Q.P 34331

Semester IV

(2 ½ Hours) [Total Marks: 75]

N.B.
1) All questions are compulsory.
2) Figures to the right indicate marks.
3) Illustrations, in-depth answers and diagrams will be appreciated.
4) Mixing of sub-questions is not allowed.

Q. 1	**Attempt All(Each of 5Marks)**	**(15M)**
(a)	Multiple Choice Questions. vi) Which of the following commands will create a list? a) list1 = list() b) list1 = [] c) list1 = ([1, 2, 3]) d) All of these **Ans:** d) All of these vii) The dot product of (1, 2, 3) and (1, −1, 0) is a) 0 b) 2 c) 1 d) −1 **Ans:** d) −1 viii) The dot product of (1, 2, 3) and (−1, 1, 0) is a) 1 b) −1 c) 0 d) 2 **Ans:** a) 1 ix) A linear equation with right hand side is equal to zero is called a) A linear System b) Saturated c) Homogeneous d) Non homogeneous **Ans:** c) Homogeneous x) A vector whose norm is 1 is called ___________ vector a) Null b) Besis c) Unit d) none of these **Ans:** a) Null.	
(b)	Fill in the blanks for the following questions vi) Two vectors are said to be orthogonal if angle between them is ___ **Ans:** 90° vii) The output when we execute list("Hello") is _____________. **Ans:** ['H', 'e', 'l', 'l', 'o'] viii) Set of all linear combinations of vectors is called _________ **Ans:** Span ix) If all the elements of a matrix have zero value is called as ________ matrix. **Ans:** Null/Zero	

	x) To add a new element to a list we use _________ command. **Ans:** list l.append	
(c)	Answer the following questions vi) If u =(1, 2, –1) and v = (3, 2, –1) find norm u and norm v. **Ans:** norm u = √6, norm v = √14 vii) Define the term Inner Product Space **Ans:** A vector space together with an inner product is called an inner product space viii) Solve (1•1) + (1•0) + (1•1) **Ans:** 0 ix) Define the term Characteristic equation **Ans:** On expanding | A- I| we get a polynomial is called characteristic polynomial. The equation obtained by solving this determinant is called characteristic equation of A. x) Find dot product of (1, 5), (4, –2) **Ans:** –6	
Q. 2	**Attempt the following (Any THREE)**	**(15M)**
(a)	Find the square root of complex number 8 – 6i **Ans:** Square root is 3 + i, –3 – i	
(b)	Determine whether v_1=(2, 2, 2), v_2=(0, 0, 3) and v_3=(0, 1, 1) span vector space R^3. **Ans:** here determinant of matrix is –4, i.e. non-zero . Hence vectors span vector space.	
(c)	Write a Python program to find conjugate of a complex number.	
(d)	Are the following vectors are linearly dependent v_1=(3, 2, 7), v_2=(2, 4, 1) and v_3=(1, –2, 6) **Ans:** Linearly dependent since $k_1 = -k_3$ and $k_2 = k_3$	
(e)	Express in polar and exponential form 1 + i √3 **Ans:** Modulus = r = 2 Argument = θ = π/3 Polar form = 2 [cos(π/3) + i sin(π/3)] Exponential form = $2\ e^{i\pi/3}$	
(f)	Check whether the set of all pairs of real numbers of the form (1, x) with operation (1, y) + (1, y′) = (1, y + y′) and k (1, y) = (1, ky) is a vector space. **Ans:** Yes, it is a vector Space.	

Q. 3	**Attempt the following (Any THREE)**	**(15M)**
(a)	angle between the two vectors $a =$ (2,3,4) and $(1,-4,3)$ in IR^3. $\text{Cos}\theta = \frac{2}{\sqrt{29}\sqrt{26}}$	
(b)	**Ans:** a) $\begin{pmatrix} 7 & 6 \\ 3 & 3 \\ 1 & 6 \end{pmatrix}$ b) $3A = \begin{pmatrix} 6 & 6 \\ 3 & 3 \\ 0 & 18 \end{pmatrix}$ c) Does not exists.	
(c)	Ans: Refer Practical.	
(d)	**Ans:** The set of functions are Linearly independent	
(e)	$U_1\{(x,y,w,z) : x-y=0\}$ ={x(1,1,0,0)+w(0,0,1,0)+z(0,0,0,1)} { (1,1,0,0),0,0,1,0),(0,0,0,1)} is linearly independent, dim U_1=3 $U_2\{(x,y,w,z) : x=w, y=z\}$={x(1,0,1,0)+y(0,1,0,1)} {(1,0,1,0),(0,1,0,1)} is linearly independent, dim$U_2 = 2$ $U_1 \cap U_2 = \{(x,y,w,z) : x-y=0, x=w, y=z\}$ ={y(1,1,1,1)} {(1,1,1,1)} is linearly independent, dim$U_1 \cap U_2 = 1$	
(f)	U^0= {f $\in V, f(x)=0\ \forall x \in U$ }, W^0={f$\in V, f(x)=0\ \forall x \in W$ } $f \in W^0 \to f(x)=0, \forall x \in W$, since U is a subset of W $\to f(x)=0, \forall x \in U \ \to x \in U^0$	
Q. 4	**Attempt the following (Any THREE)**	**(15)**
(a)	By Gaussian elimination method. y -z = 3 -2x + 4y -z = 1 -2x + 5y – 4z = -2 x=10, y=6, z=3	
(b)	$w_1 = \frac{(1,1,1,1),}{\sqrt{4}} w_2 = \frac{(-2,-1,1,2)}{\sqrt{10}}, w_3 = \frac{\left(\frac{8}{5}, \frac{-17}{10}, \frac{-13}{10}, \frac{7}{5}\right)}{\sqrt{\frac{910}{100}}}$	
(c)	Let a = (3,0), b = (2,1) vector in span $\{a\}$ that is closet to b is $b^{\|\|a}$=αa *where* $\alpha = \frac{<b,a>}{<a,a>}$=2 $b^{\|\|a}$=(6,0) , $b^{\perp a}$=b- $\alpha a = (-4,1)$ $\|\|b^{\perp a}\|\| = \sqrt{17}$.	
(d)	**Ans:** Pythagorean Theorem verified	
(e)	**Ans:** <p, q> = –13 cosθ = (–13)/ (√13√30) orthogonal	

(f)	Ans: Refer Practical.	
Q. 5	**Attempt the following (Any THREE)**	**(15)**
(a)	**Ans:** Eigen Values: 1,2,3 Eigen vectors: $\begin{bmatrix}4\\3\\2\end{bmatrix}$ $\begin{bmatrix}3\\2\\1\end{bmatrix}$ $\begin{bmatrix}2\\1\\1\end{bmatrix}$	
(b)	**Ans:** $v_1 - v_2 + v_3 = w$	
(c)	Let T :$I|R^3 \rightarrow IR^2$ be a linear map defined by f (x,y,z) = (x+2y-z, x+y-2z) Rank T=2, Nullity T = 1. 2+1=3 hence verified.	
(d)	Let S be a subset of vector space V. $S^{\perp}$ ={v$\in V: < v, s >$=0,$\forall s \in S$} <0,s>=0 → 0 ∈ $S^{\perp}$, let a,b∈ $S^{\perp}$, →<a,s>=0,<b,s>=0 let α, β are scalars. $< a\ \alpha + b\beta, s >= \alpha < a, s > + \beta < b, s >= 0$ hence $S^{\perp}$ is subspace of V.	
(e)	see table below	

Vector space	Basis	Dimension
{0}	{ }	0
IR^2	{(1,0),(0,1)}	2
$P_2(x)$	{1,x,x²}	3
$M_2(IR)$	$\left\{\begin{bmatrix}1&0\\0&0\end{bmatrix}, \begin{bmatrix}0&1\\0&0\end{bmatrix}, \begin{bmatrix}0&0\\1&0\end{bmatrix}, \begin{bmatrix}0&0\\0&1\end{bmatrix}\right\}$	4
IR	{1}	1

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