M.Sc (Mathematics) (Part-I)
Algebra - I (Revised)
(Paper – I)
(OCT-16)

QP Code : 75676

Revised] (3 Hours) **[Total Marks:80**

Instructions:
- Attempt **any two** questions from **each section**
- **All questions** carry **equal marks**
- **Answers** to **Section I** and **Section II** should be written in the **same answer book**.

SECTION I (Attempt **any two** questions)

1. (a) Let W_1 and W_2 be subspace of a finite dimensional vector V. Prove that $(W_1 \cap W_2)^\circ = W_1^\circ + W_2^\circ$, where W_1° and W_2° are annihilators of W_1 and W_2 respectively.

   (b) Using Gauss elimination method, solve of system of linear equations: $x_1 + x_2 + x_3 + x_4 = 0, 2x_1 + 3x_2 - x_3 - x_4 = 2, x_1 - x_2 + 2x_3 + 2x_4 = 3, 2x_1 + 5x_2 - 2x_3 - 2x_4 = 4$

2. (a) Let S and T be linear operators on a finite dimensional vector space V. Prove that : (i) det (ST) =det (S) det (T). (ii) T is invertible if and only if det $(T) \neq 0$.

   (b) Let $k_1, k_2, \ldots, k_n$ and $j_1, j_2, \ldots, j_n$ be positive integers. For $A \in M_n(k)$ define $D(A) = a_{j_1}k_1 a_{j_2}k_2 \ldots a_{j_n}k_n$. Show that D is n-linear if and only if $j_1, j_2, \ldots, j_n$ are distinct.

3. (a) Let $P(x)$ be the minimal polynomial of $n \times n$ matrix A. Show that the characteristic polynomial divides $(P(x))^n$.

   (b) Let V be a finite dimensional vector space over F and T be a linear operator on V than T is diagonalizable if and only if the minimal polynomial for T has the form $P(x) = (x - c_1)(x - c_2) \ldots (x - c_k)$. Where $c_1, c_2, \ldots, c_k$ are distinct elements of F.

4. (a) Let A be an invertible matrix. If v is an eigenvector of A, show it is also an eigenvector of both A^2 and A^{-2}. Find the corresponding eigen values.

   (b) Let $\langle .,. \rangle$ be a bilinear form on $\mathbb{R}^2$ defined by $\langle (x_1, x_2), (y_1, y_2) \rangle = 3x_1y_1 - 2x_1y_2 + 4x_2y_1 - x_2y_2$. Find the matrix (i) A of this bilinear form in the basis $\{u_1 = (1,1)$ and $u_2 = (1,2)\}$, (ii) B of this bilinear form in the basis $\{v_1 = (1,-1)$ and $v_2 = (3,1)\}$

SECTION II (Attempt **any two** questions)

5. (a) Compute order of each element in the following groups: (i) D_3 - Dihedral group of order 6, (ii) D_4 -Dihedral group of order 8

   (b) (i) A subgroup of a cyclic group is cyclic.

   (ii) Prove that $\mathbb{Z} \oplus \mathbb{Z}_2$ is not isomorphic to $\mathbb{Z}$

6. (a) Let G be a group acting on a set S and $\sim$ be a relation defined on S as: $a \sim b$ iff $a = g.b$ for some $g \in G$. Show that $\sim$ is an equivalence relation.

   (b) Prove that if G is a finite group and p a prime such that $p^k \mid |G|$ then G has a subgroup of order p^k

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7. (a) Let I, J be ideals of a commutative ring with unity. Show that $I \cup J$ is an ideal iff either $I \subseteq J$ or $J \subseteq I$.

   (b) Let I, J be ideals of a ring R. Define the 'product ideal' IJ. If $I + J = R$, then show that $IJ = I \cap J$.

8. (a) Prove that every principal ideal domain is unique factorization domain.

   (b) Show that $x^2 + 1$ and $x^2 + x + 4$ are irreducible polynomials in $\mathbb{Z}_{11}[x]$. Also show that $\frac{\mathbb{Z}_{11}[x]}{(x^2+1)}$ and $\frac{\mathbb{Z}_{11}[x]}{(x^2+x+4)}$ are fields having 121 elements.

M.Sc (Mathematics) (Part-I)
Algebra-I (OLD)
(Paper – I)
(OCT-16)

QP Code : 75673

Scheme A (External)]	(3 Hours)	**[Total Marks:100**
Scheme B (Internal)]	(2 Hours)	**[Total Marks: 40**

Instructions:

- **Mention** on the top of the **answer book** the **scheme** under which you are appearing
- **Scheme A** students should **attempt any five** questions
- **Scheme B** students should **attempt any three** questions
- **All questions** carry **equal marks**

1. (a) Show that a sub group of a cyclic group is cyclic.

   (b) Prove that any two right cosets of H in G are either identical or disjoint, H being a subgroup of G

2. (a) If H, K be two subgroups of a group G, then show that HK is a subgroup of G if and only if $HK = KH$

   (b) If N, M are normal subgroups of a group G, then show that $N \cap M$ is a normal subgroup of G.

3. (a) Prove that a group of order 99 is not simple.

   (b) If Z is the center of a group G such that G/Z is cyclic, then show that G is abelian.

4. (a) If F is a field, prove that it's only ideals are (0) and F itself.

   (b) Prove that a division ring is a simple ring.

5. (a) Show that $\mathbb{Z}$ is a P.I.D (Principal Ideal Domain)

   (b) If a ring R has no proper zero divisors then prove that $R[x]$ has no proper zero divisors.

6. (a) If $U(F)$ and $V(F)$ are two vector spaces and T is a linear transformation from U into V then show that the range of T is a sub space of V.

   (b) Show that every linearly independent subset of a finitely generated vector space $V(F)$ is either a basis of V or can be extended to form a basis of V.

7. (a) If A, B, C are linear transformation on a vector space $V(F)$ such that $AB = CA = I$ then show that A is invertible and $A^{-1} = B = C$

   (b) In $V3(\mathbb{R})$, examine each of the following sets of vectors for linear dependence :

      1. {(1,2,0), (0,3,1), (-1,0,1)}
      2. {(2,3,5), (4,9,25)}

8. (a) If in an inner product space $\|\alpha+\beta\| = \|\alpha\| + \|\beta\|$ then prove that the vectors α and β are linearly dependent. Give an example to show that the converse of this statement is false

   (b) Prove that the matrix $P = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ is not diagonalizable over the field $\mathbb{C}$.

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M.Sc (Mathematics) (Part-I)
Analysis - I & Topology (Revised)
(Paper – II)
(OCT-16)

QP Code : 75690

Revised] (3 Hours) **[Total Marks:80**

Instructions:
- Attempt **any two** questions from **each section**
- **All questions** carry **equal marks**
- **Answers** to **Section I** and **Section II** should be written in the **same answer book**.

SECTION I (Attempt **any two** questions)

Q. 1. (a) Show that a closed and bounded set in $\mathbb{R}$ is compact. **(10)**

(b) Show that a subset of $\mathbb{R}$ has Bolzano-Weierstrass property if and only if it is sequentially compact. **(10)**

Q. 2. (a) Show that a continuous function defined on a closed and bounded subset of $\mathbb{R}$ is uniformly continuous. **(10)**

(b) (i) Define connected set. Show that under a continuous function image of a connected set is a connected set. **(5)**

(ii) Show that $\mathbb{R}^2 \setminus \mathbb{Q} \times \mathbb{Q}$ is a path connected set. **(5)**

Q. 3. (a) Define differentiability of a function $f : E \subset \mathbb{R}^n \to \mathbb{R}^m$ at $a \in E$, where E is an open subset of $\mathbb{R}^n$. Prove that if f is differentiable at $p \in \mathbb{R}^n$, then all the partial derivatives of f exist. **(10)**

(b) Define directional derivative. Find the directional derivatives of the following function: $f(x, y) = 2xy + 3y^2$ at $p = (1, 1)$ in the direction of $v = (1, 1)$. **(10)**

Q. 4. (a) State (without proof) implicit function theorem. Show that the function $f : \mathbb{R}^2 \to \mathbb{R}^2$ given by $f(x, y) = (2xy, x^2 - y^2)$ is not invertible on $\mathbb{R}^2$, but locally invertible at every point of $E = \{(x, y) \in \mathbb{R}^2 \mid x > 0\}$. **(10)**

(b) State the Taylor's theorem for a function $f : \mathbb{R}^2 \to \mathbb{R}$. Find the Taylor expansion upto the third order for the function $f(x, y) = \sin(2x + 3y)$ at $(a, b) = (0, 0)$. **(10)**

SECTION II (Attempt **any two** questions)

Q. 5. (a) (i) Define topological space. Define interior point of a subset of a topological space. Prove that if S is a subset of a topological space X, then then the interior of S is an open subset of S. **(5)**

(ii) Let X, Y, Z be topological spaces and let $f : X \to Y$ and $g : Y \to Z$ be continous functions. Prove that $g \circ f : X \to Z$ is also continuous. **(5)**

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(b) (i) Define closure of subset of a topological space X. Prove that if $A \subset B$, then the closure of A is a subset of the closure of B. **(5)**

(ii) Define the terms: T_1 space and T_2 space. Prove that every T_2 space is a T_1 space. **(5)**

Q. 6. (a) (i) Find the connected components of $\{(x, y) \in \mathbb{R}^2 \mid x \neq 0\}$. **(5)**

(ii) Prove that the continuous image of a connected set is continuous. **(5)**

(b) Define the terms second countable space and separable space. Show that a second countable space is separable. **(10)**

Q. 7. (a) State and prove tube lemma. **(10)**

(b) Define local compactness and open function. Prove that local compactness is preserved under continuous, open functions. **(10)**

Q. 8. (a) State and prove Lebesgue covering lemma. **(10)**

(b) Define completion of a metric space. Is completion a topological property? Justify or give a counter-example. **(10)**

— PAPER ENDS —

M.Sc (Mathematics) (Part-I)
Analysis - I (Old)
(Paper – II)(OCT-16)

QP Code : 75687

Scheme A (External)] (3 Hours) [Total Marks:100
Scheme B (Internal)] (2 Hours) [Total Marks: 40

Instructions:
- **Scheme A** students should attempt **any five** questions.
- **Scheme B** students should attempt **any three** questions.
- All questions carry **equal marks**.
- **Mention** clearly the **Scheme** under which you are appearing.

Q. 1. (a) Prove that if $\sum_{n=1}^{\infty} a_n$ is an absolutely convergent series of real numbers, then every rearrangement of $\sum_{n=1}^{\infty} a_n$ converges and they all converge to the same sum. Prove by a counter example that the above result does not hold if the series is convergent but not absolutely. **(10)**

(b) State and prove the Weierstrass M-test. Use this to test the convergence of $\sum_{n=1}^{\infty} \frac{\sin nx}{n^2}$ on $\mathbb{R}$. **(10)**

Q. 2. (a) Suppose $\{f_n\}$ is a sequence of functions, differentiable on (a, b) and suppose that $f_n(x)$ converges to $f(x)$ on $[a, b]$. If $\{f_n'\}$ is uniformly Cauchy, then prove that $\{f_n\}$ converges uniformly to $f(x)$ and $f'(x) = \lim_{n\to\infty} f_n'(x)$ on (a, b). **(10)**

(b) If $\{f_n\}$ converges uniformly to f on (a, b) and if each f_n is continuous then prove that f is continuous. Prove using the sequence (x^n) that the result does not hold if the convergence is not uniform. **(10)**

Q. 3. (a) Prove that a non-empty subset X of $\mathbb{R}^n$ is connected if and only if every function $f : X \to \{0, 1\}$ is constant. **(10)**

(b) (i) Let f be a strictly increasing function on a set S in $\mathbb{R}$. Prove that f^{-1} exists and is strictly increasing on $f(S)$. **(5)**

(ii) Define the terms: open set and open interval. Prove that in $\mathbb{R}$ an open interval is an open set. **(5)**

Q. 4. (a) Let $f : E \subset \mathbb{R}^n \to \mathbb{R}^m$ be a differentiable function on an open set E. Suppose E contains the points a, b and line segment joining a and b. Then prove that there exists a point x_0 on this line segment satisfying $f(b) - f(a) = Df(x_0)(b - a)$. Show also that the result does not hold if the condition that line segment joining a and b lies in E is not satisfied. **(10)**

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(b) Show that a necessary condition for $f(x, y)$ to have an extreme value at (x_0, y_0) is that $f_x(x_0, y_0) = 0$ and $f_y(x_0, y_0) = 0$. Also show that this condition is not sufficient. **(10)**

Q. 5. (a) Find the point $P = (x, y, z)$ on the plane $2x + y - z - 5 = 0$ which lies closest to the origin. **(10)**

(b) State and prove inverse function theorem. **(10)**

Q. 6. (a) Define Riemann integrability of a function $f : [a, b] \to \mathbb{R}$. Prove that if f is Riemann integrable, then f is bounded on $[a, b]$. **(10)**

(b) State without proof Fubini's theorem. Calculate

$$\int \int_R f(x, y) dA$$

for $f(x, y) = 1 - 6x^2 y$ and R is the region given by $0 \leq x \leq 2, -1 \leq y \leq 1$. **(10)**

Q. 7. (a) State (without proof) the Taylor formula for real valued functions of a vector variable. Find the Taylor expansion of the function $f(x, y) = xe^y + \cos(xy)$ at $a = (1, 0)$. **(10)**

(b) Let f be a function of two variables x, y. Define total derivative of f at $p \in \mathbb{R}^2$. Prove that if the partial derivatives of f exist and are continuous at p, then the total derivative of f exists. **(10)**

Q. 8. (a) State the Leibnitz rule for differentiation under the integral sign. Hence or otherwise evaluate the integral $\int_0^{2\pi} e^{\cos\theta} \cos(\sin\theta) d\theta$. **(10)**

(b) State and prove chain rule. **(10)**

PAPER ENDS

M.Sc (Mathematics) (Part-I)
Topology
(Paper – III) (OCT-16)

QP Code : 75698

Scheme A (External)] (3 Hours) **[Total Marks:100**
Scheme B (Internal)] (2 Hours) **[Total Marks: 40**

Instructions:
- **Scheme A** students should attempt **any five** questions.
- **Scheme B** students should attempt **any three** questions.
- **All** questions carry **equal marks**.
- **Mention** clearly the **Scheme** under which you are appearing.

Q. 1. (a) (i) Show that the cardinality of any set is smaller than the cardinality of its power set. (5)
(ii) Show that $(0, 1)$ is an uncountable set. (5)

(b) (i) Give an example of a topological space which is T_1 but not T_2. Justify. (5)
(ii) Give an example of a connected metric space which is not path-connected. (5)

Q. 2. (a) State and prove the Baire category theorem. **(10)**

(b) (i) Define homeomorphism. Prove that (a, b) and $(0, 1)$ are homeomorphic. (5)
(ii) Define open map. Prove that $\pi_1 : X \times Y \to X$ given by $\pi_1(x, y) = x$ is an open map. (5)

Q. 3. (a) Prove that if X, Y are compact, then $X \times Y$ is also compact. **(10)**

(b) (i) Define quotient map. If $A \subset X$, a retraction of X onto A is a continnuous map $r : X \to A$ such that $r(a) = a$, for all $a \in A$. Prove that retraction is a quotient map. (5)
(ii) Prove that the union of a collection of connected subspaces of X that have a point in common is connected. (5)

Q. 4. (a) State and prove path-lifting theorem. **(10)**

(b) (i) Define the terms: covering space and covering map. (5)
(ii) Define homotopy. Prove that being homotopic is a transitive relation. (5)

Q. 5. (a) Define topological space. Define the closure of a subset A of X. Show that the closure of A is a closed subset of X. **(10)**

(b) (i) State without proof Zorn's lemma. (5)
(ii) Define complete metric space. Give an example of a metric space which is complete. Also give an example of a metric space which is not complete. (5)

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Q. 6. (a) Let X, Y be topological spaces with Y compact and Hausdorff. Prove that a map $f : X \to Y$ is continuous if its graph is closed in $X \times Y$. **(10)**

(b) (i) Define subbasis of a topological space X. Prove that open intervals of the type (a, b) with $a, b \in \mathbb{Q}$ form a subbasis for the usual topology on $\mathbb{R}$. **(5)**

(ii) Define compact set. Prove that a continuous image of a compact set is compact. **(5)**

Q. 7. (a) Define the terms: second countable space, separable space. Show that a second countable space is separable. **(10)**

(b) (i) Define local compactness. Is the set of rational numbers $\mathbb{Q}$ locally compact? **(5)**

(ii) Let X be a topological space such that X has a connected subset A such that the closure of A is X. Prove that X is connected. **(5)**

Q. 8. (a) State and prove tube lemma. **(10)**

(b) (i) Show that a closed subspace of a normal space is normal. **(5)**

(ii) Show that $\mathbb{R}$ with cofinite topology is not a first countable space. **(5)**

—PAPER ENDS—

M.Sc (Mathematics) (Part-I)
Complex Analysis (Revised)
(Paper – IV) (OCT-16)

QP Code : 75712

External (Revised) **(3 Hours)** **[Total marks: 80]**

Instructions:

1) Attempt any two questions from each section.
2) All questions carry equal marks.
3) Answer to Section I and section II should be written in the same answer book.

Section I

1) (a) Construct the Stereographic Projection Map.

(b) Find all the values of $\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)^{3/4}$

2)(a) Prove that if G is an open connected set and $f:G\to\mathbb{C}$ is differentiable with $f'(z)=0$ $\forall z\in G$, then f is constant.

(b) Find the Bilinear Transformation which maps the points $z=1,i,-1$ onto points $i,0,-i$. Also find the fixed points of the transformation.

3) (a) Suppose that f is analytic in a region G and $|f(z)|=$ constant in G. Prove that $f(z)$ is constant.

(b) Prove that $u(x,y)=x^3-3xy^2+3x^2-3y^2+2$ is a harmonic function. Also find its harmonic conjugate and the corresponding analytic function.

4) (a) State and prove Cauchy Deformation theorem.

(b) Evaluate $\int_0^{1+i} x^2+iy\,dz$ along(i) y = x(ii) Along the parabola y = x^2. Is the integral independent of path?

Section II

5) (a) State and prove Cauchy's estimate.

(b) Evaluate $\int_C \frac{z+6}{z^2-4}dz$,using Cauchy's Integral Formula where C is the circle

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(i) $|z|=1$ (ii) $|z-2|=1$ (iii) $|z+2|=1$.

6) (a) State and prove Open Mapping Theorem.

(b) Suppose f is non-constant and analytic in a domain of G. if $|f|$ attains minimum in G at α, then $f(\alpha)=0$.

7) (a) State and prove CasortiWeiestrass theorem.

(b) Find all the possible Laurent Series expansions of $f(z)=\dfrac{z-1}{(z-3)(z+1)}$.

8) (a) State and prove Rouche'stheorem.

(b) Use the residue theorem to evaluate $\displaystyle\int_0^{2\pi}\frac{\cos 2\theta}{5+4\cos\theta}\,d\theta$.

M.Sc (Mathematics) (Part-I)
Complex Analysis (Old)
(Paper – IV) (OCT-16)

QP Code : 75709

Scheme A(External) **(3 Hours)** **Total marks: 100**

N.B: 1) Scheme A students answer any five questions.
2) All questions carry equal marks.

1) (a) Define radius of convergence. Find the domain of the region of convergence of the following power series $\sum_{n=1}^{\infty} \frac{1 \cdot 2 \cdot \ldots\ldots (2n-1)}{n!} \left(\frac{1-z}{z} \right)^n$.

(b) Solve completely the equation $x^{10} + 11x^5 + 10 = 0$.

2) (a) Prove that a Mobius Transformation is a composition of translation, rotation, inversion and magnification.

(b) Prove that $u(x,y) = x^3 - 3xy^2 + 3x^2 - 3y^2 + 2$ is a harmonic function. Also find its harmonic conjugate and the corresponding analytic function.

3) (a) Let $0 \notin G$ be an open connected set in $\mathbb{C}$ and suppose that $f : G \to \mathbb{C}$ is analytic. Then prove that f is a branch of logarithm if and only if $f'(z) = \frac{1}{z}$, $\forall z \in G$ and $e^{f(a)} = a$ for atleast one $a \in G$.

(b) If $f(z) = u + iv$ is analytic, show that $\left(\frac{\partial |f(z)|}{\partial x} \right)^2 + \left(\frac{\partial |f(z)|}{\partial y} \right)^2 = |f'(z)|^2$.

4) (a) Prove that for every closed contour γ in $\mathbb{C}$ and $a \in \mathbb{C} \setminus \gamma, \eta(\gamma; a)$ is an integer, where η denotes the winding number.

(b) Find the image of the infinite strip $\frac{1}{2b} < y < \frac{1}{2a}$ under the transformation $w = \frac{1}{z}$. Show the region graphically.

5) (a) State and prove Cauchy's Theorem.

(b) Evaluate $\int_C \frac{z+6}{z^2-4} dz$,using Cauchy's Integral Formula where C is the circle i) $|z| = 1$ ii) $|z-2| = 1$

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6) (a) State and prove Morera's theorem.

(b) Find all the possible Laurent Series expansions of $f(z) = \frac{z^2 - 1}{z^2 + 5z + 6}$.

7) (a) State and prove Cauchy Residue theorem.

(b) Use the residue theorem to evaluate $\int_C \frac{z^2 + 1}{z(z-2)(z+4)^2} dz$, where C is the circle $|z| = 3$.

8) (a) Let f be meromorphic in a domain G and have only finitely many zeroes and poles. If γ is any simple closed curve in G such that no zeroes or poles of f lie on γ, prove that $\frac{1}{2\pi i} \int_\gamma \frac{f'(z)}{f(z)} dz = Z_f - P_f$, where Z_f *and* P_f denote respectively the number of zeroes and poles of f inside γ each counted according to their order

(b) Use Argument Principle to evaluate $\int_{|z|=\pi} \cot(\pi z) dz$.

M.Sc (Mathematics) (Part-I)
Soft Skills, Logic & Elementary Probability Theory (Revised) (Paper – V)
(OCT-16)

QP Code : 75725

External (Revised) **(3 Hours)** **[Total marks: 80]**

Instructions:

1) Attempt any two questions from each section.
2) All questions carry equal marks.
3) Answer to Section I and section II should be written in the same answer book.

SECTION I

Q.1.a) Prove that an equivalence relation R defined on a non empty set X induces a partition in X and a partition of X defines an equivalence relation on X. 8

b) Prove that for all sets A, B, and C,
(A - B) - C= (A - C) - (B-C). 6

c) Construct the truth table for each of the following statements. 6
$P \rightarrow (P \rightarrow Q)$, $P \vee Q \leftrightarrow Q \vee P$.

Q2)a) State and prove Schroeder-Bernstein theorem 10

b) Show that the set of all integers is countably infinite.Is the set of all rational numbers countable? Justify your answer. 10

Q3)a) State and prove Zorn's lemma 10

b) State the axiom of choice. Use it to prove the following: 10
"If $f: A \rightarrow B$ is surjective then it has a right inverse $h: B \rightarrow A$".

Q.4a) Prove that every permutation of order n can be expressed as a product of disjoint cycles. 8

b) Find the order and the signature of the permutation 6

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 \end{pmatrix}$$

.c) Describe the symmetries of a non square rectangle. Construct the corresponding composition table 6

SECTION II

Q.5)a) Prove Vander Monde Formula. 8

$$\binom{m+n}{k} = \sum_{i=0}^{k} \binom{m}{i} \binom{n}{k-i}$$

b) Three players enter a room and are given a red or a blue hat to wear. The color of each hat is determined by a fair coin toss. Players cannot see the color of their own hats, but do see the color of the other two players' hats. The game is won when at least one of the players correctly guesses the color of his own hat and no player gives an incorrect answer. In addition to having the opportunity to guess a color, players may also pass. Communication of any kind between players is not permissible after they have been given hats; however, they may agree on a group strategy beforehand. The players decided upon the following strategy. A player 7

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who sees that the other two players wear a hat with the same color guesses the opposite color for his/her own hat; otherwise, the player says nothing. What is the probability of winning the game under this strategy?

c) Is $F=\{A\subseteq\Omega/ A \text{ is a finite set}\}$ always a field? Justify 5

Q.6)a) State and prove Borel Cantelli Lemma 8

b) Let $\Omega=\{1,2,3,4,5,6\}$ with uniform probability . Show that if $A,B\subset \Omega$ are independent and A has 4 elements, then B must have 0,3 or 6 elements. 6

c) Consider $\Omega=[0,1]$ with a σ-field F of Borel sets B contained in [0,1]. Is $X(x)=|x-1/2|$ a random variable on Ω with respect to F. 6

Q.7)a) Prove that the Poison distribution is the limiting case of the Binomial distribution. 8

b) Find F_x if $\Omega=[0,1]x[0,1]$ is the unit square with uniform measure (which measure the surface area) and X(w) is the distance between $w\in \Omega$ and the nearest edge of the square. 6

c) A random variable X has the normal distribution $N(m,\sigma^2)$, where $m, \sigma\in \mathbb{R}$, if it is an absolutely continuous random variable with density 6

$$f_x(x)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(\frac{-(x-m)^2}{2\sigma^2}\right)$$

Verify that f_x is indeed a density.

Q.8.a) State and prove the weak law of large numbers 8

b) Compute the expectation of the random variable with negative binomial distribution. 6

c) We toss a coin repeatedly until head appears, hitting the numerical keypad of a calculator randomly after each toss. Find the conditional expectation of the number keyed in given that heads appear for the first time after i tosses.(Our calculator is a special model capable of displaying infinitely many digits.) 6

M.Sc (Mathematics) (Part-I)

Discrete Mathematics & Differential Equations (Revised) (Paper – IV)

(OCT-16)

QP Code : 75701

External (Revised) (3 Hours) [Total marks: 80]

Instructions:

1) Attempt any two questions from each section.
2) All questions carry equal marks.
3) Answer to Section I and section II should be written in the same answer book.

SECTION-I (Attempt any two questions)

1. (a) Use the Cardanos method to solve the cubic equation $x^3 + 3x^2 + 3x - 2 = 0$.

   (b) Let (a, b) =d then show that the Diophantine equation ax+by=c is solvable in integers if and only if d divides c.

2. (a) (i) How many different ways can we distribute n indistinguishable balls into k distinguishable boxes?

   (ii) Let X and Y be two finite sets with $|X| = n$ and $|Y| = k$, then show that total number of onto functions $f: X \to Y$ is $k!\, S(n,k)$.

   (b) Define derangement of finite objects. Let D_n denote the number of derangements of n. objects. Show that $D_n = \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots \ldots \ldots \ldots + (-1)^n \frac{1}{n!}\right)$.

3. (a) (i) 51 points are placed in an arbitrary way, into a square of side 1 unit. Prove that some 3 of these points can be covered by a circle of radius $\frac{1}{7}$.

   (ii) Show that among any n+1 numbers one can find two numbers so that their difference is divisible by n.

   (b) Show that every sequence of $n^2 + 1$ distinct real numbers contains a subsequence of length $n + 1$ that is either strictly increasing or strictly decreasing.

4. (a) (i) Define cycle in a simple graph. Show that every finite simple graph G with $|E(G)| \geq |V(G)|$ contains a cycle.

   (ii) Show that a simple finite graph G is a tree if and only if any two distinct vertices are connected by a unique path.

   (b) State and prove De-Morgan's law for Boolean expression in two variables.

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SECTION-II (Attempt any two questions)

5. (a) Prove that ' A function $\emptyset$ is a solution of the initial value problem $y' = f(x,y),\ y(x_0) = y_0$ on an interval I if and only if it is a solution of the integral equation $y = y_0 + \int_{x_0}^{x} f(t,y)dt$ on I'.

(b) Obtain approximate solution up to t^3: $\frac{dx}{dt} = 2y + t \quad x(1) = 1$

$$\frac{dy}{dt} = 3z + t^2 \quad y(1) = 2$$

$$\frac{dz}{dt} = xz \quad z(1) = 3.$$

6. (a) Let $\emptyset_1, \emptyset_2, \dots\dots\dots, \emptyset_n$ be n solution of $y^n + a_1(x)y^{n-1} + \cdots \dots\dots + a_n(x)y = 0$ on an interval I and $x_0 \in I$. Show that $\emptyset_1, \emptyset_2, \dots\dots\dots, \emptyset_n$ are linearly independent if and only if $W(\emptyset_1, \emptyset_2, \dots\dots\dots, \emptyset_n)(x_0) \neq 0$.

(b) Solve $\frac{dx}{dt} = 2x - 3y + 2 \quad x(0) = 1$

$$\frac{dy}{dt} = 3x + 2y + t \quad y(0) = 2$$

7. (a) Find the normal form of Bessel's equation $x^2y'' + xy' + (x^2 - p^2)y = 0$ use it to show that every non-trivial solution has infinitely many positive zeros.

(b) Solve $\frac{d^2x}{dt^2} + 2t\frac{dx}{dt} + 4x = 0$.

8. (a)Let the point $P(x_0, y_0, z_0)$ lie on the integral surface $S: z = f(x,y)$ of the quasi linear PDE $a(x,y,z)p + b(x,y,z)q = c(x,y,z)$. Let C be the characteristic curve through P. then show that C lies completely on S.

(b) Solve $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = u^3,\ \ u(x,0) = 4x^3$.

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M.Sc (Mathematics) (Part-I)
Combinatorics
(Paper – V)(OCT-16)

QP Code : 75722

Scheme A(External) **(3 Hours)** **Total marks: 100**

N.B: 1) Scheme A students answer any five questions.
2) All questions carry equal marks.

1. a. Define Stirling numbers of second kind S(n,k) and derive recurrence formula
for Stirling numbers of second kind S(n,k).
b. Find the numbers of non negative integer solutions of $x + y + z + w = 10$.

2. a. State and prove Mobius inversion formula.
b. Determine 10-combinations of multiset S = {4.a, 3.b, 4.c , 5.d}

3. a. A bakery boasts 8 varieties of doughnuts. If a box contains one dozen of doughnuts, how many different options are there for a box of doughnuts.
b. Find the coefficient of $x^3y^2z^4$ in $(3x - 5y + z)^9$. Also, Find sum of all coefficients in $(3x - 5y + z)^3$.

4. a. Write a note on derangement of n objects D_n. Derive formula for D_n.
b. Show that $\sum_{k=1}^{n} k\binom{n}{k}^2 = n\binom{2n-1}{n-1}$.

5. a. Prove that of any 10 points chosen within an equilateral triangle of side length 1, there are two whose distance apart is 1/3.
b. Find the number of positive integral solutions of
$(x_1 + x_2 + x_3)(y_1 + y_2 + y_3 + y_4) = 77$

6. a. In X County, 51% of the adults are males. One adult is randomly selected for a survey involving credit card usage.
i. Find the prior probability that the selected person is a male.
ii. It is later learned that the selected survey subject was smoking a cigar.

Also, 9.5% of males smoke cigars, whereas 1.7% of females smoke cigars (based on data from the Substance Abuse and Mental Health Services Administration). Use this additional information to find the probability that the selected subject is a male.

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b. Define variance of the discrete random variable. Compute the variance of the random variable with normal distribution.

7. a. If X and Y are independent random variables the show that characteristic function $\emptyset x + y(t) = \emptyset x(t).\emptyset y(t)$. Hence show that if X and Y two independently normally distributed random variables then X+Y is also normally distributed variable.

b. If X is a discrete random variable with probability mass function p(x) and sample space s. Then define expectation of X i. e. E(X). Hence prove that

i. E(X+b) = E(X) + b.

ii. E(aX) = a E(X)

iii. E(X +Y) = E(X) + E(Y)

8. a. You have three coins in your pocket, two fair one with probability of heads $\frac{1}{2}$ and the unfair one with probability of heads p and tails 1-p. A coin is selected at random and tossed, falling heads up. How likely is it that it is one of the fair coins?

b. In a sample, 2% of population has certain blood disease in a serious form; 10% have it in the mild form; and 88% don't have it at all. A new blood test is developed; the probability of testing positive is $\frac{9}{10}$ if the subject has the serious form of disease, $\frac{6}{9}$ if the subject has the mild form of disease, and $\frac{1}{10}$ if the subject doesn't have the disease. A person has just tested positive. What is the probability that the person has the serious form of disease?
