(1)

[Solution] QP code: 64060

S.Y.B.Sc. Statistics (ATKT) Sem3 - Paper 2, March 2018

Q.1) a) ① True

② False

③ False

④ True

⑤ True

b) ① Sampling errors: Errors in Statistical analysis arising from the unrepresentativeness of the sample taken.

② Parameter: Statistical constant of the population
Estimators: It is a function of sample values.

③ $\bar{Y}_h$ = Popn Mean of h^{th} stratum

$$= \frac{\sum_{j=1}^{N_h} Y_{hj}}{N_h}$$

$$S_h^2 = \frac{1}{N_h - 1} \sum_{j=1}^{N_h} (Y_{hj} - \bar{Y}_h)^2 = \text{Popn Mean Square of } h^{th} \text{ stratum}$$

④ Complete enumeration:
In data collection, we consider each and every member of the whole population.

⑤ Two stage sampling:
- The sample is taken in two steps. The procedure consist of selecting a sample of clusters and then selecting specified no. of elements from each cluster.
- It is more flexible & efficient.

Q.2) a) Simple random sample - ②

Drawing a random sample by lottery method ④

ii) P('n' specified units are selected from a popn of size N)

$$= \frac{1}{{}^{N}C_{n}} \quad \text{—(5)}$$

c) In SRSWOR,

i) $E(\bar{y}) = \bar{Y}$ —(5)

ii) $V(\bar{y}) = \frac{N-n}{N} \frac{S^2}{n}$ —(5)

Q.3) a) $\bar{y}_{str}$ —(3) ; $\bar{y}_{str} = \sum_{h=1}^{L} w_h \bar{y}_h$; $w_h = \frac{N_h}{N}$, $h = 1, 2 \cdots L$

$E(\bar{y}_{str}) = \bar{Y}$ —(3)

$$V(\bar{y}_{str}) = \sum_{h=1}^{L} w_h^2 \frac{N_h - n_h}{N_h} \frac{S_h^2}{n_h} \quad \text{—(4)}$$

b) Minimising cost fun (C) —(5)

$n_h \propto N_h S_h / \sqrt{C_h}$ —(5)

[Derivation]

c) $V(\bar{y}_{SRSWOR}) \geq V(\bar{y}_{str})_{prop} \geq V(\bar{y}_{str})_{opt}$ —(10)

Q.4) a) Estimator of popn total $= \hat{Y}_R = \frac{\bar{y}}{\bar{x}} \times X$ —(2)

$$\cdot V(\hat{Y}_R) = N^2 \frac{(1-f)}{n} \left[S_y^2 + R^2 S_x^2 - 2R S_{yx} \right] \quad \text{—(5)}$$

$$\cdot \hat{V}(\hat{Y}_R) = N^2 \frac{(1-f)}{n} \left[\frac{\Sigma y_i^2 + \hat{R}^2 \Sigma x_i^2 - 2\hat{R} \Sigma x_i y_i}{(n-1)} \right] \quad \text{—(3)}$$

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Q4) b) • Linear regression estimate of Popn Mean

$$\bar{y}_{lr} = \bar{y} + b(\bar{X} - \bar{x}) \quad \text{— (3)}$$

• $V(\bar{y}_{lr}) = \frac{1-f}{n} S_y^2 (1 - \rho^2)$ — (5)

$$\hat{V}(\bar{y}_{lr}) = \frac{1-f}{n} \frac{\sum_{i=1}^{n} [\Sigma (y_i - \bar{y}) - b(x_i - \bar{x})]^2}{(n-1)} \quad \text{— (2)}$$

c) : Systematic sampling explanation — (5)
• Advantages & Disadvantages — (5)

Q5) a) Derivation of Size of SRSWOR assuming large sample — (10)

b) Allocation : — (2)

Proportional allocation — (3)
Neyman's allocation — (5)

c) Linear regression estimate when $b = b_0$ (a fixed no)

$\bar{y}_{lr}$ — (2)

$E(\bar{y}_{lr})$ — (2)

$V(\bar{y}_{lr})$ — (5)

$\hat{V}(\bar{y}_{lr})$ — (1)