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QP code: 19834

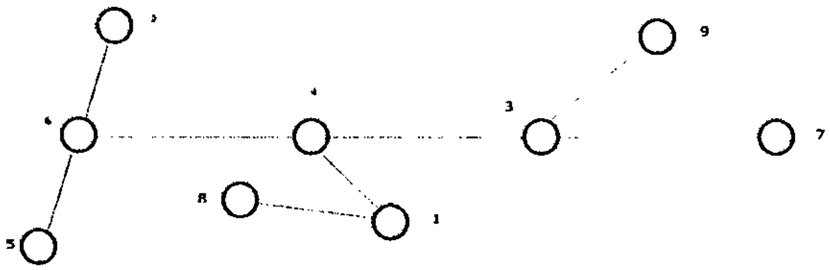
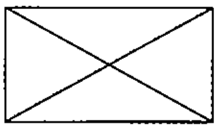
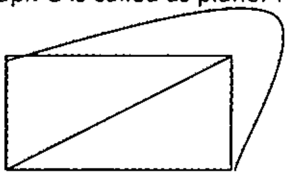
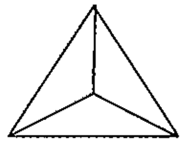
Semester III

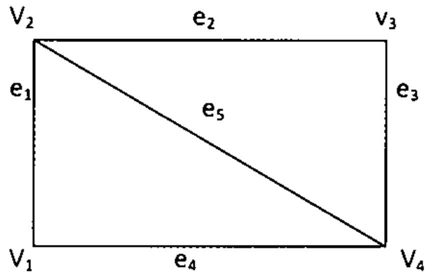
2 ½ Hours [Total Marks: 75]

N.B. 1) All questions are compulsory.
2) Figures to the right indicate marks.
3) Illustrations, in-depth answers and diagrams will be appreciated.
4) Mixing of sub-questions is not allowed.

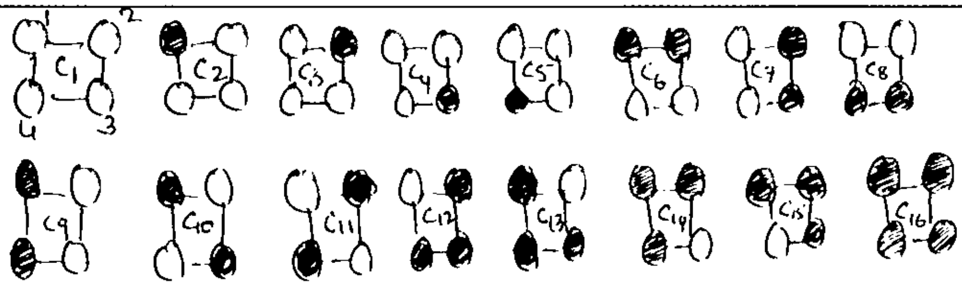
Q. 1		(15M)
(a)	Select correct answer from the following: 1. (b) 2. (b) 3. (b) 4. (c) 5. (d)	1 1 1 1 1
(b)	Fill in the blanks. 1. Combination. 2. least. 3. pseudograph. 4. 2 5. zero.	1 1 1 1 1
(c)	Define the following. Binomial Theorem 1. If x and y are variables and n is a positive integer, then $(x + y) = \binom{n}{0} x^0y^n + \binom{n}{1} x^1y^{n-1} + \binom{n}{2} x^2y^{n-2} + \cdots + \binom{n}{n} x^ny^0$ 2. A complete graph with each vertex is having same degree. 3. Clique: A clique in a graph G=(V,E) is a set K subset of V such that Subgraph induced by K is isomorphic to the complete graph $K_{\|K\|}$ 4. Source lable by triplet (S,+,∞) 5. Suppose a network G = (V,E) with a flow ∅, a path P = (x0, x1, ..., xn) of different vertices in the network such that x0 = S and xn = T is called augment path, If every forward edge of the path has excess capacity and every backward edges in the path has non zero flow.	1 1 1 1 1
Q. 2	**Attempt the following (Any THREE)**	(15M)
(a)	1. $\binom{10}{7} = \frac{10!}{7!3!}$=120ways 2. $\binom{5}{3}\binom{5}{4}$= 50 ways	2 3
(b)	1. Formula $\binom{n}{n_1, n_2, n_3, \ldots \ldots \ldots n_t}$	2

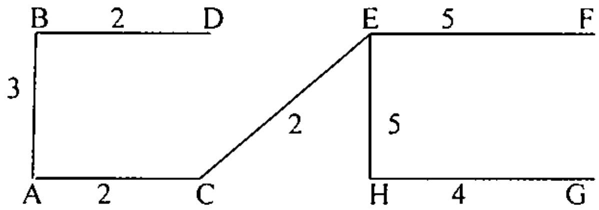
	2. $\binom{4}{1\ 1\ 2}$ 3. $\frac{4!}{2!}$ Ans 12	1 1 1
(c)	1) True for P(1) 2) Assumption n=k is true $P(k) = 7^k - 2^k = 5m$ $7^k = 2^k + 5m$ 3) True for k+1 $P(k+1) = 7^{k+1} - 2^{k+1}$ $= (2^k + 5m)\ 7 - 2^{k+1}$ $= 7.\ 2^k + 7.\ 5m - 2^{k+1}$ $= 5.\ 5^k + 7.\ 5m$ $= 5(5^k + 7m)$ $= 5.\ M$ True for all $n \in N$	1 1 3
(d)	Formula : C(n +r -1, r) $= \frac{(n+r-1)!}{r!(n-1)!}$ Calculation : (132 + 5 -1, 5) = 137!/ 5! X 131! =49,310,836,344	3 2 1
(e)	Explain the importance of combinatorics in graph theory. 1. To find the shortest path 2. To find number of diagonals 3. To find clique 4. Network problems 5. etc (with explanation)	5
(f)	Binomial Theorem $(x + y) = \binom{n}{0} x^0y^n + \binom{n}{1} x^1y^{n-1} + \binom{n}{2} x^2y^{n-2} + \cdots + \binom{n}{n} x^n y^0$ Put x= 2 and y=1 L.H.S = R. H. S	2 2 1
Q. 3	**Attempt the following (Any THREE)**	**(15M)**
(a)	Number of vertices in G_1 = number of vertices in G_2 = 6 Number of edges in G_1 = number of edges in G_2 = 9. Degree of any vertex in G_1 equal degree of any other vertex G_2. All invariants hold therefore G_1 and G_2 are isomorphic.	1 1 2 1

(b)	Table Daigram 	3 2
(c)	A Grah G in Which pairs of edges intersect only at vertices is planer graph. A drawing of a planer graph G is called as planer representation or planar embedding.  K_4  Planar representation of K_4  K_4	2 3
(d)	Eulerian graph is a graph that contain an Eulerian circuit in which all ages should be cover only once. Eg: a b  c d Eulerian path P: a b c d A graph is called Hamiltonian if it contains Hamilton circuit such that is passes through all the vertices in it only onces Eulerian path P: a b c d A graph is called Hamiltonian if it contains Hamilton circuit such that is passes through all the vertices in it only onces	2 3

<table>
<tr><td></td><td>

Hamilton circuit: $v_1 e_1 v_2 e_2 v_3 e_3 v_4 e_4 v_1$</td><td></td></tr>
<tr><td>(e)</td><td>If n+1 pigeons are entered in n holes then there exist at least one hole containing more then one pigeons.
$S_1 = \{1,8\}$, $S_2 = \{2,7\}$, $S_3 = \{3,6\}$, $S_4 = \{4,5\}$
Here pigeons = 5.
pigeons holes = 4
By pigeons hole principles their does not exist five numbers from the set $\{1,2,3,4,5,6,7,8\}$ such that two of them will add upto 9.
$\lfloor (n-1)/m \rfloor + 1 = \lfloor (5-1)/4 \rfloor + 1 = 2$</td><td>2
1
1
1</td></tr>
<tr><td>(f)</td><td>Incidence matrix
Let G = (V,E) is a graph on n vertices and m edges, then incidence matrix of G is the nxm matrix
$I(G) = [a_{ij}]_{n \times m}$
Where $a_{ij} = \begin{cases} 0 & \text{if } v_i \text{ is not incident with } e_j \\ 1 & \text{if } v_i \text{ is incident with } e_j \\ 2 & \text{if } e_j \text{ is loop at } v_i \end{cases}$
$I(G) = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$</td><td>2
3</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td>Q. 4</td><td>Attempt the following (Any THREE)</td><td>(15)</td></tr>
<tr><td>(a)</td><td>If N=(V, E) is a transport network and C is a cut set for the undirected graph associated with N, then C is called a cut or an a-z, if the removal of the edges in C from the network results in the separation of a-z.
For example(please refer page 607 of Grimaldi)Discrete and combinatorial mathematics</td><td></td></tr>
<tr><td>(b)</td><td>Steps with diagram are required
4 + 4= 8</td><td>4
1</td></tr>
</table>

(c)	Ford – Fulkerson labelling algorithm(For detail explanation refer Mitchel T. keller) page No. 242 Let G = (V,E) be a network with two specified notes sources (S) and sink (T). Edge (x,y) is the egde from node x to node y. A flow Ø across edge (x,y) is denoted by Ø (x,y) and its capacity is c (x,y). The flow must satisfy 0 ≤ Ø (x,y) ≤ c (x,y). First label the sources as S: (*, +, ∞), for labelling sink first find the augmenting path and the flow will be suitably updated. If e = edge (x,y) is forward edge then label vertex y with (x, +, p(y)), where p(y) is a potential on y and is defined as p(y) = min {p(x), c(e) - Ø (e)} If e = edge (y,x) is backward edge then label vertex y with (x,-, p(y)),where p(y) is a potential on y and is defined as p(y) = min {p(x), Ø (e)} Now sink has label with (y,+,a) and updated the flow, If e = edge (x,y) is forward edge then Ø (e) is replaced by Ø (e) + a and If e = edge (y,x) is backward edge then Ø (e) is replaced by Ø (e) – a, until the source is reached.	1 1 1 1 1
(d)	 Transformation — Fixed coloring i — All 16 (16) 90° — c1, c16 (2) 180° — c1, c10, c11, c16 (4) 270° — c1, c16 (2) Vertical — c1, c10, c11, c16 (4) Horizontal — c1, c7, c9, c16 (4) Positive slope diagonal — c1, c3, c5, c10, c11, c13, c15, c16 (8) Negative slope diagonal — c1, c2, c4, c10, c11, c12,c14, c16 (8)	2 3
(e)	Explain Cycle index Please refer Mitchel keller(page No.273) Applied Combinatorics.	
(f)	Let G= (V,E) be bipartite with V partitioned as X U Y. A complete matching of X into Y exists if and only if for every subset A of X, $\lvert A \rvert \leq \lvert R(A) \rvert$, where R(A) is the subset of Y consisting of those vertices	

	each of which is adjacent to at least one vertex in A. For example **Refer(page 617)Discrete and combinatorial mathematics by Grimaldi**	
Q. 5	**Attempt the following (Any THREE)**	(15)
(a)	Formula $\frac{n!}{n1!......n_n!}$ $\frac{11!}{3!2!2!2!1!1!}$ = 831,600 No adjacent A $\frac{8!}{3!2!2!1!1!}$ = 5040	
(b)	 Minimum weight = 2+3+2+2+5+5+4 = 23.	5
(c)	Consider a graph G ,a vertex colouring is an assignment of colour to the vertices of G such that adjacent vertices have different colour.G is said to be n-colourable if there exist a colouring of G which useses n colours. The minimum number of colour needed to paint G is called the chromatic number of G and is denoted by $\aleph(G)$. If a graph G is n colourable then $\aleph(G) \leq n$. $\aleph(C_n) = \begin{cases} 2 \text{ if n is even} \\ 3 \text{ if n is odd} \end{cases}$ chromatic number of C_5 = 3. $\aleph(P_n) = \begin{cases} 2 \text{ if } n \geq 2 \\ 1 \text{ if } n = 1 \end{cases}$ chromatic number of P_4 = 2.	2 1 1 1
(d)	1.Solve the continuous problem same as linear programming 2. If basic variables are all integers then the optimal solution has been found. 3.otherwise a LPP posed with integer coefficients and constant need not have optimal value, even when it has an optimal solution with rational value Example	1 1 2 1
(e)	1. Let P(n) be a statement associated with each positive integers n if p(1) is true, assume p(k) is true for 1< k	2.5

	and p(k+ 1) is also true then p(n)is true for all n≥ 1 2. Let P(n) be a statement associated with each positive integers n if p(1) is true, and for m $\in N$, the assumption that p(k-1) is true implies that p(k) is true. p(n)is true for all n≥ 1	2.5