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Set II

Q - P. Code 31174 Semester II

Q. 1	Attempt All (Each of 5Marks)	(15M)
(a)	Select correct answer from the following: 1. b 2. a 3. a 4. c 5. a	
(b)	Fill in the blanks: 1. $\lim_{n \rightarrow \infty} (-2x) = \underline{\hspace{2cm}}$ 2. The absolute value of continuous function is $\underline{\hspace{2cm}}$. 3. Modelling 4. $\frac{1}{4}$ 5. $5x+5y-8$	
(c)	Define the following. 1. $\epsilon - \delta$ definition of limit: $f(x)$ is said to be continuous at $x=a$ if $\forall \epsilon > 0, \exists \delta > 0$ such that $ x - a < \delta \Rightarrow f(x) - l < \epsilon$ 2. Concavity: It is the direction of bending to the curve. If the curve lies above the tangent at a point then the curve is called concave upward and if a lie below then the curve is called concave downward. 3. $I = (x^3/3) + (a^x / \log a) + c$ 4. Definite integral: Integral with limits 5. Absolute Maximum if $f(c) \geq f(x)$ Absolute Minimum if $f(c) \leq f(x)$	
Q. 2	Attempt the following (Any THREE)	(15M)
(a)	Factorization of numerator $(x-3)(x^2-x+10)/(x-3)$ $\lim_{x \rightarrow \infty} x^2 - x + 10 = 16$	3 1 1
(b)	Use definition of $ x $ to prove contiuity Left hand limit=Right hand limit	3 2
(c)	$F'(x)=2x-4$ $F'(x)<0, F$ is decreasing on $(-\infty, 2)$ $F'(x)>0, F$ is increasing on $(2, \infty)$.	1 2 2
(d)	$f'(x)=1+8x-3x^2$ when $f'(x)=0, x= 4/3$ $f''(x)=-6$ Local maxima at $=4/3$	1 2 1 1
(e)	$f(x)=x^3 - x - 1$	1

Q2

	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ interval (1, 2) $x_1 = 1.5$ $x_2 = 1.34$ $x_3 = 1.32$ $x_4 = 1.32$ Ans: 1.32	2 2
(f)	$A = xy$ $A = x(72 - x)$ $\frac{dA}{dx} = 72 - 2x$ Maximum value $x = 36$	1 2 1 1
Q. 3	Attempt the following (Any THREE)	(15M)
(a)	Divide Numerator and Denominator by $\cos^2 x$ and evaluate, we get $I = (1/6) \tan^{-1}(2 \tan x / 3) + c$	
(b)	Using Property of definite integration $I = \pi/4$	
(c)	Using variable separable method solution is $e^{2y}/2 = (e^{3x}/3) + (x^3/3) + c$	
(d)	Using the formula $x_n = x_0 + nh$, $y_n = y_{n-1} + h f(x_{n-1}, y_{n-1})$ by taking $h = 0.2$ and performing 5 iterations we get $y(1) = 2.97314$	
(e)	Using formula of linear differential equation $dy/dx + Py = Q$ Integrating factor = $\sec x$ Solution is: $y \sec x = \sin x + c$	
(f)	12.871	
Q. 4	Attempt the following (Any THREE)	(15)
(a)	Ans: 0	
(b)	Find the second order derivatives of $f(x,y) = y^2 e^x + y$ $\frac{df}{dx} = e^x y^2$, $\frac{df}{dy} = 2e^x y + 1$ $\frac{d^2 f}{dx^2} = e^x y^2$ $\frac{d^2 f}{dy^2} = 2e^x$	1 1 1

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	$\frac{d^2f}{dx dy} = 2e^x y$ $\frac{d^2f}{dy dx} = 2ye^x$	1 1
(c)	$dz/dt=0$	5
(d)	$f_x(x, y, z) = 2xy,$ $f_y(x, y, z) = x^2 - z^3$ $f_z(x, y, z) = -3yz^2 + 1$ at(1,-2,0) $f_x(x, y, z) = -4$ at(1,-2,0) $f_y(x, y, z) = 1$ at(1,-2,0) $f_z(x, y, z) = 1$ $u(\text{unit vector}) = (2i + j - 2k)/\sqrt{9}$ $D(1, -2, 0) = (-4)(2/3) + (1/3) - (2/3) = -3$	1 2 1 1
(e)	Find the gradient vector of $f(x, y)$ if $f(x, y) = x^3 + 2xy^2$. Evaluate it at $(-3, -4)$ $f_x = 3x^2 + 2y^2$ $f_y = 4xy$ $\text{grad}f(-3, -4) = (59, 48)$	1 4
(f)	Tangent: $-4x - 4y + z = -8$ Normal: $x = 2 - 4t, y = 1 - 4t, z = 4 + t$	2 3
Q. 5	Attempt the following (Any THREE)	(15)
(a)	Critical points are $(0,2), (0,-2), (2,2)$ and $(2,-2)$ $r = f_{xx} = 6x - 6, s = f_{xy} = 0$ and $t = f_{yy} = 12y$ $rt - s^2 = -144$ $(0,2)$ is saddle point $(0,-2)$ local maximum value 48 $(2,2)$ local minimum value $= -20$ F has no extreme value at $(2, -2)$	5
(b)	By substituting $y = vx$ we get the solution $\log(y/x) + (y/x) = -\log x + c$	5
(c)	Sketch the graph of the equation $y = x^3 + 3x + 2$ and identify the locations of intercepts. (draw the graph on the answer sheet itself).	5
(d)	Using trigonometric formula of $\sin A \cos B = (1/2)[\sin(A+B) + \sin(A-B)]$, evaluate we get $I = 1/2$	5
(e)	Vertical asymptotes $x = -1$ and $x = -2$ Horizontal asymptote $y = 0$	5