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QP Code: 34439

S.Y.B.Sc. (SEM-IV) Paper I - USST401

SET-1

All questions are compulsory.

Figures to the right indicate full marks.

Use of calculator is allowed.

Q.1		Attempt all sub-questions:	(20)
a.		State TRUE or FALSE and correct if necessary. 2 M each	(10)
	i.	False. Variance = 25/12	
	ii.	False. Mean = 1/4	
	iii.	False. If Z is standard Normal variable, then Z^2 follows chi-square distribution.	
	iv.	False. For Normal distribution with parameters $(\mu = 5, \sigma^2 = 100)$, μ_7 is 0.	
	v.	True	
b.		Answer the following : 2 M each	(10)
	i.	Variance = $8/36 (7) = 2/63$	
	ii.	$X \sim \text{Normal} (\mu = 10, \sigma^2 = 9)$, Mean deviation about median = $\sqrt{\frac{2}{\pi}} \sigma = 0.7979 \sigma = 3 \sqrt{\frac{2}{\pi}} = 2.3937$	
	iii.	If moment generating function (m.g.f.) of r.v. X is $e^{34t + 50t^2}$, $X \sim \text{Normal} (\mu = 34, \sigma^2 = 100)$	
	iv.	Variance = 30	
	v.	Mean = 2	
Q.2		Attempt any TWO sub-questions:	(20)
a.		M.G.F. = $[e^{bt} - e^{at}] / t (b-a)$ 3M $\mu_r^1 = [b^{r+1} - a^{r+1}] / (b-a)(r+1)$ 3M Mean = $(b+a) / 2$ 2M Variance = $(b-a)^2 / 12$ 2M	(10)
b.		M.G.F. = $(\theta / (\theta - t))$ 3M C.G.F. = $\log_e (\theta / (\theta - t))$ 3M Mean = $1/\theta$ 2M Variance = $1/\theta^2$ 2M	(10)
c.		If $X \sim \text{Gamma} (b, a)$ M.G.F. = $(1 - t/b)^{-a}$ 5M Mean = a/b 2M Variance = a/b^2 3M	(10)
Q.3		Attempt any TWO sub-questions:	(20)
a.		$X \sim \text{Normal} (\mu, \sigma^2)$ Mean = $E(X) = \mu$ 5M Variance = $E(X - \mu)^2 = \sigma^2$ 5M	(10)
b.		$X \sim \text{Normal} (\mu, \sigma^2)$	(10)

		<p>M.G.F. = $M_X(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$6M</p> <p>$X \sim \text{Normal}(\mu_1, \sigma_1^2)$</p> <p>$Y \sim \text{Normal}(\mu_2, \sigma_2^2)$</p> <p>Distribution of X+Y</p> <p>$M_{X+Y}(t) = M_X(t) \times M_Y(t) = e^{\mu_1 t + \frac{1}{2}\sigma_1^2 t^2} \times e^{\mu_2 t + \frac{1}{2}\sigma_2^2 t^2}$</p> <p style="text-align: center;">$= e^{(\mu_1 + \mu_2)t + \frac{1}{2}(\sigma_1^2 + \sigma_2^2)t^2}$</p> <p>$X+Y \sim \text{Normal}(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$2M</p> <p>Distribution of X-Y</p> <p>$M_{X-Y}(t) = M_X(t) \times M_Y(-t) = e^{\mu_1 t + \frac{1}{2}\sigma_1^2 t^2} \times e^{\mu_2(-t) + \frac{1}{2}\sigma_2^2 t^2}$</p> <p style="text-align: center;">$= e^{(\mu_1 - \mu_2)t + \frac{1}{2}(\sigma_1^2 + \sigma_2^2)t^2}$</p> <p>$X-Y \sim \text{Normal}(\mu_1 - \mu_2, \sigma_1^2 + \sigma_2^2)$2M</p>	
	c.	<p>i. $X \sim \text{Normal}(\mu, \sigma^2)$</p> <p>Mode of Normal distribution = μ5M</p>	(05)
		<p>ii. p.d.f. of Log Normal distribution with parameters (μ, σ^2)</p> <p style="text-align: center;">.....2M</p> <p>$E(X) = e^{\mu + \frac{1}{2}\sigma^2}$3M</p>	(05)
	Q.4	Attempt any TWO sub-questions:	(20)
	a.	<p>Pdf 1M</p> <p>M.G.F. = $(1 - 2t)^{-n/2}$ 4M</p> <p>C.G.F.1M</p> <p>Mean = n 2M</p> <p>Variance = $2n$ 2M</p> <p>Where n is degrees of freedom</p>	(10)
	b.	<p>$\mu_r^k = \mu_{2k}^k = [f^k \Gamma(k + \frac{1}{2}) \Gamma(f/2 - k)] / \Gamma(1/2) \Gamma(f/2) \dots$ 5M</p> <p>$\mu_r^k = \mu_{2k+1}^k = 0$2M</p> <p>Mean = 01m</p> <p>Variance = $f/f - 2$ 2M</p> <p>f: degrees of freedom</p>	(10)
	c.	<p>$\mu_r^1 = (f_2/f_1)^r \Gamma(f_1/2 + r) \Gamma(f_2/2 - r) / \Gamma(f_1/2) \Gamma(f_2/2)$7 M</p> <p>Mean = $f_2 / f_2 - 2$ 3M</p> <p>Where r.v. follows F distribution with (f_1, f_2) degrees of freedom.</p>	(10)
	Q.5	Attempt any TWO sub-questions:	(20)
	a.	$X \sim \beta_I(m, n)$	(10)

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		$\mu_r = \Gamma(m+n) \Gamma(m+r) / \Gamma(m) \Gamma(m+n+r)$ 5M Mean $m/m+n$ 2M Variance $= mn / (m+n)^2 (m+n+1)$ 3M	
b.	i.	$X \sim \text{Normal}(\mu_1=15, \sigma_1^2=16)$ $Y \sim \text{Normal}(\mu_2=10, \sigma_2^2=9)$ $2X+Y \sim \text{Normal}(\mu=40, \sigma^2=73)$2M $P(2X+Y < 40) = 0.5$3M	(05)
	ii.	Any five properties of Normal distribution with parameters of (μ, σ^2)5M	(05)
c.		$f(x,y)$ 3M let $u = x/y$ $v = y$ 1M $f(u,v)$ 2M $f(u) = \int f(u,v) dv$ 1M $f(u)$ 2M $u \sim \text{beta}$ 1M	(10)
