(2½ hours)

Total Marks: 75

N. B.: (1) **All** questions are **compulsory**.
(2) Make **suitable assumptions** wherever necessary and **state the assumptions** made.
(3) Answers to the **same question** must be **written together**.
(4) Numbers to the **right** indicate **marks**.
(5) Draw **neat labeled diagrams** wherever **necessary**.
(6) Use of **Non-programmable** calculators is **allowed**.

1.	**Attempt *any three* of the following:**	**15**
a.	Find the Adjoint of the given matrix and hence find Inverse if exist $\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ Ans Adjoint $\begin{bmatrix} -11 & -4 & 4 \\ -8 & 1 & 4 \\ -16 & 8 & 5 \end{bmatrix}$ Inverse $\frac{1}{3}\begin{bmatrix} -11 & -4 & 4 \\ -8 & 1 & 4 \\ -16 & 8 & 5 \end{bmatrix}$	
b.	Find the Characteristic values and characteristic vectors of the given matrix. $\begin{bmatrix} -17 & 18 & -6 \\ -18 & 19 & -6 \\ -9 & 9 & -2 \end{bmatrix}$ **Ans** **Characteristic Values : 1,1,-2** **Characteristic Vectors: X1 = $[1,1,0]^T$, X2 = $[-1,0,3]^T$, X3 = $[2,2,1]^T$**	
c.	Discuss the consistency of the following systems of equations and solve them whenever possible. $X_1 + 2X_2 + 2X_3 = 1$ $2X_1 + 2X_2 + 3X_3 = 3$ $X_1 - X_2 + 3X_3 = 5$ **Ans** **Consistent** **$X_1 = 1, X_2 = -1, X_3 = 1$**	
d.	Express in a + ib form cot (x + iy) **Ans** **(sin 2x + i sinh 2y)/(cos 2x + cosh 2y)**	
e.	Solve the Equation $x^7 + x^4 + x^3 + 1 = 0$ **Ans** **-1, cos π/3 ± isinπ/3,±(cos π/4 + isin π/4) and ±(cos 3π/4 + isin 3π/4)**	
f.	Prove that $(1 + \cos x + i \sin x)^n = 2^n \cos^n x/2 (\cos nx/2 + i \sin nx/2)$ **Ans Hint** **$1 + \cos x = 2 \cos^2 x/2$ and $\sin x = 2 \sin x/2 \cos x/2$** **Apply De Movrie's Theorem**	
2.	**Attempt *any three* of the following:**	**15**
a.	Solve the Differential Equation $(1 - 2xy - x^3)\, dy - (1 + y^2 + 3x^2y)\, dx = 0$	

	Ans **$x^3y + xy^2 + x – y = c$**	
b.	Solve the Differential Equation $x^2dy/dx = 3x^2 – 2xy + 1$ **Ans** **$x^2y = c + x^3 + x$**	
c.	Solve the following Equation sec x dy/dx = y + sin x **Ans** **$Y = ce^{\sin x} – \sin x - 1$**	
d.	Solve the following Equation $p^2x(x-2) + p(2y – 2xy – x + 2) + y^2 + y = 0$ **Ans** **$(y – cx + 2c) (y – cx +1) = 0$**	
e.	Find the Complementary and Particular Solution of the equation $(D^3 + D^2 + D + 1)y = \sin 2x$ **Ans** **Complementary Solution : $c_1e^{-x} + c_2 \cos x + c_3 \sin x$** **Particular Solution : $(2 \cos 2x – \sin 2x)/15$**	
f.	Find the General Solution of the equation $(D^3 + 3D)y = \cos x$ **Ans** **General Solution: $y = c_1 + c_2 \cos x + c_3 \sin x – x /2 \cos x$**	
3.	**Attempt *any three* of the following:**	**15**
a.	Evaluate $\int_0^\infty e^{-3t} t \cos 2t \, dt$ **Ans** **0**	
b.	Find the inverse Laplace transform for the function $F(s) = \frac{5s+3}{(s-1)(s^2+2s+5)}$ **Ans** **$e^t - 1/2e^{-t} \cos 2t + 2/13 \sin 2t$**	
c.	Find Laplace transformation of the function $f(t) = t(2\sin 3t + e^{2t})$ **Ans** **$12s/(s^2 + 9)^2 + 1/(s-2)^2$**	
d.	Obtain the Laplace transform of each of the given function $F(t) = e^{-2t}\cos 4t + e^{3t} \sin 6t$ **Ans** **$(s-2)/[(s-2)^2 + 16] + 6 / [(s-3)^2 + 36]$**	
e.	Find Inverse Laplace Transformation by convolution theorem for $F(s) = \frac{s^2}{(s^2 + a^2)^2}$ **Ans** **(sin at + at cos at)/2a**	
f.	Using Laplace transform method solve the following differential equations with the given condition. $(D^2 + 3d + 2)y = 4t + e^{3t}$ if y = 1, Dy = -1 at t = 0 **Ans** **$Y = -1/2\ e^t – 2\ e^{2t} + ½\ e^{3t} + 2t + 3$**	
4.	**Attempt *any three* of the following:**	**15**
a.	$Evaluate \int_0^2 \int_x^{4-x} \int_0^1 e^{2x+2y} dxdy$ **Ans 43/3**	

b.	$Evaluate \int_0^4 \int_0^{\sqrt{4x-x^2}} \frac{ydxdy}{(x^2+y^2)^{1/2}}$ **Ans** **8/3**	
c.	$Evaluate \int_0^1 \int_{y^2}^1 \int_0^{1-x} x\, dxdydz$ **Ans** **4/35**	
d.	$Evaluate \int_0^a \int_0^{(a^2-x^2)^{1/2}} \int_0^{(a^2-x^2-y^2)^{1/2}} (xyz)\, dxdydz$ **Ans** **$4a^6/3$**	
e.	Change the order of integration and evaluate $\int_{-1}^{2} \int_{x^2}^{x+2} dxdy$ **Ans** **3/2**	
f.	Change to polar coordinates and evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dxdy$ **Ans put x = rcos θ, y = r sin θ** **π/4**	
5.	**Attempt *any three* of the following:**	**15**
a.	Evaluate $\int_0^\pi sin^2x\,(1+\cos x)^4\ dx$ **Ans** **21π/16**	
b.	Evaluate $\int_0^\infty \frac{x^2dx}{(1+x^6)^{7/2}}$ **Ans** **8/45**	
c.	Evaluate $\int_0^\infty e^{-\alpha x} \sin x / x\ \ dx$ **Ans** **Cot^{-1} α**	
d.	Evaluate $\int_0^{\pi/2} \frac{\log(1+a\,sin^2x)dx}{sin^2x}$ **Ans** **$\pi(\sqrt{(1+a)}-1)$**	
e.	Evaluate $\int_0^1 x^m(\log x)^n\, dx$ **Ans** **$(-1)^n \lfloor\overline{(n+1)}/(m+1)^{n+1}$**	
f.	Define error function. Evaluate erf(√x) **Ans** **$2/\sqrt{\pi}(x^{1/2} - x^{3/2}/3*1! + x^{5/2}/5*2! - x^{7/2}/7*3! + \ldots)$**	