that an orthogonal set of non-zero vectors is linearly independent.

(b) Find real orthogonal matrix P such that $P^T A P$ is diagonal for the following matrix A.

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

SECTION II (Attempt any Two Questions)

5. (a) Any finite cyclic group of order n is isomorphic to Z_n, the group of integer residue classes modulo under addition.

(b) State and prove First Isomorphism Theorem i.e., let f: G to $\overline{G}$ be a homomorphism of groups. If f is onto, $\frac{G}{kerf} \approx \overline{G}$ or $(\frac{G}{Kerf} \approx Imf)$

6. (a) Let H be a subgroup of a group G. Then the following statements are equivalent:

i) $H \Delta G$ $(i.e. aHa^{-1} \subseteq H \ \forall \ a \in G)$

ii) $aHa^{-1} = H$ $for\ each\ a \in G$

iii) $aH = Ha,$ $for\ each\ a \in G$

iv) $H_aH_b = H_{ab}$ $for\ each\ a, b \in G$

(b) A group G of order p^n where p is prime and $n \geq 1$ has non-trivial centre.

7. (a) M is maximal ideal if and only if R/M is field. (Prove it)

(b) Let R = $\left\{\begin{bmatrix} a & b \\ b & a \end{bmatrix} : a, b \in Z\right\}$. $Let\ \emptyset : R \to Z\ be\ defined\ by$

$$\emptyset\left\{\begin{bmatrix} a & b \\ b & a \end{bmatrix}\right\} = a - b\ then\ \emptyset\ is\ ring\ homomorphism.$$

8. (a) In unique factorization domain, irreducible polynomials are prime.

(b) Let F be a field. If $f(x) \in F[x]$ $and\ deg\ f(x) = 2\ or\ 3$ then $f(x)$ is reducible over F if and only if $f(x)$ has zero in F.

— End —

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M.Sc (Maths) [Part - I)

Algebra - I

(Paper- I) (Old)

(May-2017)

QP Code : 75674

Scheme A(External) **(3 Hours)** **Total marks: 100**

N.B: 1) Scheme A students answer any five questions.
2) All questions carry equal marks.

Q1. (a) Show that the intersection of two subgroups of a group G is a subgroup of G. Give an example to show that the union of two subgroups of a group G need not to be a subgroup of G.

(b) P rove that a finite semi-group G is a group if and only if G satisfies both the cancellation laws.

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Q2. (a) Let H be a subgroup of a group G and a,b ε G. Show that either

$Ha \cap Hb = \emptyset$ or $Ha = Hb$.

(b) Prove that every group of prime order is cyclic.

Q3. (a) Show that every quotient group of a group is a homomorphic image of the group.

(b) Prove that a group of order 99 is not simple.

Q4. (a) Define Integral Domain and prove that every field is an integral domain.

(b) Prove that order of a finite field F is P^n, for some prime p and some positive integer n.

Q5. (a) Prove that every integral domain can be imbedded in a field.

(b) Show that Z [√ -5] is not a principal ideal domain.

Q6. (a) Prove that the union of two subspaces is a subspace if and only if one is contained in the other.

(b) Let U and V be the vector spaces over the field F and let T be a linear transformation from U into V. Suppose that U is finite dimensional then prove that

Rank (T) + Nullity (T) = dim (U)

[TURN OVER

Q7. (a) Let V be a finite dimensional vector space over the field F, and let W be a subspace of V.

Then prove that

$\dim W + \dim W^0 = \dim V$

Where W^0 is the annihilator of W.

(b) In $V_3(R)$, where R is the field of Real numbers, examine each of the following sets of vectors for linear dependence

i. { (1,2,1), (3,1,5), (3,-4.7) }
ii. { (2,1,2), (8,4,8) }

Q8. (a) Suppose that α and β are vectors in an inner product space. Then show that

$\| \alpha + \beta \|^2 + \| \alpha - \beta \|^2 = 2 \| \alpha \|^2 + 2 \| \beta \|^2$

(b) Show that every square matrix satisfies its characteristic equation.

M.Sc (Maths) [Part - I]
Analysis - I & Topology
(Paper- II) (Revised)
(May-2017)

QP Code : 75688

Revised] (3 Hours) [Total Marks:80

Instructions:

- Attempt **any two** questions from **each section**
- **All questions** carry **equal marks**
- **Answers** to **Section I** and **Section II** should be written in the **same answer book**.

SECTION I (Attempt **any two** questions)

1. (a) Define convergence of a sequence in a metric space (X, d). Define Cauchy sequence in a metric space (X, d). Prove that every convergent sequence in (X, d) is Cauchy. Does the converse of the above statement hold?

   (b) State and prove Lebesgue covering lemma.

2. (a) Let $(X, d_1), (Y, d_2)$ be metric spaces. Let $f : X \to Y$ be a function. Prove that f is continuous on X if and only if inverse image of an open set in Y is an open set in X.

   (b) Define compact set. Define uniform continuity of a function $f : (X, d_1) \to (Y, d_2)$, where $(X, d_1), (Y, d_2)$ are metric spaces. Prove that if K is a compact subset of X, and f is continuous on K then f is uniformly continuous on K.

3. (a) Define partial derivative. Find partial derivatives of all possible orders for the function $f(x, y, z) = (x^2y^2, 3xy^3z, xz^3)$.

   (b) State (without proof) chain rule. Write the matrices for $f', g', (f \circ g)'$ for the following functions and evaluate them at the point $(2, 5)$: $f(x, y) = (x + y, x^2 + y^2, 2x + 3y)$ and $g(u, v) = (u^2, v^3)$.

4. (a) State and prove mean value theorem.

   (b) State implicit function theorem. Examine whether the function $f(x, y) = x^2 + y^2 - 4$ can be expressed as a function $y = g(x)$ in a neighbourhood of the point $(0, -2)$.

SECTION II (Attempt **any two** questions)

5. (a) Define base of a topological space. Define product topology. Prove that if $\mathcal{B}$ is a basis for the topology on X and $\mathcal{C}$ is a basis for the topology on Y then prove that the collection $\mathcal{D} = \{B \times C \mid B \in \mathcal{B}, C \in \mathcal{C}\}$ is a basis for the topology on $X \times Y$.

   (b) Let (X, τ) be a topological space and $A \neq \emptyset$ be a subset of X. Define interior of A. Prove that if $A \subset B$, then $i(A) \subset i(B)$ and for all subsets A, B of X, $i(A \cap B) = i(A) \cap i(B)$.

6. (a) Define first countable topological space. Define second countable topological space. Prove that a second countable topological space is first countable.

   (b) Define T_1 topological space. Prove that a topological space is a T_1 space if and only if every one point subset of it is a closed subset.

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PA-Con.1158-17.

7. (a) State (without proof) tube lemma. Let f be a continuous real-valued function on $[a, b]$. Prove that the set $\{(x, f(x)) \mid x \in [a, b]\}$ is a compact subset of $\mathbb{R}^2$.

   (b) Define local compactness of a topological space (X, τ). Define regular topological space. Prove that if X is a regular space such that X is locally compact at $x \in X$, then x has a local base of compact neighbourhoods in X.

8. (a) Define complete metric space (X, d). Assume that for each $n \in \mathbb{N}$, F_n's are closed and bounded subsets of X such that $F_1 \supset F_2 \supset \cdots \supset F_n \supset F_{n+1} \supset \cdots$ and $\text{diam}(F_n) \to 0$ and $n \to \infty$. Prove that $\bigcap_{n=1}^{\infty} F_n$ contains precisely one point.

   (b) Define total boundedness of a metric space (X, d). Prove that if (X, d) is a compact metric space, then (X, d) is complete and totally bounded.

M.Sc (Maths) [Part - I]

Analysis - I

(Paper- II) (Old)

(May-2017)

QP Code : 75685

Scheme A (External)]	(3 Hours)	**[Total Marks:100**
Scheme B (Internal)]	(2 Hours)	**[Total Marks: 40**

Instructions:

Instructions:

- **Mention** on the top of the **answer book** the **scheme** under which you are appearing
- **Scheme A** students should **attempt any five** questions
- **Scheme B** students should **attempt any three** questions
- All **questions** carry **equal marks**

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1. (a) State and prove Nested Intervals theorem.

   (b) Define the supremum and infimum of a non-empty subset S of $\mathbb{R}$ and state Supremum property (axiom) of $\mathbb{R}$. SHow that a real number M is the supremum of S iff $M \geq x, \forall x \in S$ and for any $\epsilon > 0, \exists y \in S$ such that $M - \epsilon < y \leq M$, where $\phi \subseteq S \subseteq \mathbb{R}$.

2. (a) If S is a nonempty, open subset of $\mathbb{R}^n$ and $f : S \to \mathbb{R}$, define continuity of f at $a \in S$. Show that if $f, g : S \to \mathbb{R}$ are both continuous at a and α, β are real numbers then $(\alpha f + \beta g)$ is continuous at a.

   (b) Examine the continuity and differentiablity of f at $(0,0)$ given that $f(x,y) = \frac{x^3 y}{x^6+y^2}$ for $(x,y) \neq (0,0)$ and $f(0,0) = 0$

3. (a) Define a real valued Cauchy sequence in $\mathbb{R}^n$ and show that a convergent (real valued) sequence in $\mathbb{R}^n$ is Cauchy.

   (b) Examine the pointwise and uniform convergence of the sequence $\{f_n(x)\}$ defined by $f_n(x) = x^n$ on $[0,1]$. Justify your answers.

4. (a) State and prove Weirstrass test for uniform convergence of a series $\sum f_n(x)$ defined on a non-empty subset S of $\mathbb{R}$.

   (b) State Root test for convergence of a positive term series $\sum a_n$. Hence or otherwise discuss the convergence of $\sum \frac{3^n}{n^n . x^n}$, where $x \in \mathbb{R}^+$.

5. (a) Let S be a non-empty open subset of $\mathbb{R}^n$ and $a \in S$. Suppose $f : S \to \mathbb{R}$. Define the total derivative of f at a. Find the total derivative of $f(x,y,z) = xy + yz + zx$ at $(1,-2,3)$. State the result used.

   (b) If $f : \mathbb{R}^2 \to \mathbb{R}^3$ is given by $f(x;y) = (x-y, 2y^2, x+y)$ and $g : \mathbb{R}^3 \to \mathbb{R}^2$ is given by $g(u,v,w) = (u - rv, w^2 u)$ then find the jacobians of f and g respectively at $(2,-3)$ and at $f(2,-3)$. Also find the jacobian of $g \circ f$ at $(2,3)$.

6. (a) If $x = se^{\sin t}, y = te^{\cos t}$ and $s = r\cos\theta, t = r\sin\theta$, use chain rule to find $\frac{\partial x}{\partial r}, \frac{\partial x}{\partial \theta}, \frac{\partial y}{\partial r}, \frac{\partial y}{\partial \theta}$ in terms of functions of r, θ.

   (b) State Taylor's theorem and use it to expand the function $f(x,y) = e^x \cos y$ near $(0, \pi/4)$ upto and including degree two terms.

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PA-Con.1157-17.

7. (a) State and prove Fubini's theorem for the double integral of a bounded real-valued function $f(x, y)$ over a rectangle D in xy-plane.

   (b) Evaluate the double integral of $f(x, y) = x^2 + y^2$ over the disc $x^2 + y^2 \leq 4$ in the xy-plane

8. (a) Define the convergence of an improper integral $\int_a^\infty f(x)dx$ and show that an improper integral $\int_a^\infty \frac{dx}{x^p}, p > 0$, converges iff $p > 1$. Hence show that $\int_1^\infty \frac{dx}{5x^5}$ converges.

   (b) Discuss the convergence of (i) $\int_0^1 \frac{dx}{x^3\sqrt{1-x^2}}$, (ii) $\int_2^3 \frac{dx}{(x-2)^2 x^3}$

M.Sc (Maths) [Part - I]
Topology
(Paper- III) (Old)
(May-2017)

Q. P. Code: 11740

External (Scheme A) (3 Hours) Total Marks: 100
Internal (Scheme B) (2 Hours) Total Marks: 40

N.B.:Scheme A students should attempt any five questions.
Scheme B students should attempt any three questions.
Write the scheme under which you are appearing, on the top of the answer book.

Q.1. a) Prove that the set of all real numbers is uncountable. 10
b) Let f : X→ Y . Prove that f is bijective iff f(X\ A) = Y \ f(A) for A⊏ X. 10

Q.2 a) Define subspace topology. If $\mathcal{B}$ is a basis for the topology of X then prove that the collection $B_Y = \{B \cap Y / B \epsilon \mathcal{B}\}$ is a basis for the subspace topology on Y . 10
b) Let X be a topological space and A ⊂_ X. Prove that the following statements are equivalent: 10
(i) A is open (ii) A $= A°$.

Q.3 a) Define a connected topological space. Let X be a topological space and A be a subset of X. Show that if A is connected then its closure A is also connected. 10
b) Define a path connected topological space. Prove or disprove: Every connected space is path connected. Justify your answer. 10

Q.4. a) Define connected topological space. Prove that the cartesian product of two connected spaces is connected. 10
b) Give an example of a continuous bijection from one topological space to the other, which is not a homeomorphism. 10

Q.5 a) Show that a limit point compact metric space is sequentially compact. 10
b) Prove that the continuous image of a compact metric space is compact. 10

Q.6 a) Define a dense set. Let X be a topological space with countable basis. Show that: 10
(i) Every open covering of X has a countable subcollection covering X.
(ii) There exists a countable subset of X that is dense in X.

b) Show that every open subset of ℝ is the union of disjoint sequence of open intervals. 10

Q.7 a) Define a quotient map. Prove that if p : X → Y is a continuous, surjective and open map then it is a quotient map. 10
b) Prove that every compact subset of a Hausdroff space is closed. 10

Q.8. a) State and prove path lifting lemma 10
b) Let f and g be two paths in a topological space with same initial point and same end point. Define the path homotopy relation $\sim_p$ and show that it is an equivalence relation. 10

Q.P. Code :10066

[Time: 2 Hours] **[Marks:60]**

Please check whether you have got the right question paper.

N.B:
1. All questions carry equal marks
2. There is internal choice in each question

Q.1 a. Trace the trajectory in the meanings of the word 'Development' over time. **15**

Or

b. Critically analyze modernization theory as elaborate by W.W. Rostow in "The Stages of economic growth".

Q.2 a. Critically analyze the dependency theory and elaborate on the work of any one of the theorist. **15**

Or

b. Discuss Amartya Sen's Capability and Rights based approach to Development.

Q.3 a. Critically elaborate on the shifts in the meaning of Gender and Development. **15**

Or

b. Critically analyze the Structure Adjustment and Liberalization Programme in India.

Q.4 Write short notes on any TWO of the following **15**

a) Development and Social Justice
b) Mixed Economy Model
c) Development as Discourse-Escobar
d) Manuel Castells & Globalization

[वेळ :दोन तास] [गुण : ६०]

सूचना : १. प्रश्नास दिलेला पर्याय विचारात घेऊन चारही प्रश्न सोडवा. २. सर्व प्रश्नांना समान गुण आहेत.

प्र.१ अ. विकास संकल्पनेच्या अर्थाचा इतिहास स्पष्ट करा. **१५**

किंवा

ब. डब्लु, डब्लु रोष्टोच्या आर्थिक विकासाच्या टप्याचा सिद्धांत आधुनिकरणाचा सिद्धांत म्हणून टिकात्मक विश्लेषण करा.

प्र.२ अ. अवलंबित्वाच्या सिद्धांताचे कोणत्याही एका सिद्धांतकाराचे कार्य लक्षात घेऊन टिकात्मक विश्लेषण करा. **१५**

किंवा

ब. अमर्त्य सेन यांच्या क्षमता आणि विकासाचा अधिकार आधारित दृष्टीकोनाची चर्चा करा.

प्र.३ अ. लिंगभाव आणि विकास यांच्या अर्थात होणाऱ्या बदलाचे चिकित्सक विश्लेषण करा- **१५**

किंवा

ब. भारतातील रचनात्मक समायोजन आणि उदारीकरण कार्यक्रमाचे चिकित्सक विश्लेषण करा.

प्र.४ खालीलपैकी कोणत्याही दोनावर थोडक्यात टिप लिहा. **१५**

अ. विकास आणि सामाजिक न्याय
ब. संमिश्र अर्थ व्यवस्था प्रारूप
क. विकास एक चर्चाविश्व इस्कोबार
ड. मॅन्युअल कॅसेल आणि जागतिकीकरण

M.Sc (Maths) [Part - I)

Complex Analysis

(Paper- IV) (Old)

(May-2017)

QP Code : 10065

External (Scheme A)	(3 Hours)	Total Marks: 100
Internal (Scheme B)	(2 Hours)	Total Marks: 40

N.B.:Scheme A students should attempt any five questions.
Scheme B students should attempt any three questions.
Write the scheme under which you are appearing, on the top of the answer book.

1) (a) State and prove Weierstrass M test.
(b) Find the roots common to $x^4+1=0$ and $x^6-i=0$.

2) (a) State and prove the Cauchy Riemann Equation.

(b) Prove that the circle $|z-2|=3$ is mapped onto a circle $\left|w+\frac{2}{5}\right|=\frac{9}{25}$ under the transformation $w=\frac{1}{z}$.

3) (a) Prove that a branch of logarithms is analytic and find its derivative.

(b) Prove that $u(x,y)=e^{2x}(x\cos 2y-y\sin 2y)$ is a harmonic function. Also find its harmonic conjugate and the corresponding analytic function.

4) (a) Let γ be such that $\gamma(t)=\gamma_1(t)+i\gamma_1(t)$ be a smooth curve and suppose that f is a continuous function on an open set containing $\{\gamma\}$. Then prove that

(i) $\int_{-\gamma} f(z)dz=-\int_{\gamma} f(z)dz$

(ii) $\left|\int_{\gamma} f(z)dz\right|\le\int_{\gamma}|f(z)||dz|$

(iii) If $M=\underset{t\in[a,b]}{Max}|f(\gamma(t))|$ and $L=L(\gamma)$ (length of γ) then $\left|\int_{\gamma} f(z)dz\right|\le ML$

(b) If O is the origin, L is the point $z=3$, M is the point $z=3+i$, evaluate $\int z^2dz$ along

(i) the path OM

(ii) the path OLM

5) (a) State and prove Liouville's theorem.

(b) Evaluate $\int_C \frac{z^2}{z^4+1}$ where C is (i) $|z|=1/2$ (ii) $|z+i|=1$.

6) (a) State and prove Open Mapping Theorem.

(b) State and prove the Minimum Modulus Principle.

7) (a) Define (i) Singularity (ii) Removable singularity (iii) Pole. Determine whether $f(z)=\frac{1-\cos z}{z}$ has a removable singularity using definition and Laurent series expansion.

(b) Find all the possible Laurent Series expansions of $f(z)=\frac{1}{z^2(z-1)(z+2)}$.

8) (a) State and prove Rouche's theorem.

(b) Using contour integration evaluate $\int_{-\infty}^{\infty}\frac{x^2+x+2}{x^4+10x^2+9}dx$.

M.Sc (Maths) [Part - I)

Set Theory & Logic & Elementary probability theory

(May-2017)

Q. P. Code: 11749

[Total marks: 80]

1) Attempt any two questions from each section.
2) All questions carry equal marks.
3) Answer to Section I and section II should be written in the same answer book.

SECTION-I (Attempt any two questions)

Q.1(A) Determine whether the relation R on a set A is reflexive, transitive, symmetric or antisymmetric. **(8)**

A= set of all positive integers aRb iff $|a - b| \leq 2$.

(B) i) Explain the meaning of Partition of a set and Congruent Modulo M Relation, with examples. **(6)**

ii) Determine whether each of the following is a tautology:

a) $(\neg P \wedge \neg Q) \rightarrow (P \rightarrow Q)$ **(6)**

b) $(P \rightarrow Q) \wedge (P \wedge \neg Q)$

Q.2(A) i) Prove that R not countable. **(8)**

ii) Explain the meaning of injective function, composition of functions. Also prove that the inverse of one-to-one and onto function is one-to-one and onto. **(6)**

(B) Let X= {1, 2, 3, 4, 5, 6}, Y={1, 2, 3, 4, 5} and function f is defined by f(1) = f(4) = f(6)=3; f(2) =5 and f(3) = f (5) = 4.Find **(6)**

a) $f([1,2]) \wedge [2,6]$; $f([1,2]) \wedge f([2,6])$.

b) $f([1,2,3] \vee [4,5])$; $f([1,2,3]) \vee f([4,5])$.

Q.3(A) i) Show by mathematical induction that for all $n \geq 1$, **(8)**

$$1 + 2 + 3 + \cdots .. + n = \frac{n(n+1)}{2}$$

ii) Explain partially ordered set with example. Let R be a set of real numbers and a relation ≤ defined on R, then prove that (R, ≤) is a partially ordered set. **(6)**

(B) By using Zorn's lemma prove that every vector space has a basis. **(6)**

Q.4(A) i)Let A= $\{a_1, a_2, a_3, \ldots\ldots, a_n\}$ be a finite set with n elements, $n \geq 2$. Prove that there are $\frac{n!}{2}$ even permutation and $\frac{n!}{2}$ odd permutation. **(8)**

ii) Find inverse of each of the following- **(6)**

Turn Over

a) $P = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{bmatrix}$

b) A= (3 2) (1 4)

c) H= (1 2 3) (4 1 2) (3 4)

(B) List all permutations in S_4, each factored into disjoint cycles. Factor following permutation M, as a product of disjoint cycles. **(6)**

$$M = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ 5 & 12 & 2 & 1 & 9 & 11 & 4 & 3 & 7 & 10 & 13 & 8 & 6 \end{bmatrix}$$

SECTION-II (Attempt any two questions)

Q.5(A) i) Give mathematical definition of probability .State its limitations. **(4)**

ii) A, B, C forms partition of Ω, find smallest field containing A, B, C. **(4)**

(B) (i) A class C of subsets of Ω, such that either A or $\bar{A}$ are finite. Is C a sigma field? **(4)**

(ii){A_n} is a sequence of events $A_n = \begin{cases} A & n = 1,3,5 \ldots \\ B & n = 2,4,6 \ldots . \end{cases}$ **(4)**

Find Limit superior and limit inferior. Show that lim A_n does not exist.

(iii) In a random arrangement of alphabets in word CHILDREN, find probability that i) All vowels are together. ii) No two vowels are together. **(4)**

Q.6(A) i) Define following **(5)**

a) Conditional probability of an event A given B.

b) Pair wise Independence

c) Mutual independence (for three events)

ii) State and prove monotone and subtractive property of probability. **(7)**

(B) i) In a population 55% are males and45% are females. If 4% of males and 1% of females are colour blind, find the probability that a randomly selected person is colour blind. **(4)**

ii) Find constant K ,if following id density function. **(4)**

f(x) = kx(1-x) 0 < x < 1 .Hence P(-1,0.9].

Turn Over

Q.7(A) i) Explain the concept of a discrete and continuous random variable (r.v) and give one example of each . **(4)**

ii) Define a distribution function (d.f) of a continuous r.v and state and prove its any two properties. **(6)**

(B) i) X has Binomial with (n = 8, p=0.4). Find mean and variance if X. **(5)**

ii) The joint p.d.f of X,Y is f(x,y) = 2 for $0 < x < y < 1$; find conditional p.d.f of X given Y. **(5)**

Q.8(A) i) For X, Y independent r.v.s , show that E[XY] = E[X]E[Y]. **(3)**

ii) For any r.v.s X,Y show that $E^2[XY] \le E[X^2]E[Y^2]$. **(5)**

(B) i) Examine whether the Strong law of large numbers holds for sequence of independent r.v.s { X_k } . **(4)**

$$X_k = \begin{cases} \pm k & with - prob \quad \frac{1}{2\sqrt{k}} \\ 0 & with - prob \quad 1 - \frac{1}{\sqrt{k}} \end{cases}$$

ii) A r.v X has mean 40 and variance is 12 find bound on P[X< 32] + P[X>48]. **(4)**

iii) Let $\{X_i\}$ be a sequence of independent r.v.s with mean 0 and variance 0.25. If S_n denotes sum of such n r.vs. Find $P[S_{100} > 5]$ .Given P[Z< 1] =0.8413 where Z has N(0,1). **(4)**

Q. P. Code: 11726

External (Revised) (3 Hours) [Total marks: 80]

Instructions:

1) Attempt any two questions from each section.
2) All questions carry equal marks.
3) Answer to Section I and section II should be written in the same answer book.

SECTION-I (Attempt any two questions)

1. A) Solve the linear Diophantine equations 172x + 20y =1000. [10]
   B) If $ca \equiv cb(mod\ n)$ then show that $a \equiv b\ (mod\ \frac{n}{d})$, where $d = \gcd(c, n)$. [10]

2. A) i) How many ways are there to distribute four identical balls and six distinct balls into five distinct boxes? [05]
   ii) Find the number of distribution of indistinguishable objects into distinguishable boxes provide no box is empty. [05]

   B) Using combinatorial argument prove that $S(n,k) = \sum_{r=0}^{n-1} {}^{n-1}_{r}C\ S(r, k-1)$. [10]

3. A) Give any sequence of $mn + 1$ distinct real numbers then prove that there exist either an increasing sequence of length $m + 1$ or decreasing sequence of length $n + 1$ or both. [10]
   B) i) Fifteen children gathered 100 nuts. Prove that some pairs of children gathered the same number of nuts. [05]
   ii) Assume that in a group of 6 people each pair of individuals consists of two friends or two enemies. Show that there are either 3 mutual friends or 3 enemies in group. [05]

4. A) Prove that 'A connected graph G is Euler if and only if the degree of every vertex is even'.
   B) i) Express $E(X, Y, Z) = X * (y' + Z)'$ in complete sum of product form. [05]
   ii) Let L be a bounded distributive lattice, then show that complements are unique if they exist. [05]

P.T.O….

SECTION-II (Attempt any two questions)

5. A) Prove that 'If $\emptyset$ is a fundamental matrix for $Y' = A(x)Y$ then the function φ is defined by $\varphi(x) = \emptyset(x) \int_{x_0}^{x} \emptyset^{-1}(t)b(t)dt$, $x \in I$, is a solution of $Y' = A(x)Y + B(x)$ satisfying $\varphi(x_0) = 0$. [10]

   B) Obtain approximate solution to with in t^5 of the initial value problem
   $\frac{dx}{dt} = xt + t^2, x(0) = 2.$ [10]

6. A) If $\emptyset_1(x)$ is a solution of $L_2(y) = 0$ on an interval I and $\emptyset_1(x) \neq 0$ on I then show that the other linearly independent solution of $L_2(y) = 0$ is
   $$\emptyset_2(x) = \emptyset_1(x) \int_{x_0}^{x} \left[\frac{1}{\emptyset_1(t)^2} e^{-\int a_1 t dt}\right] dt. \text{[10]}$$

   B)Solve the following IVP.
   $$\frac{dx}{dt} = 2x + y + z, \quad x(1) = 1$$
   $$\frac{dy}{dt} = 2y + 2z, \quad y(1) = 2$$
   $$\frac{dz}{dt} = 2z \quad z(1) = 3.$$ [10]

7. A)Show that the Legendre polynomial $P_n(x)$ of degree n is given by

   $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n.$ [10]

   B) Obtain solution in the form of power series of the following Differential equation:
   $\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + (4t^2 - 2)x = 0.$ [10]

8. A) Solve $\frac{\partial u}{\partial y} + c\frac{\partial u}{\partial x} = 0,\ u = u(x,y)$ with $u(x,0) = h(x)$ for a given $h: \mathbb{R} \to \mathbb{R}$. [10]

   B) Solve $u_x . u_y = u,\ u(x,0) = x^2.$ [10]

Q. P. Code: 11727

External (Revised) (3 Hours) [Total marks: 80]

Instructions:

1) Attempt any two questions from each section.
2) All questions carry equal marks.
3) Answer to Section I and section II should be written in the same answer book.

SECTION-I (Attempt any two questions)

Q.1] A) Let p be an odd price and $\gcd(a,p) = 1$. Prove that " 'a' is a quadratic residue of p if and only if $a^{(p-1)/2} \equiv 1(mod\ p)$". [10]

B) Use Cardanos method to find the roots of the cubic equation $64x^3 - 48x^2 + 12$. [10]

Q.2] A) i) Determine the number of ways to put k indistinguishable balls into n indistinguishable boxes. [05]

ii) How many non-negative integer solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 67$? [05]

B) In a survey of students it was found that 80 students knew Marathi, 60 knew English, 50 knew Hindi , 30 knew Marathi and English, 20 knew English and Hindi, 15 knew Marathi and Hindi and 10 knew all the three languages. How many students knew a) At least one language , b) Only Marathi and c) English but not both English and Hindi. [10]

Q.3] A) Let m and n be relative prime positive integer then prove that the system $x \equiv a(mod\ m)$ and $x \equiv (mod\ n)$ has a solution. [10]

B) A chess master who has 11 weeks to prepare for a tournament decides to play at least one game every day but, order not to tire himself, he decides not to play more than 12 games during any calendar weeks. Show that there exist a succession of consecutive days during which the master will have played exactly 21 games. [10]

Turn Over

Q.4] A) State and prove De-Morgans law for Boolean expressions in two variables. [10]

B) Show that a simple graph G is a tree if and only if any two distinct vertices are connected by a unique path. [10]

SECTION-II (Attempt any two questions)

Q.5] A) Verify the conditions of the existence and uniqueness theorem to conclude that the initial value problem $\frac{dy}{dx} = 3y + 1,\ y(0) = 2$ has a solution. Find the domain where the solution exists and use Picard iteration scheme to compute the first four approximations of the solution. [10]

B) Prove that " If $f: I \times \Omega \to \mathbb{R}^n$ has the locally Lipschitiz property then the initial value problem $\frac{dy}{dx} = f(t, x),\ x(t_0) = x_0$ has a solution $X: (t_0 - \delta, t_1 + \delta) \to \Omega$". [10]

Q.6] A) Solve the initial value problem :

$\frac{dx}{dt} = 3x + 4y,$ $\quad x(1) = 2$

$\frac{dy}{dt} = -4x + 3y\ ,$ $\quad y(1) = 3$

$\frac{dz}{dt} = yz + 3\ ,$ $\quad z(1) = 4.$ [10]

B) Show that $\{\emptyset_1, u_2\emptyset_1, u_3\emptyset_1, \dots \dots, u_n\emptyset_1\}$ is a basis for solution of $L_n(y) = 0$ on I where $\emptyset_1(x) \neq 0$ on I and $V_k = u'_k$ (k=1,2,3,……,n) are linearly independent solution of $\emptyset_1 V^{n-1} + \cdots \dots + (n{\emptyset_1}^{n-1} + a_1{\emptyset_1}^{n-2} + \cdots \dots + a_{n-1}\emptyset_1)V = 0.$ [10]

Q.7] A) Solve $y'' - 2xy' + y = 0,\ y(0) = 0,\ y'(0) = 1$ using power series. [10]

B) Solve Bessel's equation $x^2y'' + xy' + (x^2 - p^2)y = 0, \quad p \geq 0.$ [10]

Q.8] A) For the partial differential equation $u_y = {u_x}^3$ find the solution satisfying

$u(x,0) = 2x^{3/2}$. [10]

B) Find the integral surface of the equation $(2xy-1)p + (z-2x^2)q = 2(x-yz)$,

which passes through the line $x_0(t) = 1,\ y_0(t) = 0,\ z_0(t) = t$. [10]

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M.Sc (Maths) [Part - I)
Combinatorics
(May-2017)

Q. P. Code: 11747

(3 Hours) Total marks: 100

Scheme B(Internal/External) (2 Hours) Total marks: 40

N.B: 1) Scheme A students answer **any five** questions.
2) Scheme B students answer **any three** questions.
3) All questions carry equal marks.
4) Write on the top of your answer book the scheme under which you are appearing.

1. (a) How many two digit numbers have distinct and nonzero digits?

   (b) Determine number of 10-combinations of the multiset T = {3.a, 4.b, 5.c}

2. (a) Define D_n, derangement of n objects. Use combinatorial technique to show that $D_n = n!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots\ldots\ldots + (-1)^n \frac{1}{n!}\right)$, n ≥1. Hence find D_5.

   (b) If S(n,k) denotes Stirling numbers of second kind then show that

   (i) S(n,1) = 1 = S(n,n), (ii) $S(n,2) = 2^{n-1} - 1$,

   (iii) $S(n,n-1) = \binom{n}{2}$, for n ≥ 2.

3. (a) How many positive integers between 100 and 999 both inclusive are not divisible by either 3 or 4?

   (b) Give one application of Pigeon hole principle by stating and proving strong form of Pigeon hole principle.

4. (a) Show that $\binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \cdots = 2^{n-1}$

   (b) Compute the Möbius function of partially ordered set $(P(X_n),\subseteq)$ where $X_n = \{1,2,3,\ldots\ldots,n\}$.

5. (a) Solve the recurrence relation $a_n = 5a_{n-1} - 6a_{n-2}$ subject to the initial condition $a_0= 1$ and $a_1= -2$; n ≥ 2.

   (b) Prove that the family $A = \{ A_1,A_2,\ldots\ldots,A_n\}$ of sets has an SDR if and only if marriage condition holds true.

6. (a) State and prove Baye's theorem.

   (b) Define expectation of random variable. Prove that

   [i] E(c) = c where c is constant.

   [ii] E(aX + b) = aE(X) + b where a ,b are constants.

7. (a) In a bolt factory, machines A,B,C manufactures respectively 25%, 35% and 40% of the total. Of their output, 5%, 4% and 2% are defective bolts. One bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by machines A,B and C?

   (b) Find the sum of all coefficients in $(-3x + y - 4z)^4$.

8. (a) What is discrete random variable? Give two examples.

   (b) Show that the family of sets $A_1,A_2,\ldots,A_n$ has a system of distinct representatives if and only if for each k=1,2,….,n and for each choice of $i_1, i_2, \ldots, i_k$ with $1 \leq i_1 < i_2 < \ldots < i_k \leq n$; $|A_{i_1} \cup A_{i_2} \cup \ldots \cup A_{i_k}| \geq k$.

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