Q1   QP code: 23178

a $L(\sin^2 3t) = L\left(\left(\frac{1-\cos 6t}{2}\right)^2\right)$

$= \frac{1}{4}\left\{ \frac{3}{2s} - \frac{2s}{s^2+36} + \frac{s}{2(s^2+144)} \right)$

b. $u = \log\sqrt{x^2+y^2}$ $v = \tan^{-1}\left(\frac{y}{x}\right)$

$u_x = v_y = \frac{x}{x^2+y^2}$ $u_y = -v_x$

$u_y = \frac{y}{x^2+y^2}$ $v_x = \frac{-y}{x^2+y^2}$

C-R Equations

c. $f(x)$ is an even function $b_n = 0$

$a_0 = \frac{8}{3}$ $a_n = \frac{8(-1)^n}{n^2\pi^2}$

$f(x) = \frac{a_0}{2} + \sum_1^\infty a_n \cos\frac{n\pi x}{2}$ $l = 2$

d. $z\{\cos 2k\} = \sum_{k=0}^{\infty}\left(\frac{e^{i2k}+e^{-2ik}}{2}\right) z^{-k} = \frac{z(z-\cos 2)}{z^2 - 2z\cos 2 + 1}$

Q2 a

$$\nabla \times \bar{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xyz^3 & x^2z^3 & 3x^2yz^2 \end{vmatrix} = 0 \quad \text{irrotational}$$

Scalar potential is $x^2yz^3 + C$

b. $L^{-1}\left(\frac{1}{((s+3)^2+9)^2}\right)$

$= e^{-3t} L^{-1}\left(\frac{1}{(s^2+9)^2}\right)$

$L^{-1}\left(\frac{1}{s^2+9}\right) = \frac{1}{3}\sin 3t = f(t)$ $g(t) = \frac{1}{3}\sin 3t$

$f(u) = \frac{1}{3}\sin 3u$ $g(t-u) = \frac{1}{3}\sin 3(t-u)$

$L^{-1}\left(\frac{1}{(s^2+6s+9)^2}\right) = \frac{e^{-3t}}{9}\int_0^t \sin 3u \sin 3(t-u)\,du$

$= \frac{e^{-3t}}{18}\left[\frac{\sin 3t}{3} - t\cos 3t\right]$

e. $a_0 = 0 \quad a_n = 0 \quad b_n = \frac{1}{n}$

$f(x) = \sum_{1}^{\infty} \frac{\sin nx}{n}$ for deduction put $x = \frac{\pi}{2}$

Q3 a. $f(z)(i+1) = u - v + i(u+v)$

$F(z) = U + iV$

$F'(z) = V_y + iV_x$ C-R Equs

$F'(z) = \cos x \sinh y - \sin x \cosh y - i(\sin x \cosh y + \cos x \sinh y)$

$F'(z) = (1+i)(-\sin z)$ by Milne Thomson meth

$F(z) = (1+i)(\cos z) + c$

$f(z) = \cos z + c$

b. $Z^{-1}\left(\frac{2z^2 - 10z + 13}{(z-3)^2(z-2)}\right)$

$F(z) = \frac{1}{z-2} + \frac{1}{z-3} + \frac{1}{(z-3)^2}$

$z^{-1}(f(z)) = 2^{k-1} \quad k \geq 1$

$= \frac{-(k+2)}{3^{-k+2}} \quad k \leq 0$

c. $L[y''(t) + 2y'(t) + y(t)] = \frac{t^3}{(s+1)^2}$

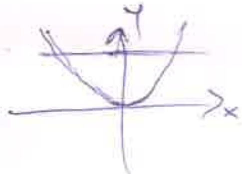
$\bar{y}(s) = \frac{3}{(s+1)^4} + \frac{4s+4}{(s+1)^2} + \frac{6}{(s+1)^2}$

$= e^{-t}\frac{t^3}{2} - 4e^{-t} + 6te^{-t}$

Q4 a. $V = -2xy - \frac{3x^2}{2} - \frac{3y^2}{2} = C$

b. Green's theorem $\int_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$

RHS $= 2\int_0^4 \int_0^{\sqrt{y}} dx\,dy = 2\int_0^4 \sqrt{y}\,dy = \frac{32}{3}$



Q5 a. $L^{-1}\left(\frac{2s^2 - 6s + 5}{s^3 - 6s^2 + 11s - 6}\right) = \frac{1}{2}e^t - e^{2t} + \frac{5}{2}e^{3t}$

b. $w = \frac{i-z}{i+z}$

c. $\int_C \bar{F} \cdot d\bar{r} = \iint_S (\nabla \times \bar{F}) \cdot \hat{n}\, ds \qquad \nabla \times \bar{F} = 0 \quad \therefore \iint \nabla \times \bar{F} \cdot \hat{n}\, ds = 0$

$\int_C \bar{F} \cdot d\bar{r} = 0$