Examination : SYBSc Semester IV **Subject : Mathematics**

Exam Date : 27th April, 2019 **Q.P.Code : 66041**

(3 Hours) [Total Marks: 100]

Note: (i) All questions are compulsory.

(ii) Figures to the right indicate marks for respective parts.

Q.1	Choose correct alternative in each of the following (20)			
$i.$	Let $a, b \in D_3$, where a and b denotes rotation and reflection then $\lvert ab \rvert=$			
	(a)	2	(b)	3
	(c)	6	(d)	None of the above
	Ans	(2)		
$ii.$	Let H and K be the subgroup of a group G. Then $H \cup K$			
	(a)	Is always a subgroup of G		
	(b)	Is never a subgroup of G		
	(c)	Is a subgroup of G if and only if $H \subseteq K$ or $K \subseteq H$		
	(d)	None of the above		
	Ans	(c)		
$iii.$	The set $\mathbb{Z}_n$ is forms a group under the binary operation			
	(a)	'+'	(b)	$'-'$
	(c)	'•'	(d)	None of the above
	Ans	(a)		
$iv.$	In the group $(\mathbb{Z}_{18}, +)$, order of $\overline{10}$ is			
	(a)	10	(b)	9
	(c)	6	(d)	18
	Ans	(b)		
$v.$	Let H is a proper subgroup of $\mathbb{Z}$ under addition and 12, 14, 18 ∈ H then			
	(a)	$H = 756\mathbb{Z}$	(b)	$H = 2\mathbb{Z}$
	(c)	$H = 4\mathbb{Z}$	(d)	$H = \mathbb{Z}$
	Ans	(b)		
$vi.$	Let a be an element of a group and let order of a in G be infinite then how many generators does the group $< a >$ have?			
	(a)	Only one	(b)	Exactly 2
	(c)	Infinitely many	(d)	none
	Ans	(b)		
$vii.$	If $G = (\mathbb{Z}, +)$ and $H = \{0, \pm 3, \pm 6, \pm 9, \cdots\cdots\}$ then			
	(a)	$11 + H = 17 + H$	(b)	$11 + H = 7 + H$

	(c)	$7 + H = 23 + H$	(d)	None of these
	Ans	(a) $11 + H = 17 + H$		
$viii.$	Let G be a group of order 8 then G must have an element of order			
	(a)	2	(b)	4
	(c)	8	(d)	None of these
	Ans	(a) 2		
$ix.$	Let $\phi: \mathbb{C}^* \longrightarrow \mathbb{C}^*$ given by $\phi(x) = x^4$ be a homomorphism then $ker\phi =$			
	(a)	$\{1, -1\}$	(b)	$\{1, -1, i, -i\}$
	(c)	$\{i, -i\}$	(d)	None of these
	Ans	(b) $\{1, -1, i, -i\}$		
$x.$	Let G be an abelian group which has no element of order 2 and $\phi: G \longrightarrow G$ given by $\phi(x) = x^2$, then			
	(a)	ϕ is an automorphism.		
	(b)	ϕ is a group homomorphism which may not be one $-$one.		
	(c)	ϕ is an automorphism if G is finite.		
	(d)	ϕ is not a group homomorphism.		
	Ans	(a) ϕ is an automorphism.		
Q2.	Attempt any **ONE** question from the following: (08)			
$a)$	i.	Show that $U_n = \{\bar{a} \in \mathbb{Z}_n \mid 1 \le a \le n-1, (a, n) = 1\}$, form a group under the binary operation '•'.		

	Ans	We first prove that closure property is satisfied. Let $\bar{a}, \bar{b} \in \mathbb{Z}_n$ $\Rightarrow (a, n) = 1$ and $(b, n) = 1$ $\Rightarrow (ab, n) = 1$ $\Rightarrow \overline{ab} \in \mathbb{Z}_n \quad \Rightarrow \bar{a} \cdot \bar{b} \in \mathbb{Z}_n$ 2 Hence U (n) is closed with respect to multiplication. Clearly $\bar{a} \cdot (\bar{b} \cdot \bar{c}) = \bar{a} \cdot \overline{bc} = \overline{abc}$ and $(\bar{a} \cdot \bar{b}) \cdot \bar{c} = \overline{ab} \cdot \bar{c} = \overline{abc}$ 1 Hence associative property holds in $\mathbb{Z}_n$ with respect to multiplication. Further $\bar{1} \in \mathbb{Z}_n$ is such that $\bar{1} \cdot \bar{a} = \bar{a} \cdot \bar{1} = \overline{1a} = \bar{a}$ 1 and this is the identity element in $\mathbb{Z}_n$. Let $\bar{a} \in U(n) \Rightarrow (a, n) = 1$ $\Rightarrow \exists$ integers $b, c \in \mathbb{Z}$ such that $ab + nc = 1$ 2 $\Rightarrow ab = 1 - nc = 1 + (-c)n$ $\Rightarrow ab \equiv 1 \pmod{n} \Rightarrow \bar{a} \cdot \bar{b} = \bar{1}$ Further $ab + nc = 1 \Rightarrow (b, n) = 1 \Rightarrow \bar{b} \in U(n)$ This for $\bar{a} \in U(n), \exists\, \bar{b} \in U(n)$ such that $\bar{a} \cdot \bar{b} = \bar{b} \cdot \bar{a} = \bar{1} \Rightarrow$ Multiplication inverse exists for every $\bar{a} \in U(n) \Rightarrow$ U (n) is a group with respect to multiplication modulo n. 2	
	ii.	Define Centre of Group G. Hence or otherwise prove that the Centre of any group is a subgroup of the group.	
	Ans	Clearly $ea = ae \quad \forall a \in G$ $\Rightarrow e \in H \Rightarrow H \neq \phi$ 1 Consider any $x, y \in H$ $x \in H \Rightarrow xa = ax \quad \forall a \in G$ ... (1) $y \in H \Rightarrow ya = ay \quad \forall a \in G$ ... (2) $\Rightarrow y^{-1}(ya)y^{-1} = y^{-1}(ay)y^{-1}$ ... by (2) $\Rightarrow (y^{-1}y)(ay)^{-1} = (y^{-1}a)(yy^{-1})$ ... associativity of G $\Rightarrow e(ay^{-1}) = (y^{-1}a)e$ $\Rightarrow ay^{-1} = y^{-1}(a) \; \forall a \in G$ ... (3) 3 $\Rightarrow y^{-1} \in H$ Consider any $a \in G$ Consider, $(xy^{-1})a = x(y^{-1}a)$ associativity $= x(ay^{-1})$ by (2) and (3) $= (xa)y^{-1}$ associativity $= (ax)y^{-1}$ ... by (1) $= a(xy^{-1})$ 2 Thus, $\forall a \in G$ $(xy^{-1})a = a(xy^{-1})$ $\Rightarrow xy^{-1} \in H$ Thus for any $x, y \in H$ $xy^{-1} \in H$ $\Rightarrow$ H is a subgroup of G 2	

Q.2	Attempt any **TWO** questions from the following:		(12)
$b)$	i.	Let G be a group. Prove that, $(aba^{-1})^n = ab^n a^{-1}, \quad \forall\, a, b \in G$ and $\forall\, n \in \mathbb{Z}$	
	Ans	By induction $n = 0$ $(aba^{-1})^0 = e = ab^0 a^{-1}$ Assume for $n > 0$ $(aba^{-1})^{n+1} = (aba^{-1})^n . aba^{-1}$ $= ab^n a^{-1} . aba^{-1}$ $= ab^{n+1} a^{-1}$ For $n < 0,\ -n > 0$ $(aba^{-1})^n (aba^{-1})^{-n} = e$ $(aba^{-1})^n ab^{-n} a^{-1} = e$ $(aba^{-1})^n = ab^n a^{-1}$	2 2 2
	ii.	Let G be a group and $a \in G$. Show that $H = \{a^{2n} \mid n \in \mathbb{Z}\}$ is a subgroup of G.	
	Ans	Let $x, y \in H \therefore x = a^{2n}$ and $y = a^{2m}$, for some $n, m \in \mathbb{Z}$ $xy^{-1} = a^{2n}(a^{2m})^{-1} = a^{2n-2m} = a^{2(n-m)}$ $\therefore xy^{-1} \in H$ by 1-step test .	2 2 2
	iii.	Let G be a group and $a \in G$ with $O(a) = n$ then show that if and only if a^m=e then $n\|m$.	
	Ans	(⇒) $O(a) = n$ T.P.T if a^m=e then $n\|m$. Let $m = nq + r\ , r = 0$ or $r < n$ $e = a^m = a^{nq+r} = a^{nq} . a^r = a^r \quad \Rightarrow \quad a^r = e$ If $r < n, a^r = e$ is a contradiction As $O(a) = n$. $\therefore r = 0$ $\therefore n\|m$ (⇐) Given $n\|m \quad \therefore m = nq$ $a^m = a^{nq} = (a^n)^q = e$	4 2

	iv.	Let α=(1 2 5)(6 13 5)and $\beta = (1\ 3\ 4)(2\ 6\ 5)(2\ 3\ 4)$ .Write α and β as a product of disjoint cycles. Further, verify the following. p) $O(\alpha) = O(\alpha^{-1})$ q) $O(\alpha\beta) = O(\beta\alpha)$ r) $O(\alpha\beta\alpha^{-1}) = O(\beta)$	
	Ans	$\alpha = (1\ 3)(2\ 5\ 6), \beta = (1\ 3)(2\ 4\ 6\ 5)$ $\alpha^{-1} = (1\ 3)(2\ 6\ 5),\ \ O(\alpha) = O(\alpha^{-1}) = 6$ $\alpha\beta = (2\ 4), \beta\alpha = (4\ 6), O(\alpha\beta) = O(\beta\alpha) = 2$ $\alpha\beta\alpha^{-1} = (1\ 3)(2\ 6\ 5\ 4), \quad O(\alpha\beta\alpha^{-1}) = O(\beta) = 8$	 2 2 2
Q3.	Attempt any **ONE** question from the following:		(08)
$a)$	i.	Prove that $\mathbb{Z}_n$ the set of residue classes modulo n is a group under addition. Also determine all the generators for $\mathbb{Z}_n$	
	Ans	$\mathbb{Z}_n$ is closed under addition Addition is associative in $\mathbb{Z}_n$ 0 is additive identity in $\mathbb{Z}_n$ For any m ϵ $\mathbb{Z}_n$, - m ϵ $\mathbb{Z}_n$ is the additive inverse The element a ϵ $\mathbb{Z}_n$is a generator of $\mathbb{Z}_n$ whenever gcd of a & n is 1.since by result gcd 1 happens if and only if there exists x, y ϵ$\mathbb{Z}$ so that ax+ny=1. Modulo n this becomes ax=1. Now as 1 is a generator of ϵ $\mathbb{Z}_n$So is ax modulo n i.e a ϵ $\mathbb{Z}_n$is a generator of $\mathbb{Z}_n$ whenever gcd of a & n is 1.	1 1 1 1 4
	ii.	Let G be a finite cyclic group of order n then prove that G has a unique subgroup of order d for every divisor d of n.	
	Ans	Let G = (a) be cyclic group of order n. We observe that n is the smallest positive integer such that a^n= e, for if m is smallest positive integer with a^m= e, then G = { e, a, a^2,…., a^{m-1}} and since o(G)=n hence m=n. Let d be a divisor of n, so that n=dd^1,Let H= (d^1) , then o(H)=d, as d is the smallest positive integer such that $(a^{d^1})^d = a^{dd^1} = a^n$=e. Thus G has a subgroup of H of order d. Suppose H^1 is another subgroup of order d. Since H^1 is cyclic, , H^1= (a^k) for some k ϵ $\mathbb{Z}$ and a^{kd}=e. By division algorithm k=qd^1 + r, 0≤r≤ $d^1 - 1$, then dk=qn +rd and e=a^{dr} . But dr< n hence r=0 hence a^kϵ H .i.e.	4 4

		$H^1 \subseteq H$. But as they are of same order hence $H=H^1$.	
Q3.	Attempt any **TWO** questions from the following:		(12)
b)	i.	Let G=(a) be a finite cyclic group of order 12 then what are all the generators of G. Also determine all the generators of the subgroup $H=(a^3)$.	
	Ans	Generators are a , a^5, a^7, a^{11} $(a^3) = \{a^3, a^6, a^9, e\}$ Generators of (a^3) are a^3 and a^9 (Using the result G=(b) of order n then the generators of G are b^m where gcd of m and n is 1.)	3 3
	ii.	Determine all the subgroups of the cyclic group $\mathbb{Z}_{11}^*$	
	Ans	$\mathbb{Z}_{11}^* = U(11) = \{1,2,3,4,5,6,7,8,9,10\} = (2) = (2^3) = (2^7) = (2^9)$ $(2^2)=(2^4)=(2^6)=(2^8)$ =\{4,5,9,3,1\}which is of order 5 (2^5)=\{10,1\} is of order 2 Hence(2),(2^2),(2^5) , (1) are all the four cyclic subgroups of U(11) of orders 10,5,2 and 1 respectively..	2 2 2
	iii.	Show that H = $\left\{\begin{bmatrix}1 & n\\0 & 1\end{bmatrix} / n \in \mathbb{Z}\right\}$ is an infinite cyclic subgroup of $GL_2(\mathbb{R})$.	
	Ans	By first principle of mathematical induction, $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}^n = \begin{bmatrix}1 & n\\0 & 1\end{bmatrix}$ for positive integers n $\begin{bmatrix}1 & -1\\0 & 1\end{bmatrix}$is the inverse of $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$ in $GL_2(\mathbb{R})$. $\begin{bmatrix}1 & -1\\0 & 1\end{bmatrix}^n = \begin{bmatrix}1 & -n\\0 & 1\end{bmatrix}$ for positive integers n. Matrix multiplication is associative. $\begin{bmatrix}1 & m\\0 & 1\end{bmatrix} \times \begin{bmatrix}1 & n\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & m+n\\0 & 1\end{bmatrix}$for m,n ϵ $\mathbb{Z}$ hence closure. $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ is the identity $\begin{bmatrix}1 & -n\\0 & 1\end{bmatrix}$ is the inverse of $\begin{bmatrix}1 & n\\0 & 1\end{bmatrix}$ $\therefore \left(\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\right)$is an infinite cyclic subgroup of $GL_2(\mathbb{R})$.	1 1 1 1 1 1

iv. Consider the set {4,8,12,16}. Show that this set is a group under multiplication modulo 20 by constructing a Cayley table. What is the identity element? Is the group cyclic?. If so find all its generators.

Ans

x	16	4	8	12
16	16	4	8	12
4	4	16	12	8
8	8	12	4	16
12	12	8	16	4

{4,8,12,16}=(8) modulo 20 with identity 16 Another generator is 12.i.e. 8 & 12 are the generators of the group H={4,8,12,16} under multiplication mod 20

4
2

Q4. Attempt any **ONE** question from the following: (08)

$a)$ i. Let H is a subgroup of a group G then $aH = H$ if and only if $a \in H$. Further aH is subgroup of G if and only if $a \in H$.

Ans

For $e \in H \Longrightarrow ae \in aH = H \Longrightarrow a \in H$

Conversely,

Let $x \in aH \Longrightarrow x = ah\,,\ h \in H \Longrightarrow x = ah \in H$ as $a \in H \Longrightarrow aH \subseteq H$

For $a \in H$ and $e \in H \Longrightarrow a = ae \in aH \Longrightarrow H \subseteq aH$. Hence $aH = H$ 4M

Further aH is subgroup of $G \Longrightarrow e \in aH \Longrightarrow e = ah\,,\ h \in H \Longrightarrow eh^{-1} = a$

As $e\,, h^{-1} \in H \Longrightarrow eh^{-1} = a \in H$

Conversely, as $a \in H \Longrightarrow aH = H$

Hence aH is subgroup of G as H is subgroup of G. 4M

ii. Let $f: G \longrightarrow G'$ is onto group homomorphism. Prove that

(p) If H is subgroup of G then $f(H) = \{f(h)/h \in H\}$ is subgroup of G'.

(q) If H' is subgroup of G' then $f^{-1}(H') = \{a \in G/f(a) \in H'\}$ is subgroup of G and $kerf \subseteq f^{-1}(H')$.

Ans

(p) Since $H \subseteq G$ and $e \in H \Longrightarrow f(H) \subseteq G'$ and $f(e) = e' \in f(H)$

Claim : $xy^{-1} \in f(H)$ where $x, y \in f(H)$

For $a, b \in H$ such that $x = f(a), y = f(b)$

Now $xy^{-1} = f(a)(f(b))^{-1} = f(ab^{-1}) \in f(H)$ as $ab^{-1} \in H$

		$\therefore f(H)$ is subgroup of G'. (q) Since $H' \subseteq G'$ and $f(e) = e' \in H' \Rightarrow f^{-1}(H') \subseteq G$ and $e \in f^{-1}(H') \Rightarrow f^{-1}(H')$ is non−empty subset of G. Claim : $ab^{-1} \in f^{-1}(H')$ where $a, b \in f^{-1}(H')$ As $a, b \in f^{-1}(H')$ gives $f(a) = x \in H', f(b) = y \in H' \Rightarrow xy^{-1} \in H'$ Now $f(ab^{-1}) = f(a)(f(b))^{-1} = xy^{-1} \in H' \Rightarrow ab^{-1} \in f^{-1}(H')$ $f^{-1}(H')$ is subgroup of G. Let $a \in kerf \Rightarrow f(a) = e' \in H' \Rightarrow a \in f^{-1}(H') \Rightarrow kerf \subseteq f^{-1}(H')$	3M 4M 1M
Q4.	Attempt any **TWO** questions from the following: (12)		
$b)$	i.	State Lagrange's theorem for finite group. If H and K are subgroups of G such that $o(H) = 12$ and $o(K) = 35$ then show that $H \cap K = \{e\}$.	
	Ans	Statement : Let G be a finite group and H is subgroup of G then $o(H) \mid o(G)$. Since H and K be two subgroups of $G \Rightarrow H \cap K$ is also subgroup of G. Further $H \cap K \subseteq H$ and $H \cap K \subseteq K \Rightarrow H \cap K$ is also subgroup of H and K. By Lagrange's theorem , $o(H \cap K) \mid o(H)$ and $o(H \cap K) \mid o(K)$ $\Rightarrow o(H \cap K) \mid 12$ and $o(H \cap K) \mid 35 \Rightarrow o(H \cap K) \mid \gcd(12, 35)$ $\Rightarrow o(H \cap K) \mid 1 \Rightarrow o(H \cap K) = 1 \Rightarrow H \cap K = \{e\}$	1M 5M
	ii.	Let G be a finite group then show that (p) $o(a) \mid o(G), \forall a \in G$ (q) $a^{o(G)} = e, \forall a \in G$	
	Ans	Since $a \in G \Rightarrow H = \langle a \rangle$ is cyclic subgroup of G. Also $o(H) = o(a)$ By Lagrange's theorem $o(H) \mid o(G) \Rightarrow o(a) \mid o(G)$ Let $o(G) = n$ and $o(a) = m \Rightarrow a^m = e$, also $o(a) \mid o(G) \Rightarrow m \mid n$ $\Rightarrow n = mk, k \in \mathbb{N}$ Now $a^{o(G)} = a^n = (a^m)^k = e^k = e$	3M 3M
	iii.	Show that $f: G \longrightarrow G$ given by $f(x) = x^{-1}$ is a automorphism if and only if G is abelian.	
	Ans	$(\Rightarrow)$ Consider $f(xy) = (xy)^{-1} = y^{-1} x^{-1} = f(y)f(x) = f(yx)$ as f is homomorphism Since f is injective, $xy = yx \Rightarrow G$ is abelian. Conversely, Consider, as f is abelian $f(xy) = (\boldsymbol{xy})^{-1} = \boldsymbol{x}^{-1} \boldsymbol{y}^{-1} = f(x)f(y) \Rightarrow f$ is homomorphism Let $f(x) = f(y) \Rightarrow x^{-1} = y^{-1} \Rightarrow (x^{-1})^{-1} = (y^{-1})^{-1} \Rightarrow x = y \Rightarrow f$ is	 2M

		injective Let $y \in G \implies y^{-1} \in G$, Now $f(y^{-1}) = (y^{-1})^{-1} = y \implies f$ is sujective Therefore f is automorphism.	4M
	iv.	Show that $G = \{a + b\sqrt{2} \,/\, a, b \in \mathbb{Q}\}$ and $H = \left\{\begin{pmatrix} a & 2b \\ b & a \end{pmatrix} / a, b \in \mathbb{Q}\right\}$ are isomorphic groups under addition.	
	Ans	Define $f: G \to H$ by $f(a + b\sqrt{2}) = \begin{pmatrix} a & 2b \\ b & a \end{pmatrix}$, $a, b \in \mathbb{Q}$ f is well defined and injective. f is onto. f is homomorphism Therefore G is isomorphic to H.	1M 2M 1M 1M 1M

Q5.	Attempt any **FOUR** questions from the following: (20)	
$a)$	Construct composition table of $\mathbb{Z}_5^*$ under multiplication modulo 5. Also find the order of each of its elements.	

Ans

	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

3

$O(1) = 1, O(2) = 4, O(3) = 4, O(4) = 2$ 2

$b)$	Define Abelian group. If $(ab)^2 = a^2b^2$ for every a, b in a group G, show that G is Abelian.	
Ans	G is abelian if $ab = ba, \forall a, b \in G$ $(ab)^2 = a^2b^2$ $abab = aabb$ $bab = abb$ $ba = ab \therefore G$ is abelian	1 2 2
$c)$	Prove that a group of order 3 must be cyclic.	
Ans	A group of order 3 can have subgroups of order 1 and 3 since order of a subgroup divides order of the group. The only element of order 1 is the identity element e.	2 1

	If a∈G and a$\neq e$ then o(a)=3 as no element of G can have order 2 as 2 does not divide 3. ∴ G=(a) . Hence G is cyclic.	1 1
$d)$	Let G be a group and let 'a' be an element of G. (i) If $a^{12} = e$, what can you say about order of a. (ii) Suppose that G is cyclic and o(G) = 24 . Further if $a^8 \neq e$ and $a^{12} \neq e$ then show that $< a >$=G.	
Ans	a^{12} = e ∴o(a) can be 1,2,3,4,6 and 12 which are the divisors of 12. Now if o(G) =24 and a∈G , $a^8 \neq e$, $a^{12} \neq e$. ∴$a \neq e$, $a^2 \neq e$, $a^4 \neq e$. But order of a is a divisor of o(G) ∴o(a)=24. ∴ G=(a) i.e. G is cyclic.	2 2 1
$e)$	Give an example of a group G and a subgroup H of G such that $aH = bH$ but $Ha \neq Hb$ for some $a, b \in G$.	
Ans	$G = S_3$, $H = \{e\ ,(12)\}$ then for $a = (13)$ and $b = (123)$ $aH = bH = \{(13),(123)\}$ but $Ha = \{(13),(132)\}$ and $Hb = \{(23),(123)\} \Longrightarrow$ $Ha \neq Hb$	 5M
$f)$	Find the number of group homomorphism from $\mathbb{Z}_{12}$ to $\mathbb{Z}_{30}$.	
Ans	Let $f: \mathbb{Z}_{12} \rightarrow \mathbb{Z}_{30}$ is a group homomorphism. Let $\bar{1} \in \mathbb{Z}_{12}$ then $f(\bar{1})$ defines all homomorphism. Also $o(\bar{1}) = 12$ $\bar{1} \in \mathbb{Z}_{12}$ and $f(\bar{1}) \in \mathbb{Z}_{30} \Longrightarrow o\big(f(\bar{1})\big) \mid o(\mathbb{Z}_{30}) \Longrightarrow o\big(f(\bar{1})\big) \mid 30$ ------- (1) As f is homomorphism $\Longrightarrow o\big(f(\bar{1})\big) \mid o(\bar{1}) \Longrightarrow o\big(f(\bar{1})\big) \mid 12$ ------- (2) (1) and (2) $\Longrightarrow o\big(f(\bar{1})\big) \mid \gcd(12,30) \Longrightarrow o\big(f(\bar{1})\big) \mid 6 \Longrightarrow o\big(f(\bar{1})\big) = 1, 2, 3\ or\ 6$ Since $f(\bar{1}) \in \mathbb{Z}_{30} \Longrightarrow$ there are 6 elements $\{\bar{0}\}$, $\{\overline{15}\}$, $\{\overline{10}\ ,\overline{20}\}$, $\{\bar{5}\ ,\overline{25}\}$ of order 1, 2, 3, 6 respectively. Hence there are **six** homomorphism.	 5M

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