QP code: 24510

Q1 ⓐ $L[\sin t] = \frac{1}{s^2+1}$, $L[e^{-t}\sin t] = \frac{1}{(s+1)^2+1}$

$$\therefore L\left[\frac{1}{t}e^{-t}\sin t\right] = \int_s^\infty \frac{1}{(s+1)^2+1}\,ds = \left[\tan^{-1}(s+1)\right]_s^\infty = \frac{\pi}{2} - \tan^{-1}(s+1) = \cot^{-1}(s+1).$$

ⓑ $$L^{-1}\left[\frac{1}{\sqrt{2s+1}}\right] = L^{-1}\left[\frac{1}{\sqrt{2}}\frac{1}{\sqrt{s+\frac{1}{2}}}\right]$$

$$= \frac{1}{\sqrt{2}}e^{-\frac{1}{2}t}L^{-1}\left[\frac{1}{\sqrt{s}}\right]$$

$$= \frac{1}{\sqrt{2}}e^{-\frac{1}{2}t}\frac{t^{-1/2}}{\Gamma\frac{1}{2}} = \frac{1}{\sqrt{2\pi}}e^{-t/2}t^{-\frac{1}{2}}$$

ⓒ $f(z) = \sinh z = \sinh(x+iy) = \sinh x\cos y + i\cosh x\sin y$

$\therefore u = \sinh x\cos y$, $v = \cosh x\sin y$

$\therefore u_x = \cosh x\cos y$, $u_y = -\sinh x\sin y$

$v_x = \sinh x\sin y$, $v_y = \cosh x\cos y$

$\therefore u_x = v_y$, $u_y = -v_x$

Further, u_x, u_y, v_x, v_y are continuous and Cauchy-Riemann equations are satisfied.
Hence, $\sinh z$ is analytic.

Now, $f'(z) = u_x + iv_x$

$$= \cosh x\cos y + i\sinh x\sin y = \cosh(x+iy) = \cosh z.$$

ⓓ Let $f(x) = a_0 + \sum_{n=1}^\infty a_n\cos nx + \sum_{n=1}^\infty b_n\sin nx$

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} x\,dx = \frac{1}{2\pi}\left[\frac{x^2}{2}\right]_0^{2\pi} = \frac{1}{2\pi}\left[\frac{4\pi^2}{2}\right] = \frac{1}{2\pi}\cdot 2\pi^2 = \pi.$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} x\cos nx\,dx = \frac{1}{\pi}\left[\frac{x\sin nx}{n} + \frac{\cos nx}{n^2}\right]_0^{2\pi} = \frac{1}{\pi}\left[\frac{\cos 2n\pi}{n^2} - \frac{1}{n^2}\right]$$

$$= \frac{1}{\pi}\left[\frac{(-1)^{2n}}{n^2} - \frac{1}{n^2}\right] = \frac{1}{\pi}\left[\frac{1}{n^2} - \frac{1}{n^2}\right] = 0$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} x\sin nx\,dx = \frac{1}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) + \frac{\sin nx}{n^2}\right]_0^{2\pi} = \frac{1}{\pi}\left[-\frac{2\pi\cos(2n\pi)}{n}\right] = \frac{-2(-1)^{2n}}{n} = -\frac{2}{n}$$

$$\therefore \boxed{f(x) = \pi - 2\sum_{n=1}^\infty \frac{\sin nx}{n}}$$

Q.2 ⓐ $$\int_0^\infty e^{-t}\frac{\sin^2 t}{t}\,dt = L\left[\frac{\sin^2 t}{t}\right]_{s=1}$$

Now $\sin^2 t = \frac{1-\cos 2t}{2}$, $L[\sin^2 t] = L\left[\frac{1-\cos 2t}{2}\right] = \frac{1}{2}\frac{1}{s} - \frac{1}{2}\frac{s}{s^2+4}$

$$= \frac{1}{2}\left(\frac{1}{s} - \frac{s}{s^2+4}\right)$$

$$\therefore L\left[\frac{\sin^2 t}{t}\right] = \int_s^\infty \frac{1}{2}\left(\frac{1}{s} - \frac{s}{s^2+4}\right) ds$$

$$= \left[\frac{1}{2}(\log s) - \frac{1}{4}\log(s^2+4)\right]_s^\infty$$

$$= \left[\frac{1}{4}\log\frac{s^2}{s^2+4}\right]_s^\infty = -\frac{1}{4}\log\left(\frac{s^2}{s^2+4}\right)$$

$$= \frac{1}{4}\log\left(\frac{s^2+4}{s^2}\right)$$

$$\therefore \int_0^\infty e^{-t}\frac{\sin^2 t}{t}\,dt = L\left[\frac{\sin^2 t}{t}\right],\ s=1$$

$$= \frac{1}{4}\log\left(\frac{1+4}{1}\right) = \frac{1}{2}\log 5\ ,$$

(b) The sequence is $\{f(k)\} = \{\cdots\ 4^{-4}, 4^{-3}, 4^{-2}, 4^{-1}, 3^0, 3^1, 3^2, 3^3 \cdots\}$

Z-transform of $f(k)$ is

$$Z\{f(k)\} = \{\cdots\ 4^{-4}z^4 + 4^{-3}z^3 + 4^{-2}z^2 + 4^{-1}z + 3^0 z^0 + 3^1 z^{-1} + 3^2 z^{-2} + 3^3 z^{-3} + \cdots\}$$

$$\therefore Z\{f(k)\} = \left[\frac{z}{4} + \frac{z^2}{4^2} + \frac{z^3}{4^3} + \cdots\right] + \left[1 + \frac{3}{z} + \frac{3^2}{z^2} + \cdots\right]$$

$$= \frac{z}{4}\left[1 + \frac{z}{4} + \left(\frac{z}{4}\right)^2 + \cdots\right] + \left[1 + \frac{3}{z} + \left(\frac{3}{z}\right)^2 + \cdots\right]$$

$$= \frac{z}{4}\,\frac{1}{1-\frac{z}{4}} + \frac{1}{1-\frac{3}{z}} \quad \text{if } \left|\frac{z}{4}\right| < 1,\ \left|\frac{3}{z}\right| < 1$$

$$= \frac{z}{4}\,\frac{4}{4-z} + \frac{z}{z-3} = \frac{z}{4-z} + \frac{z}{z-3}, \quad |z| < 4,\ |z| > 3$$

$$\therefore Z\{f(k)\} = \frac{z}{(4-z)(z-3)} \quad \text{if } 3 < |z| < 4$$

(c) $\because u = \cos x \cosh y$

$u_x = -\sin x \cosh y$, $u_{xx} = -\cos x \cosh y$

$u_y = \cos x \sinh y$, $u_{yy} = \cos x \cosh y$

$$\therefore \nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = u_{xx} + u_{yy} = 0$$

$\therefore u = \cos x \cosh y$ is a harmonic function.

Now, we use Milne-Thompson method.

$u_x = \phi_1(x,y) = -\sin x \cosh y$, $\therefore \phi_1(z,0) = -\sin z$

$u_y = \phi_2(x,y) = \cos x \sinh y$ $\therefore \phi_2(z,0) = 0$

$$\therefore f'(z) = \phi_1(z,0) - i\,\phi_2(z,0) = -\sin z$$

$\therefore f(z) = \int -\sin z\,dz = \cos z + C$ is the required analytic function.

Now, $f(z) = \cos z + c$

$$= \cos(x+iy) + c$$

$$= \cos x \cosh y - i \sin x \sinh y + c$$

$\therefore u + iv = \cos x \cosh y - i \sin x \sinh y + c$

$\therefore v = -\sin x \sinh y$ is the required harmonic conjugate.

Q3 (a) Calculations of regression

Sr. No	x	x^2	y	y^2	xy
1	5	25	11	121	55
2	6	36	14	196	84
3	7	49	14	196	98
4	8	64	15	225	120
5	9	81	12	144	108
6	10	100	17	289	170
7	11	121	16	256	176
N=7	56	476	99	1427	811

Now, the line of regression of y on x is $y = a + bx$

The normal equation are

$\Sigma y = Na + b\Sigma x$ $\quad\therefore\ 99 = 7a + 56b$ — (1)

$\Sigma xy = a\Sigma x + b\Sigma x^2$ $\quad\therefore\ 811 = 56a + 476b$ — (2)

(1) × 56 − (2) × 7

$\Rightarrow$ $5544 = 392a + 3136b$

$5677 = 392a + 3332b$

$-133 = -196b$

$\therefore b = \frac{133}{196} = 0.6789$

from (1) $a = 8.7143$

$\therefore$ The equation of the line of regression of y on x is

$$\boxed{y = 8.7143 + 0.6786x}$$

(b) Let the transformation be $w = \dfrac{az+b}{cz+d}$ — (1)

Putting the given values,

$$0 = \frac{a+b}{c+d}, \quad 2 = \frac{-ai+b}{-ci+d}, \quad -i = \frac{2a+b}{2c+d}$$

$\therefore$ we get $a+b=0$ — (2)

$(a-2c)i + (2d-b) = 0$ — (3)

$(2c+d)i + (2a+b) = 0$ — (4)

from (2) $b=-a$, put in (3) & (4), we get

$(a-2c)i + (2d+a) = 0$ — (5)

$(2c+d)i + a = 0$ — (6)

(5) + (6) $\Rightarrow$ $(a+d)i + 2(a+d) = 0$

$\therefore (a+d)(i+2) = 0$

$\therefore d = -a \quad (\because i \neq -2)$

$\therefore$ ~~from~~ (II) $\therefore 2 = \dfrac{-ai+b}{-ci+d}$

$\Rightarrow 2 = \dfrac{-ai-a}{-ci-a} = \dfrac{a(1+i)}{ci+a}$

$\therefore 2ci + 2a = a + ai$

$\therefore 2ci = -a + ai$

$\therefore 2ci = ai^2 + ai = ai(1+i)$

$\therefore c = \left(\dfrac{1+i}{2}\right)a$

$\therefore$ ~~w~~ $= \dfrac{az+b}{cz+d} = \dfrac{az-a}{\left(\frac{1+i}{2}\right)az - a} = \dfrac{z-1}{\left(\frac{1+i}{2}\right)z - 1} = \dfrac{2(z-1)}{(1+i)z-2}$

$\therefore w = \dfrac{2(z-1)}{(1+i)z-2}$

(c) Let $f(x) = \sum b_n \sin nx$

$$\therefore b_n = \frac{2}{\pi}\left[\int_0^{\pi/2} x \sin nx\, dx + \int_{\pi/2}^{\pi} (\pi - x)\sin nx\right] dx$$

$$= \frac{4}{\pi} \frac{\sin\left(\frac{n\pi}{2}\right)}{n^2}$$

$$\therefore b_1 = \frac{4}{\pi}\frac{1}{1^2}, \quad b_2 = 0, \quad b_3 = -\frac{4}{\pi}\frac{1}{3^2}, \quad b_4 = 0 \cdots\cdots$$

$$\therefore f(x) = \frac{4}{\pi}\left[\frac{1}{1^2}\sin x - \frac{1}{3^2}\sin 3x + \frac{1}{5^2}\sin 5x - \cdots\right] \quad \text{— (1)}$$

By Parseval's Identity.

$$\frac{1}{\pi}\int_0^{\pi} [f(x)]^2 dx = \frac{1}{2}\left[b_1^2 + b_2^2 + b_3^2 + \cdots \infty\right]$$

$$\therefore \frac{1}{\pi}\left[\int_0^{\pi/2} x^2 dx + \int_{\pi/2}^{\pi} (\pi - x)^2 dx\right] = \frac{1}{2}\left[b_1^2 + b_2^2 + \cdots\right]$$

$$\text{L.H.S} = \frac{\pi^3}{12}$$

$$\therefore \frac{\pi^3}{12} = \frac{1}{2}\left[\frac{16}{\pi^2}\frac{1}{1^4} + \frac{16}{\pi^2}\frac{1}{3^4} + \frac{16}{\pi^2}\frac{1}{5^4} + \cdots\right]$$

$$\therefore \frac{\pi}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots\cdots$$

$$\boxed{\therefore \sum_{(2n-1)}^{n} \frac{1}{n^4} = \frac{\pi}{96}}$$

Q.4 (a). Let $F(s) = \frac{1}{s-a}$, $G(s) = \frac{1}{(s+a)^2}$

$$\therefore f(t) = L^{-1}[F(s)] = e^{at}, \quad g(t) = L^{-1}[G(s)] = t\, e^{-at}$$

$$\therefore L^{-1}\left[F(s)\, G(s)\right] = L^{-1}\left[\frac{1}{(s-a)(s+a)^2}\right] = \int_0^t f(u)\, g(t-u)\, du$$

$$= \int_0^t e^{au} e^{-a(t-u)} (t-u)\, du$$

$$= \int_0^t e^{2au - at} (t-u)\, du$$

$$= e^{-at} \int_0^t e^{2au} (t-u)\, du$$

$$= \frac{1}{4a^2}\left[e^{at} - 2at\, e^{-at} - e^{-at}\right]$$

Q4 (b) Calculation of r

x	x^2	y	y^2	xy
8	64	3	9	24
8	64	4	16	32
7	49	10	100	70
5	25	13	169	65
6	36	22	484	132
2	4	8	64	16
$\Sigma x=36$	$\Sigma x^2=242$	$\Sigma y=60$	$\Sigma y^2=842$	$\Sigma xy=339$

$\bar{x}=6,\ \bar{y}=10$

$$r=\frac{\Sigma xy-\frac{\Sigma x\Sigma y}{n}}{\sqrt{\left(\Sigma x^2-\frac{(\Sigma x)^2}{n}\right)\left(\Sigma y^2-\frac{(\Sigma y)^2}{n}\right)}}$$

$$=\frac{339-\frac{(36)(60)}{6}}{\sqrt{\left(242-\frac{(36)^2}{6}\right)\left(842-\frac{(60)^2}{6}\right)}}=\frac{339-360}{\sqrt{242-216}\sqrt{842-600}}$$

$$=\frac{-21}{\sqrt{26}\sqrt{242}}=\frac{-21}{\sqrt{6292}}=\frac{-21}{22\sqrt{13}}$$

$$=-0.2646$$

$$\therefore \boxed{r=-0.2646}$$

(c) (i) If $|z|<a$

$$F(z)=\frac{1}{a^2\left[1-\left(\frac{z}{a}\right)\right]^2}=\frac{1}{a^2}\left[1-\frac{z}{a}\right]^{2}=\frac{1}{a^2}\left[1+2\cdot\frac{z}{a}+3\cdot\frac{z^2}{a^2}+\cdots+(n+1)\frac{z^n}{a^n}+\cdots\right]$$

$$=\frac{1}{a^2}+2\cdot\frac{z}{a^3}+3\cdot\frac{z^2}{a^4}+\cdots+(n+1)\frac{z^n}{a^{n+2}}+\cdots$$

coefficient of $z^n=\frac{n+1}{a^{n+2}},\ n\geq 0$

$\therefore$ coefficient of $z^{-k}=\frac{-k+1}{a^{-k+2}},\ -k\geq 0$ ie $k\leq 0$

$$\therefore z^{-1}[F(z)]=z^{-1}\left[\frac{1}{(z-a)^2}\right]=\left\{\frac{-k+1}{a^{-k+2}}\right\},\ k\leq 0$$

(ii) If $|z|>3$, clearly $|z|>2$ ie $\left|\frac{z}{3}\right|>1$ & $\left|\frac{z}{2}\right|>1$ ie $\left|\frac{3}{z}\right|<1$ & $\left|\frac{2}{z}\right|<1$, Hence,

$$F(z)=\frac{1}{(z-3)(z-2)}=\frac{1}{z-3}-\frac{1}{z-2}$$

$$=\frac{1}{z}\frac{1}{\left(1-\frac{3}{z}\right)}-\frac{1}{z\left(1-\frac{2}{z}\right)}$$

$$=\frac{1}{z}\left(1-\frac{3}{z}\right)^{-1}-\frac{1}{z}\left(1-\frac{2}{z}\right)^{-1}$$

$$=\frac{1}{z}\left(1+\frac{3}{z}+\frac{3^2}{z^2}+\frac{3^3}{z^3}+\cdots+\frac{3^{k-1}}{z^{k-1}}+\cdots\right)-\frac{1}{z}\left(1+\frac{2}{z}+\frac{2^2}{z^2}+\cdots+\frac{2^{k-1}}{z^{k-1}}+\cdots\right)$$

$$=\left(\frac{1}{z}+\frac{3}{z^2}+\frac{3^2}{z^3}+\cdots+\frac{3^{k-1}}{z^k}+\cdots\right)-\left(\frac{1}{z}+\frac{2}{z^2}+\frac{2^2}{z^3}+\cdots+\frac{2^{k-1}}{z^k}+\cdots\right)$$

$\therefore$ coefficient of $z^{-k}=3^{k-1}-2^{k-1},\ k\geq 1$

$$\therefore z^{-1}[F(z)]=\left\{3^{k-1}-2^{k-1}\right\},\ k\geq 1$$

Q5 (a) $\int_0^{\infty} e^{-t}(1+2t-t^2+t^3)H(t-1)\,dt$

$= L\left[(1+2t-t^2+t^3)H(t-1)\right], \ s=1$

$\cancel{\text{Now } L\left[(1+2t-t^2+t^3)H(t-1)\right]} = \cancel{e^{-s}\left[L\left[1+2(t+1)-(t+1)^2+(t+1)^3\right]\right]}$

$= \cancel{L\left[t^3+2t^2+3t+3\right]}$

Here, $f(t) = 1+2t-t^2+t^3$

$\therefore f(t+1) = 1+2(t+1)-(t+1)^2+(t+1)^3$

$\therefore L\left[f(t+1)\right] = L\left[t^3+2t^2+3t+3\right]$

$= \frac{3!}{s^4} + 2\cdot\frac{2!}{s^3} + 3\cdot\frac{1}{s^2} + 3\cdot\frac{1}{s}$

$\therefore L\left[(1+2t-t^2+t^3)H(t-1)\right] = e^{-s}\left[\frac{6}{s^4}+\frac{4}{s^3}+\frac{3}{s^2}+\frac{3}{s}\right]$

$\therefore \int_0^{\infty} e^{-t}(1+2t-t^2+t^3)H(t-1)\,dt = e^{-s}\left[\frac{6}{s^4}+\frac{4}{s^3}+\frac{3}{s^2}+\frac{3}{s}\right], \ s=1$

$= e^{-s}\left[\frac{3}{1}+\frac{3}{1}+\frac{4}{1}+\frac{6}{1}\right] = \frac{16}{e}$ //

Q5 (b) we have $f_n(x) = \cos nx$

$\therefore \int_{-\pi}^{\pi} f_n(x) f_m(x)\,dx = \int_{-\pi}^{\pi} \cos mx \cos nx\,dx$

$= \begin{cases} 0, & m \neq n \\ \pi, & m = n \end{cases}$

Here the set of function $\cos x, \cos 2x, \cos 3x, \ldots$ is a set of orthogonal function over $[-\pi, \pi]$.

The required orthonormal set of function is $\frac{\cos x}{\sqrt{\pi}}, \frac{\cos 2x}{\sqrt{\pi}}, \frac{\cos 3x}{\sqrt{\pi}}, \ldots$ in $[-\pi, \pi]$

Q-5 (c) $L\left[D^3 - 2D^2 + 5D\right] = L[0]$

$s^3 L[y(t)] - s^2 y(0) - s y'(0) - y''(0) - 2\left[s^2 L[y(t)] - s y(0) - y'(0)\right] + 5\left[s L[y(t)] - y(0)\right] = 0$

$\therefore L[y(t)](s^3 - 2s^2 + 5s) - 1 = 0$

$\therefore L[y(t)] = \frac{1}{s^3 - 2s^2 + 5s}$ $\quad (\because y(0) = 0, y'(0) = 0, y''(0) = 1)$

$[y(t)] = L^{-1}\left[\frac{1}{s^3 - 2s^2 + 5s}\right] = L^{-1}\left[\frac{1}{s[(s-1)^2 + 2^2]}\right]$

$$y(t) = L^{-1}[F(s)\,G(s)] \quad , \quad F(s) = \frac{1}{(s-1)^2+2^2} \quad , \quad G(s) = \frac{1}{s}$$

$$f(t) = \frac{1}{2} e^{t} \sin 2t \quad , \quad g(t) = 1$$

$$\therefore y(t) = \int_0^t f(u)\, g(t-u)\, du \quad \text{by Convolution theorem.}$$

$$= \int_0^t \frac{1}{2} e^{u} \sin 2u\, du = \frac{1}{2}\cdot\frac{1}{5}\left[e^{u}(\sin 2u - 2\cos 2u)\right]_0^t$$

$$= \frac{1}{10}\left[e^{t}(\sin 2t - 2\cos 2t) + 2\right]$$

$$\boxed{\therefore y(t) = \frac{1}{5} - \frac{1}{5} e^{t}\cos 2t + \frac{1}{10} e^{t}\sin 2t.}$$

Q6 (a). $f(x) = \sum_{-\infty}^{\infty} c_n e^{inx}$

where, $c_n = \frac{1}{2\pi}\int_0^{2\pi} f(x)\, e^{-inx}\, dx$

$$= \frac{1}{2\pi}\int_0^{2\pi} 2x\, e^{-inx}\, dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} x\, e^{-inx}\, dx$$

$$= \frac{1}{\pi}\left[-\frac{2\pi}{in} + \frac{1}{n^2} - \frac{1}{n^2}\right] \text{ for } n \neq 0 \quad \left[\because e^{-i2n\pi} = \cos(2n\pi) - i\sin(2n\pi) = 1\right]$$

$$= \frac{-2}{in} = \frac{-2i}{i^2 n} = \frac{2i}{n} \quad \text{for } n \neq 0$$

For $n=0$

$$c_n = \frac{1}{2\pi}\int_0^{2\pi} 2x\, e^{0}\, dx = \frac{1}{2\pi}\int_0^{2\pi} 2x\, dx = 2\pi$$

Hence $f(x) = 2\pi + 2i \sum_{n=-\infty}^{\infty} \frac{1}{n} e^{inx}$ for $n \neq 0$

Putting $n = \pm 1, \pm 2, \pm 3 \cdots$, we get

$$f(x) = 2\pi + 2i\left[\frac{e^{ix}}{1} + \frac{e^{2ix}}{2} + \frac{e^{3ix}}{3} + \cdots - \frac{e^{-ix}}{1} - \frac{e^{-2ix}}{2} - \frac{e^{-3ix}}{3} \cdots\right]$$

$$= 2\pi + 2i\left[\frac{e^{ix} - e^{-ix}}{1} + \frac{1}{2}\left(\frac{e^{2ix} - e^{-2ix}}{1}\right) + \frac{1}{3}\left(\frac{e^{3ix} - e^{-3ix}}{1}\right) \cdots\right]$$

$$= 2\pi + 4i^2\left[\left(\frac{e^{ix} - e^{-ix}}{2i}\right) + \frac{1}{2}\left(\frac{e^{2ix} - e^{-2ix}}{2i}\right) + \frac{1}{3}\left(\frac{e^{3ix} - e^{-3ix}}{2i}\right) + \cdots\right]$$

$$= 2\pi - 4\left[\sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x + \cdots\right]$$

Q.6 (b) Let $f(z) = u+iv$ then $\overline{f(z)} = u-iv = u+i(-v)$

$\because f(z)$ is analytic, $u_x = v_y$ & $u_y = -v_x$ by C-R-equation.

$\because \overline{f(z)}$ is analytic, $u_x = -v_y$ & $u_y = -(-v_x)$ by C-R equation.

Adding $u_x = v_y$ & $u_x = -v_y$, we get, $u_x = 0$

Adding $u_y = -v_x$ & $u_y = v_x$, we get, $u_y = 0$

$\because u_x = 0$ & $u_y = 0$, $u =$ a constant.

Similarly by subtraction we can prove that $v_x = 0$ & $v_y = 0$

$\therefore v =$ a constant.

Hence, $f(z) = u+iv =$ a constant.

Q.6 (c). Let the equation of the curve be $y = ab^x$

Taking logarithm of both sides, we get

$\log y = \log a + x \log b$

Let $\log y = Y$, $\log a = A$, $\log b = B$, $X = x$

$\therefore Y = A + BX$

Sr. No	$X = x$	y	$Y = \log y$	X^2	XY
1	1	151	2.1790	1	2.1790
2	2	100	2.0000	4	4.0000
3	3	61	1.7853	9	5.3559
4	4	50	1.6990	16	6.7960
5	5	20	1.3010	25	6.5050
6	6	8	0.9031	36	5.4186
N=6	21		9.8674	91	30.2545

Now, the normal equations are

$\Sigma Y = NA + B\Sigma X$, $\Sigma XY = A\Sigma X + B\Sigma X^2$

$\therefore$ we get $9.8674 = 6A + 21B$ — (1)

$30.2545 = 21A + 91B$ — (2)

(1) $\times 7$ − (2) $\times 2$ ⇒

$B = -0.2447$

From (1) $A = 2.5010$

$\therefore a = \text{antilog } A = 316.96$

$b = \text{antilog } B = 0.5692$

$$\boxed{\therefore y = 316.96\,(0.5692)^x.}$$

—×—×—