00063978

A.T.K.T Theory Examination - 2019.

S.Y.B.A - Set - I

(1)

Statistics Paper - II

solution

Q1

(a) (i) $P(A \cup B) = P(A) + P(B)$

(ii) True

(iii) False

(iv) $\mu_2 = \mu_2' - (\mu_1')^2 = 12 - 3^2 = 12 - 9 = 3$

∴ Statement is false.

(v) false. for a Poisson distribution Mean = Var

(vi) False.

(vii) True.

(b)

(i) Definition — (2 marks)

(ii) $E[2x+5] = 2E(x) + 5 = 2 \times 10 + 5 = 25$ — (2 marks)

(iii) modes = 0 and 1

(iv) $V(x+y) = V(x) + V(y) + 2\,cov(x, y)$ — (2 marks)

(v) 2 examples — 2 marks.

(vi) statement — 2 marks.

(vii) Definition — 2 marks.

Q2

(a)(1) statement & Proof — 5 marks.

statement for three events — 2 marks.

(ii) $S = \{HH, HT, TH, TT\}$ — (1/2)

$P(\text{Single head}) = 2/4 = 0.5$ — (1/2)

$P(\text{more than one head}) = 1/4$ — (1)

2) P(atleast one tail) = 3/4 — (1)

(b)(i) 4 definitions (1 $\frac{1}{2}$ marks each)

(ii) A – Passing in B.Sc
B – Passing in IIT entrance

$P(A) = \frac{2}{3}$ $P(A \cap B) = \frac{14}{45}$ $P(A \cup B) = \frac{4}{5}$

$P(B) = P(A \cup B) - P(A) + P(A \cap B)$

$= \frac{4}{5} - \frac{2}{3} + \frac{14}{45} = 0.44$ — (02)

$P(A) \cdot P(B) = 0.296$
$P(A \cap B) = 0.311$ $\therefore P(A \cap B) \neq P(A) \cdot P(B)$
A and B are not independent.

— (02)

(c) (i) Definition — (2 marks)
Limitations — (2 marks).

(ii)

6	3
B	R

9

4	5
B	R

9

2	8
B	R

10

$P(\text{Blue}) = \frac{1}{3} \times \frac{6}{9} + \frac{1}{3} \times \frac{4}{9} + \frac{1}{3} \times \frac{2}{10}$

$= 0.437$ — (3m)

$P(I/B) = \frac{P(B \cap I)}{P(B)} = \frac{P(B) \cdot P(B/I)}{P(B)}$

$= \frac{\frac{1}{3} \times \frac{6}{9}}{0.437} = 0.5085$

— (3m)

3

Q3(a)

(i) Defn — 3 marks

Properties — 2 marks.

(ii) $\Sigma P(x) = 1 \Rightarrow 15K = 1 \Rightarrow K = 1/15$ — (2m)

$P(X > 0) = P(X = 1 \text{ or } 2) = \frac{9}{15} = 0.6$ — $(1\frac{1}{2})$

$P(-1 < X < 2) = P(X = 0, 1) = \frac{7}{15} = 0.47$ — $(1\frac{1}{2})$

(b)

(i)

X	1	2	3	4	5	
P(x)	0.1	0.15	0.2	0.3	0.25	
xP(x)	0.1	0.3	0.6	1.2	1.25	3.45
$x^2P(x)$	0.1	0.6	1.8	4.8	6.25	13.55

$E(X) =$ mean $= 3.45$ units — (2)

$V(X) = E(X^2) - (E(X))^2 = 13.55 - (3.45)^2$

$= 1.6475$ units — (2)

X	1	2	3	4	5
$F_X(x)$	0.1	0.25	0.45	0.75	1.0

— (2)

(ii) Statement & Proof — (04)

(c)

(i) Definition & example — (2)

(ii)

X \ Y	-1	0	1	$P_X(x)$	$xP_X(x)$	$x^2P_X(x)$
10	0.1	0.2	0.3	0.6	6	60
20	0.2	0.1	0.1	0.4	8	160
P(y)	0.3	0.3	0.4		14	220
yP(y)	-0.3	0	0.4	0.1		

(4)

Y:	-1	0	1	
$Y^2P(Y)$:	0.3	0	0.4	0.7

$E(X) = 14$ $\quad E(Y) = 0.1$ — (2)

$V(X) = 220 - (14)^2 = 24$

$V(Y) = 0.7 - (0.1)^2 = 0.69$ } — (3)

$E(XY) = -10 \times 0.1 + 10 \times 0.3 + (-20) \times 0.2 + 20 \times 0.1$

$= 0$ — (1)

$Cov(X,Y) = 0 - 14 \times 0.1 = -1.4$

$$\rho_{X,Y} = \frac{-1.4}{\sqrt{24}\sqrt{0.69}} = -0.3136 \quad \text{— (2)}$$

(Total)

Q 4(a)

(i) Statement — (1m)

3 applications — (3m)

(ii) $P = 1/4$ $\quad n = 10$ — (1)

$P(X=3) = {}^{10}C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^7 = 0.25028$ — (2)

$P(X \geq 1) = 1 - P(X < 1) = 1 - P(X=0)$

$= 1 - {}^{10}C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{10}$

$= 0.9437$ — (3)

(b) Statement of P.m.f — (2)

Mean — (4)

Var — (4)

(c)

(i) Statement — (2)

Proof — (4)

(ii) | 5 Prizes | 95 rem |

100

Prob = 1 − P(no prize)

$= 1 - \frac{(^5C_0)(^{95}C_3)}{(^{100}C_3)} = 0.144$

Mean $= \frac{nM}{N} = \frac{3 \times 5}{100} = 0.15$

Q5

(a) (i) Definition — (02)

Properties — (03)

(ii) $P(A \cap B) = P(A) \cdot P(B)$ — (1)

(iii) $P(A \cup B) = 0.5 + 0.4 - 0.5 \times 0.4$ — (1)

$= 0.7$

$P(A \cap B') = P(A) \cdot P(B') = 0.5 \times 0.6$

$= 0.3$ — (1)

$P(A' \cap B) = 0.5 \times 0.4 = 0.2$ — (1)

$P(A' \cap B') = 0.5 \times 0.6 = 0.3$ — (1)

b(i) Definitions — (2 + 2)

formulae — (1 + 1)

(ii) $V(X) = 10 \quad V(Y) = 10 \quad S_{xy} = 0.5$

(PTO)

6

$$\rho_{xy} = \frac{\text{Cov}(x,y)}{\sqrt{V(x) \cdot V(y)}}$$

$\text{Cov}(x,y) = 5$ — (2)

$V(2x+3y) = 4V(x) + 9V(y)$

$= 130$ — (2)

(c) mean — (4)

Var — (6)

— X —