(J) P.P. code - 53211

Semester III

(2½ Hours) [Total Marks: 75]

N.B. 1) All questions are compulsory.
2) Figures to the right indicate marks.
3) Illustrations, in-depth answers and diagrams will be appreciated.
4) Mixing of sub-questions is not allowed.

Q. 1	**Attempt All (Each of 5Marks)**	**(15M)**
(a)	Select correct answer from the following: 1. (b) 2. (b) 3. (b) 4. (b) 5. (a)	 1 1 1 1 1
(b)	Fill in the blanks 1. Coefficients 2.one 3. degree 4. Chromatic 5. equal	 1 1 1 1 1
(c)	Short Answers 1. If P(n) be a statement and it is true for n=1, n=m-1 and n=m-2 then it is true for n=m. If x and y are variables and n is a positive integer, then $(x+y)=\binom{n}{0}x^0y^n+\binom{n}{1}x^1y^{n-1}+\binom{n}{2}x^2y^{n-2}+\cdots+\binom{n}{n}x^ny^0$ 2. A complete graph with each vertex is having same degree. 3. A tree with its nodes labelled is called labelled tree 4. graph is said to be planar if it can be drawn in a plane such that no edges in G intersect each other except possibly a vertex. 5. If N=(V, E) is a transport network, a function f from E to the nonnegative integers is called a flow for N if if f(e) ≤ c(e)	 1 1 1 1 1
Q. 2	**Attempt the following (Any THREE)**	**(15M)**
(a)	Two letters followed by four digits 26^2x10^4 First two digits followed by 4 letters 10^2x26^4	2.5 2.5

(2)

	By sum rule $26^2 \times 10^4 + 10^2 \times 26^4$ =5,24,57,600	
(b)	Determine the coefficient on $x^2y^3z^2$ in the expansion of $(x + y + z)^7$. 1. Formula $\binom{n}{n_1, n_2, n_3, \ldots\ldots\ldots n_t}$ 2. $\binom{7}{2\,3\,2}$ 3. $\frac{7!}{2!3!2!}$ Ans210	2 1 1 1
(c)	1) True for P(1) (proof) 2) Assumption n=k is true $1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + k^2 = k(k+1)(2k+1)/6$ 3) For n =k+1 $1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + k^2 + (k+1)^2 = 1 = (k+1)(k+2)(2k+3)/6$ L.H.S $1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + k^2 + (k+1)^2$ $= k(k+1)(2k+1)/6 + (k+1)^2 = (k+1)(k+2)(2k+3)/6$ R.H.S	1 1 3
(d)	Formula : $C(n + r - 1, r) = \frac{(n+r-1)!}{r!(n-1)!}$ Calculation : $C(65 + 5 - 1, 5) = c(69,5) = 69!/ 5!\ X64!$	2 2 1
(e)	The word Sudoku is an abbreviation of aphrase which means that "the digit must occur only once". Method: select the row or column which has less number of empty boxes. Use elimination method or solve by breaking the grid in squares or rectangles of equal number of boxes. Benefits: Logical thinking, Analysis, Cognitive Assessment etc (with explanation)	5
(f)	Binomial Theorem $(x + y)^n = \binom{n}{0} x^0y^n + \binom{n}{1} x^1y^{n-1} + \binom{n}{2} x^2y^{n-2} + \cdots + \binom{n}{n} x^ny^0$ Put x= 1 and y=-1 L.H.S= R.H.S	2 2 1
Q. 3	**Attempt the following (Any THREE)**	**(15M)**
(a)	Not isomorphic	
(b)	No. of faces or region + no. of vertices –No. of edges = 2	
(c)	A graph G is said to be bipartite if its vertices V can be partitioned in to two subsets V1 and V2 such that each edge of G connects a vertex of V1 to a	2

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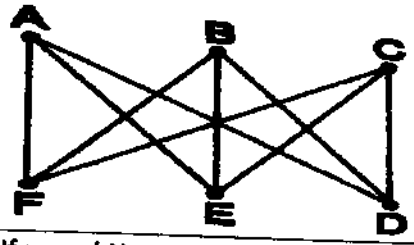
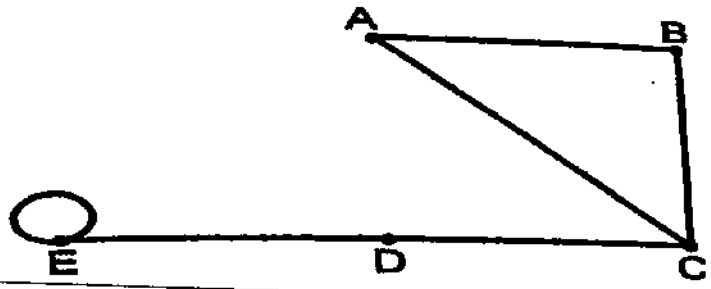
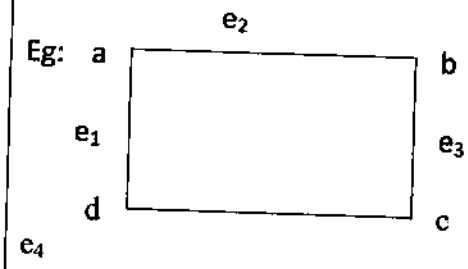
	vertex of V2.  	2 1
(d)	If m and N are positive integers, then there exists a least integers R(m,n) so that if G is a graph and G has at least R(m,n) vertices then either G contains a complete subgraph on m vertices or graph contains an independent set of size n. R(m,n)= C(m+n-2, m-1) R(2,4)= 4 and R(3,5)=13	2 1 2
(e)	Adjacency matrix Let G = (V,E) is a graph on n vertices , then adjacency matrix of G is the nxn matrix A (G) = $[a_{ij}]_{nxn}$ Where aij = number of edges between v_i and v_j = 1 for self loop  	2 3
(f)	Eulerian graph is a graph that contain an Eulerian circuit in which all edges should be cover only once. A graph is called Hamiltonian if it contains Hamilton circuit such that is passes through all the vertices in it only onces  Eulerian path P: a b c d a Hamilton path: a e_2 b e_3 c e_3 d e_1 a.	1 1 1 1 1
Q. 4	**Attempt the following (Any THREE)**	**(15)**

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(a)	**Refer(page 740)Discrete and combinatorial mathematics by Grimaldi**	3 2
(b)	Transformation — Fixed coloring i — All 16 (16) 90^0 — c1, c16 (2) 180^0 — c1, c10, c11, c16 (4) 270^0 — c1, c16 (2) Vertical — c1, c10, c11, c16 (4) Horizontal — c1, c7, c9, c16 (4) Positive slope diagonal — c1, c3, c5, c10, c11, c13, c15, c16 (8) Negative slope diagonal — c1, c2, c4, c10, c11, c12,c14, c16 (8) No. of equivalence class=$\frac{\sum Fixed\ Transformation}{No.of\ groups}$ State Burnside's theorem. Let S be a set of configurations on which a finite group of permutations acts. The number of equivalence classes into which S is partitioned by the action of G is given by $$\frac{1}{\lvert G\rvert}\sum_{c\in G}\lvert sx\rvert$$ **(For detail explanation refer Mitchel T. keller) page No. 271**	2 3
(c)	First augmented path a-b-c-d Second augmented path a-f-b-c-d Third augmented path a-f-b-e-d Fourth augmented path a-f-c-d Fifth augmented path a-f-e-d Maximum flow=35	5
(d)	Ans:15Diagram for cuts required with calculation **Refer(page 607)Discrete and combinatorial mathematics by Grimaldi**	5
(e)	solution: π_1= (1 3 5 2)(4 8 7)(6) π_2 = (1 3)(2 7)(4 6)(5 8)	1 1

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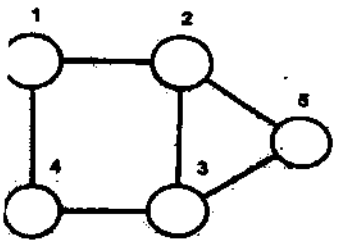
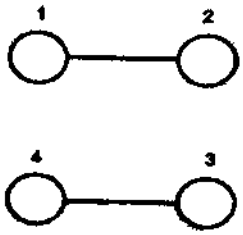
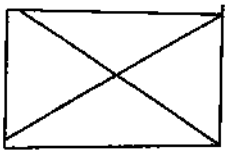
	$\pi_1\pi_2 = \begin{pmatrix}1&2&3&4&5&6&7&8\\1&8&5&7&4&6&2\end{pmatrix}$ $Inv(\pi_1) = \begin{pmatrix}1&2&3&4&5&6&7&8\\2&5&1&7&3&6&8&4\end{pmatrix}$	1 2
(f)	**Refer(page 616)Discrete and combinatorial mathematics by Grimaldi**	5
Q. 5	**Attempt the following (Any THREE)**	**(15)**
(a)	Formula $\frac{n!}{n1!.....n_n!}$ ANS: $\frac{11!}{2!2!2!1!1!}$	2 3
(b)	Consider a graph G ,a vertex colouring is an assignment of colour to the vertices of G such that adjacent vertices have different colour.G is said to be n-colourable if there exist a colouring of G which useses n colours. The minimum number of colour needed to paint G is called the chromatic number of G and is denoted by $\aleph(G)$. $\aleph(G) = 3$ Clique=3	2 1 2
(c)	**(For detail explanation refer Mitchel T. keller) page No. 248**	5
(d)	 $G = G_1 \cup G_2$  $G = G_1 \cap G_2$	3 1
(e)		2.5 2.5