(1)

00067057

(1)

Set 2

Q. 1. (a) 2 marks each

(i) False, $P(X=5)=0$

(ii) $F(Q_3) = 0.75$

(iii) $F(0) = \frac{x-a}{b-a} = \frac{1}{2}$

(iv) One application of exponential distribution

(v) False, second central moment $\mu_2 = 256$

(vi) False, function of sample observations

(vii) True, α = maximum P(Type I error)

(b) 2 marks each

(i) r^{th} raw moment about origin 'a'

$E(X-a)^r = \int_S (x-a)^r f(x)\,dx$

(ii) $\int_1^\infty \frac{1}{x^2}\,dx = 1$

$\therefore f(x)$ is a p.d.f.

(iii) variance $= \frac{(b-a)^2}{12} = \frac{(20-8)^2}{12} = 12$

(iv) $\hat{p} \sim N\left(P, \frac{PQ}{n}\right)$

(v) interval estimation

(vi) Simple and composite hypothesis – one difference

(vii) Test statistic,

$$Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$$

Q. 2 (a)

(i) E(X) – (1)

Median – (1)

Mode – (1)

Quartiles – (2)

(ii) Cumulative distribution function –

$F(x) = 0 \qquad x < 1$

$= 1 - \frac{1}{x^2} \qquad 1 < x < \infty$ – (3)

$P(X > 3) = \int_3^\infty f(x)\,dx = 0.1111$ – (2)

Q. 2. (b)

(i) $E(aX+b) = a\,E(X)+b$ — 2

$V(aX+b) = a^2 \cdot V(X)$ — 3

(ii) $\int_1^3 k(x-1)^3\,dx = 1$

$k\left(\frac{(x-1)^4}{4}\right)_1^3 = 1$

$k = \frac{1}{4}$ — 2

Let M be the median

$\int_1^M \frac{1}{4}(x-1)^3\,dx = \frac{1}{2}$

$\frac{1}{4}\left(\frac{(x-1)^4}{4}\right)_1^M = \frac{1}{2} \Rightarrow (M-1)^4 = 8$

$M = -0.682$ or $M = 2.682$

$M = -0.682$ not possible

$\therefore$ median $= 2.682$ — 3

(c) mean $= E(X)$

$= \int_0^1 x\,f(x)\,dx + \int_1^4 x\,f(x)\,dx$

$= \int_0^1 x \cdot \frac{x}{3}\,dx + \int_1^4 x \cdot 5\frac{(4-x)}{27}\,dx$

$= \frac{1}{3}\left(\frac{x^3}{3}\right)_0^1 + \frac{5}{27}\left(4\left(\frac{x^2}{2}\right)_1^4 - \left(\frac{x^3}{3}\right)_1^4\right)$

$= \frac{1}{9} + \frac{5}{27}(9)$

$= 1.7778$ — 5

$E(X^2) = \int_0^1 x^2 f(x)\,dx + \int_1^4 x^2 f(x)\,dx$

$= 3.8333$ — 3

Variance of X

$V(X) = E(X^2) - [E(X)]^2$

$= 0.6728$ — 2

Q. 3. (a)

cumulative distribution function — ④

mean — ③

variance — ③

(b)

(i) Lack of memory property — ③

interpretation — ②

(ii)

$f(x) = 0.01\, e^{-0.01x} \qquad x \geq 0$

$= 0 \qquad o.w.$

$\lambda = 0.01$

1. $\text{mean} = \frac{1}{\lambda} = 100$ — ②

2. P(device will fail in first 40 hrs)

$= P(x < 40)$

$= \int_0^{40} f(x)\cdot dx$

$= 0.3297$ — ③

(c)

(i) P.d.f of normal variate — ②

important properties — ③

(ii) X : income

$N = 10000$

$\mu = 750$

$\sigma = 50$ — ①

1. $P(X > 668) = P\left(Z > \frac{668-750}{50}\right)$

$= P(Z > -1.64)$

$= 0.9495$

$= 94.95\%$ — ②

2. $P(X > 832) = P(Z > 1.64)$

$= 0.0505$

$= 5.05\%$ — ②

(4)

Q 4. (a)

(i) Confidence interval for population mean (5)

(ii) $P = 0.30$ $q = 0.70$

$n = 100$

$\hat{p} \sim N(P, \sqrt{\frac{pq}{n}})$

$\sqrt{\frac{pq}{n}} = 0.04583$ — (1)

1. $P(0.27 < \hat{p} < 0.32) = P(-0.6546 < Z < 0.4364)$

$= 0.4122$ — (2)

2. $P(\hat{p} < 0.31) = P(Z < 0.2182)$

$= 0.5871$ — (2)

(b) Each term – 2 marks.

(c)

(i) $H_0 : p = 0.5$

$H_1 : p = 0.6$

$n = 5$

Critical region $= \{x \mid x > 3\}$

x : no. of heads

$P(\text{Type I error}) = P(\text{reject } H_0 \mid H_0 \text{ true})$

$= P(x > 3 \mid p = 0.5)$

$= 0.1875$ — (2)

$P(\text{Type II error}) = P(\text{accept } H_0 \mid H_1 \text{ true})$

$= P(x \leq 3 \mid p = 0.6)$

$= 0.66304$ — (2)

Power of the test $= 1 - P(\text{reject } H_0 \mid H_1 \text{ true})$

$= 0.33696$ — (1)

(ii) $n = 30$

$\bar{x} = 112$

$H_0 : \mu = 100$

$H_1 : \mu > 100$ — (1)

test statistic $Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$ — (1)

$= \frac{112 - 100}{15/\sqrt{30}} = 4.38$ — (2)

At 5% l.o.s. $Z_{table} = 1.64$

Reject H_0 as $Z_{cal} > Z_{table}$ — (1)

Q.5. (a)

1. $F(x) = 0 \quad x < 0$

$= \frac{x^2}{4} \quad 0 \le x < 1$

$= \frac{1}{2}x - \frac{1}{4} \quad 1 \le x < 2$

$= \frac{3}{2}x - \frac{x^2}{4} - \frac{5}{4} \quad 2 \le x < 3$

$= 1 \quad x \geq 3$ — (6)

2. $P(X < 1.4) = F(1.4) = 0.45$ — (2)

3. $P(X > 2.5) = 1 - P(X \le 2.5)$

$= 1 - F(2.5)$

$= 0.0625$ — (2)

(b)

(i) $X \sim$ Rectangular $(0, 100)$

$f(x) = \frac{1}{100} \quad 0 < x < 100$ ← (1)

$= 0 \quad$ otherwise

1. $P(X > 80) = \int_{80}^{100} f(x) \cdot dx$

$= \frac{1}{100} \int_{80}^{100} dx$

$= 0.2$ — (2)

2. $P(X < 50) = \int_{0}^{50} f(x) \cdot dx$

$= 0.5$ — (1)

3. $P(60 < X < 70) = \int_{60}^{70} f(x)\, dx$

$= 0.1$ — (1)

(ii) central limit theorem — (2)

application — (2)

(c)

(i) Distinguish — $2\frac{1}{2}$ marks each

(ii) Test statistic — (2)

critical region — (1)

assumption — (2)