Ans

1(a) $L\left\{e^{-2t}\frac{\sin 2t \cosh t}{t}\right\} = L\left\{e^{-2t}\frac{\sin 2t}{t}\cdot\frac{(e^{t}+e^{-t})}{2}\right\}$

$\because L\left\{\frac{\sin t}{t}\right\} = \cot^{-1}s$

$L\left\{\frac{\sin 2t}{2t}\right\} = \frac{1}{2}\cot^{-1}\left(\frac{s}{2}\right)$

$\Rightarrow L\left\{e^{-2t}\frac{\sin 2t \cosh t}{t}\right\} = L\left\{e^{-t}\frac{\sin 2t}{2t} + e^{-3t}\frac{\sin 2t}{2t}\right\}$

$= \frac{1}{2}\left[\cot^{-1}\left(\frac{s+1}{2}\right) + \cot^{-1}\left(\frac{s+3}{2}\right)\right]$

Ans

1(b) Cayley - Hamilton Theorem

$|A-\lambda I| = 0 \Rightarrow \lambda^2 - 4\lambda = 0$

To show $A^2 - 4A = 0$ $\quad A^2 = \begin{bmatrix} 8 & 16 \\ 4 & 18 \end{bmatrix}$

A^{-1} doesn't exist

$A^3 - 5A^2 = A^2$

1(c) $\int_0^1 f(x)\,dx = 1$

(i) $k = 6$

(ii) $P(0<x<1/2) = \frac{15}{64} = 0.234$

Ans 1(d) $w = \frac{2}{z+i} \Rightarrow z+i = \frac{2}{w}$

$z = \frac{2}{w} -$

$x + i(y+1) = \frac{2(u-iv)}{u^2+v^2}$

$\Rightarrow y+1 = \frac{-2v}{u^2+v^2}$

Real axis is $y=0$

$\therefore$ The map is $\underline{u^2+v^2+2v=0}$

circle eqn.

Ans 2(a) Given $u = e^{-x}(y\cos y - x\sin y)$

$f'(z) = u_x + iv_x$

$= u_x - i u_y$

$u_x = e^{-x}[-y\cos y + x\sin y + \sin y] = \phi_1(x,y)$

$u_y = e^{-x}[\cos y - y\sin y - x\cos y] = \phi_2(x,y)$

Put $x=z,\ y=0$

$f'(z) = i(z-1)e^{-z}$

$\boxed{f(z) = -ize^{-z} + C}$ Ans.

Ans 2(b) $L\{\cos u\} = \frac{s}{s^2+1}$

$L\{e^{-4u}\cos u\} = \frac{s+4}{(s+4)^2+1}$

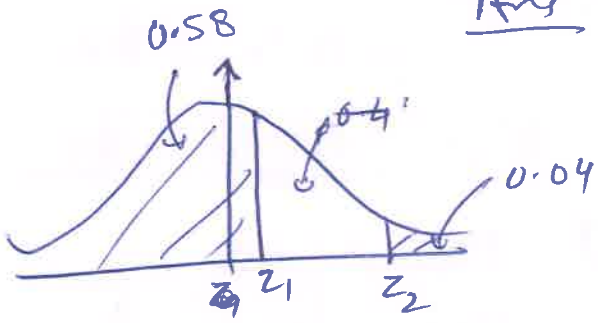
$$L\left\{\int_0^t e^{-4u}\cos u\,du\right\} = \frac{s+4}{s(s^2+8s+17)}$$

$$L\left\{t\int_0^t e^{-4u}\cos u\,du\right\} = (-1)\frac{d}{ds}\left[\frac{s+4}{s(s^2+8s+17)}\right]$$

$$\int_0^\infty e^{-t}\left(t\int_0^t e^{-4u}\cos u\,du\right)dt = \frac{77}{338} = \underline{\underline{0.227}}$$

Ans

2(c)



$$z_1 = \frac{x-m}{\sigma}$$

$$0.2 = \frac{75-m}{\sigma} \quad (1)$$

$$1.8 = \frac{80-m}{\sigma} \quad (2)$$

Solve (1) & (2)

$\sigma = 3.125$, $m = 74.4$ marks

3(a). $|A-\lambda I| = 0$

$\lambda = -1, -1, 3$ eigen values of matrix A

For $\lambda = -1$ eigen vectors are

$X_1 = [1, 0, 2]'$ $X_2 = [1, 2, 0]'$

For $\lambda = 3$ eigen vectors are

$X_3 = [1, 1, 2]'$

$$D = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \qquad M = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 2 & 0 & 2 \end{bmatrix}$$

Q3 (c) $f(s) = \frac{s}{s^2+4}$ $g(s) = \frac{1}{s^2+4}$

By Convolution $\int_0^t \cos 2u \, \frac{1}{2}\sin 2(t-u)\,du = \underline{\frac{t\sin 2t}{4}}$ Ans

(3)

4(a) $p = 1/4 \quad q = 3/4$

(i) $n = 7$

$P(X \geq 2) = 1 - [P(X=0) + P(X=1)] = \underline{\underline{0.555}}$ Ans

$P(X = x) = {}^{n}C_x \, p^x q^{n-x}$

(ii)

$1 - P(0) > 2/3$

$n > 3.82$

$\underline{\underline{n = 4}}$ Ans

4(b) Mean $= 47$ $\quad$ $H_0 = \sigma = \sqrt{20}$

Var $= \Sigma(x_i - \bar{x})^2 = 280$ $\quad$ $H_a = \sigma \neq \sqrt{20}$

$$\chi^2_{cal} = \frac{\Sigma(x_i - \bar{x})^2}{\sigma^2} = \frac{280}{20} = 14$$

$\chi^2_{cal} < \chi^2_{9\,dof}$

H_0 is accepted

4(c)

$$\frac{(w-1)(i+1)}{(1-i)(-1-w)} = \frac{(z-2)(i+2)}{(2-i)(-2-z)}$$

$$w = \frac{3z + 2i}{zi + 6}$$

Ans

5(a) Eigen values are $\lambda = 1, 2, 3$

For $\lambda = 1$ $\quad X_1 = t[0, 1, 1]'$

$\lambda = 2$ $\quad X_2 = t[1, 1, 1]'$ $\quad$ For $\lambda = 3$ $\quad X_3 = t[1, 0, 1]'$

5(b) $H_0 = \mu_b = \mu_g$

$H_a = \mu_b > \mu_g$ (right tailed test)

$$Z_{cal} = \left| \frac{\mu_b - \mu_g}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \right| = 1.15$$

$Z_\alpha = 2.326 \Rightarrow Z_{cal} < Z_\alpha$

H_0 is accepted. Boys do not perform better than girls

5 ~~(c)~~ ~~$y = 4e^{3t} - e^{-t}$ Ans~~

~~6(a)~~

5(c) $y = \frac{1}{6}\left[e^{4t} + 2e^{-2t} - 3\right]$

6(a) $\frac{3s+7}{s^2 - 2s - 3} = \frac{4}{s-3} - \frac{1}{s+1}$

Ans $= \underline{\underline{4e^{3t} - e^{-t}}}$

6(b) $R = 1 - \frac{6 \sum d_i^2}{n^3 - n} = -0.879$

6(c) $A = \begin{bmatrix} 1 & -1 & 1/2 \\ -1 & 2 & -1 \\ 1/2 & -1 & 2 \end{bmatrix} \Rightarrow A = IAI$

On Transformation

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & \frac{1}{\sqrt{6}} & \sqrt{2/3} \end{bmatrix} A \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1/\sqrt{6} \\ 0 & 0 & \sqrt{2/3} \end{bmatrix}$$

$X = PY$

$x = u + v$; $y = v + w/\sqrt{6}$; $z = \sqrt{2/3}\, w$;

Rank is 3, Signature $= 3 - 0 = 3$

(5)