(1) Physics - I

Duration :- 3hrs. MKs :- 100

Q 1A) i) $\bar{A} \cdot \bar{B} = 2(4) + a(-2) + 1(-2) = 8 - 2a - 2 = 0$

$2a = 6 \quad \therefore a = 3$ Ans :- (a) = 3

ii) ______ waves do not transmit energy from one point to another. Ans:- (d) stationary

iii) Solution of homogeneous differential equations possesses the property of ______

Ans :- (d) Superposition

iv) Curl of electrostatic field is ______

Ans :- (c) zero

v) Work done = $\vec{F} \cdot \vec{S} = 1(4) + 3(-2) - 2(4) = 4 - 6 - 8 = -10$

Ans :- (a) -10

vi) A vector field $(\bar{A})$ is said to be solenoidal if ______. Ans :- (c) Div of $(\bar{A}) = 0$

Q1 B) i) $\frac{dy}{dx} + P(x) y = Q(x)$ and $Q(x) = 0$ for homogeneous.

ii) ~~$\tau = CR$ is define as the time in which the capacitor gets charged to 63% of its maximum value.~~ For a non dispersive medium, $V_p = V_g$

iii) For $\bar{A}$, $\bar{B}$ and $\bar{C}$ to be coplanar $\bar{A} \cdot (\bar{B} \times \bar{C}) = 0$

Q1 C) i) Time interval during which the oscillation repeats itself is known as ______ of oscillation

Ans :-

ii) if $|\bar{A} + \bar{B}| = |\bar{A} - \bar{B}|$ only if $\bar{A}$ $\perp$lar $\bar{B}$

iii) The ______ in a series LR circuit is the tim pg ②

iii) taken for the current to rise to 63% of its final maximum value. Ans:- Time Constant $\tau = L/R$

iv) A differential equation which contains only one independent variable is called as ________ differential equation

Ans:- ordinary

v) The Principle behind group velocity is the concept of ________ packet. Ans:- wave

Q2 A) i) Projection of vector $\bar{A}$ on $\bar{B} = \frac{\bar{A}\cdot\bar{B}}{|\bar{B}|}$

$\bar{A} = 4\hat{i} - 8\hat{j} + \hat{k}$ $\quad |\bar{A}| = \sqrt{16+64+1} = \sqrt{81} = 9$

$\bar{B} = 3\hat{i} + 3\hat{j} + \hat{k}$

$\bar{A}\cdot\bar{B} = 4(3) - 8(3) + 1 = 12 - 24 + 1 = -11$

$|\bar{B}| = \sqrt{3^2+3^2+1^2} = \sqrt{9+9+1} = \sqrt{19}$

$\therefore$ Projection of $\bar{A}$ on $\bar{B} = \frac{-11}{\sqrt{19}}$

Angle made by $\bar{A}$ with x-axis $= \cancel{\cos}\,\theta$?

$\bar{A}\cdot\bar{B} = AB\cos\theta$

$$\cos\theta = \frac{\bar{A}\cdot\bar{B}}{A\cdot B} = \frac{-11}{9\times\sqrt{19}}$$

$$\theta = \cos^{-1}\left(-\frac{11}{9\sqrt{19}}\right)$$

$$\bar{A}\times\bar{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -8 & 1 \\ 3 & 3 & 1 \end{vmatrix}$$

$= (-8-3)\hat{i} - (4-3)\hat{j} + (12+24)\hat{k}$

$= -11\hat{i} - \hat{j} + 36\hat{k}$

$|\bar{A}\times\bar{B}| = \sqrt{121+1+1296} = \cancel{37.65}$

$= \sqrt{1418}$

$\sin\theta' = \frac{|\bar{A}\times\bar{B}|}{AB}$

$\theta' = \sin^{-1}$ [illegible] $\frac{}{9\times\sqrt{19}}$

(2) Q2 A) ii) $\vec{V} = \vec{\omega} \times \vec{r}$ (given)

let $\vec{\omega} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$$V = \vec{\omega} \times \vec{r} = (a_2 z - a_3 y)\hat{i} - (a_1 z - a_3 x)\hat{j} + (a_1 y - a_2 x)\hat{k}$$

$\nabla \times \vec{V} = 2w$

(b) $\nabla \times V = \nabla \times (\vec{\omega} \times \vec{r})$

$$= \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ w & w & w \\ x & y & z \end{vmatrix}$$

$$\vec{\omega} \times \vec{r} = (wz - wy)\hat{i} + (wz - wx)\hat{j} + \hat{k}(wy - wx)$$

$$\bar{\nabla} \times \bar{V} = \bar{\nabla} \times (\bar{\omega} \times \bar{r})$$

$$= w\left[\frac{\partial}{\partial x}(z-y)\right] + w\left[\frac{\partial}{\partial y}(x-z)\right] + w\left[\frac{\partial}{\partial z}(y)\right]$$

$$= w[0+0+0] = 0$$

Q2B)i) For vectors $\vec{A}, \vec{B}, \vec{C}$

Scalar triple product is $\vec{A}\cdot(\vec{B}\times\vec{C})$ / $[ABC]$

Vector triple product is $\vec{A}\times(\vec{B}\times\vec{C})$

<u>properties</u>:- ① $(A\cdot B)\cdot C \neq A\cdot(B\cdot C)$

② $A\cdot(B\times C) = B\cdot(C\times A) = C\cdot(A\times B) =$ Volume of parallelepiped

③ $A\times(B\times C) \neq (A\times B)\times C$

④ $A\times(B\times C) = (A\cdot C)B - (A\cdot B)C = (A\cdot C)B - (B\cdot C)A$

⑤ $A\cdot(B\times C) = 0$ for vectors to be coplanar

Bii) <u>Operator DEL $\vec{\nabla}$</u> is written as.

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

<u>Divergence</u> :- If $\vec{V}(x,y,z) = V_1\hat{i} + V_2\hat{j} + V_3\hat{k}$ is differentiable at each point (x,y,z) then div. of $\vec{V} = \vec{\nabla}\cdot\vec{V} =$

$$\vec{\nabla}\cdot\vec{V} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot(V_1\hat{i} + V_2\hat{j} + V_3\hat{k})$$

$$= \frac{\partial V_1}{\partial x} + \frac{\partial V_2}{\partial y} + \frac{\partial V_3}{\partial z}$$

<u>physical Interpretation</u> :- In case of fluid flow, Rate of flow of fluid per unit volume

$$= \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = \vec{\nabla}\cdot\vec{V}$$

equation of Continuity $\vec{\nabla}\cdot\vec{V} = 0$

$\vec{V} \rightarrow$ Solenoid vector function c Q2 C) i) let $\vec{F} = (y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$

For solenoid, $\vec{\nabla} \cdot \vec{F} = 0$

$$\vec{\nabla} \cdot \vec{F} = \left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right] \cdot \left[(y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}\right]$$

$$= -2 + 2x - 2x + 2 = 0$$

For irrotational field, curl $\vec{F} = 0$

$$\text{curl } \vec{F} = (3x - 3x)\hat{i} - (-2z + 3y - 3y + 2z)\hat{j} + (3z + 2y - 2y - 3z)\hat{k}$$

$$= 0\hat{i} + 0\hat{j} + 0\hat{k} = 0$$

c) ii) $\phi = 3y^2x - y^3z^2$

$$\text{grad } \phi = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(3y^2x - y^3z^2)$$

$$= (3y^2)\hat{i} + (6yx - 3y^2z^2)\hat{j} + (-2y^3z)\hat{k}$$

grad ϕ at $(1, -1, -2) = 3\hat{i} - 18\hat{j} - 4\hat{k}$

Q3 A) i) First order linear differential equation is $\frac{dy}{dx} + P(x)y = Q(x)$

$Q(x) = 0 \rightarrow$ homogeneous

$Q(x) \neq 0 \rightarrow$ Inhomogeneous

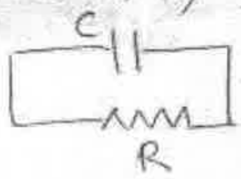
Complementary function $y_n = c_1 e^{-\int P(x)dx}$

$e^{\int P(x)dx} \rightarrow$ Integrating factor

Particular Integral

$$y = e^{-\int P(x)dx} \int e^{\int P(x)dx} Q(x)\,dx + c\, e^{-\int P(x)dx}$$

Q3 A) ii) Discharging of capacitor :-



$i = \frac{dq}{dt}$, $R\frac{dq}{dt} + \frac{q}{C} = 0$, $\ln q - \ln q_m = -\frac{t}{CR}$

$\boxed{q = q_m e^{-t/\tau}}$, $i = -\frac{q_m}{CR} e^{-t/\tau}$

$V_R = iR = -\frac{q_m}{C} e^{-t/\tau}$, $V_C = \frac{q}{C} = \frac{q_m}{C} e^{-t/\tau}$

Q3 B) ii) let $M(x,y)$ and $N(x,y)$ then

$M(x,y)\,dx + N(x,y)\,dy = 0$ for exact diff. equation

$\therefore (4x^3 + 6xy + y^2)\,dx + (3x^2 + 2xy + 2)\,dy = 0 \rightarrow (1)$

$M = 4x^3 + 6xy + y^2$; $N = 3x^2 + 2xy + 2$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\therefore$ eqn (1) is exact equation

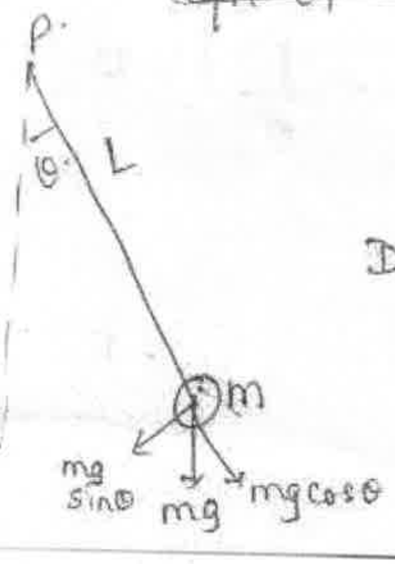
$C_1(y) = 2y$, $C_2(x) = x^4$

Solution $F(x,y) = x^4 + 3x^2y + xy^2 + 2y = C$

Q3 B.

i) Differential equation for simple pendul

~~eqn of motion~~ $\tau = I\alpha$



$-mg\sin\theta\, L = mL^2\frac{d^2\theta}{dt^2}$

Diff. eqn is $\frac{d^2\theta}{dt^2} + \frac{g}{L}\sin\theta = 0$

its Solution $\theta(t) = \theta_0\cos(\omega t + \phi)$

$\omega = 2\pi f = \sqrt{\frac{g}{L}}$, $T = 2\pi\sqrt{\frac{L}{g}}$

Q3 c) i) $\frac{dy}{dx} - \frac{y}{x+3} = \frac{2}{x+3}$ → non-homogeneous linear diff. eqn.

$P(x) = -\frac{1}{x+3}$, $Q(x) = \frac{2}{x+3}$.

$\phi = \int P(x)\,dx = -\ln(x+3)$

$e^{\phi} = \frac{1}{x+3}$, $e^{-\phi} = x+3$

Solution $y = e^{-\phi}\int e^{\phi} Q(x)\,dx + c\,e^{-\phi} = c(x+3) - 2$

Q3 C ii) $I = 5A$ $\quad T = \frac{L}{R} = \frac{200\times10^{-3}}{20} = 10^{-2}$ sec

$L = 200mH$

$R = 20\Omega$ $\quad i = i_m e^{-t/T}$

$t = 8$ sec $\quad = 5\,e^{-\frac{8}{10^{-2}}} = 5\,e^{-800}$

Q4 A) i) $x = A\sin(2\omega t + \phi_1)$

$y = B\sin(\omega t + \phi_2)$

$\frac{y}{B} = \sin\omega t$; $\frac{x}{A} = 2\sin\omega t\cos\omega t\cos\delta + (1 - 2\sin^2\omega t)\sin\delta$

$= 4\sin^2\omega t\cos^2\delta\,(1-\sin^2\omega t)$

$\left[\left(\frac{x}{A} - \sin\delta\right) + 2\frac{y^2}{B^2}\sin\delta\right]^2 + 4\left[\frac{y^4}{B^4} - \frac{y^2}{B^2}\right]\cos^2\delta = 0$

For $\delta = 0$ or π , $\frac{x^2}{A^2} + 4\left(\frac{y^4}{B^4} - \frac{y^2}{B^2}\right) = 0$

$\delta = \pi/2$, $y^2 = -\frac{B^2}{2A}(x - A)$

4 A ii) Particle velocity = simple harmonic velocity of the oscillator about its equilibrium position. $u = \frac{dy}{dt}$

wave velocity → velocity with which the planes of the equal phase progress thr. the medium. $\therefore V = \frac{dx}{dt}$

$$\frac{u}{V} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{dy}{dx} = \text{slope of displacement curve}$$

$$\frac{dy}{dx} = -Ka \sin(Kx - \omega t) = -\frac{K}{\omega}\frac{dy}{dt}$$

$$\frac{dy}{dt} = -V \frac{dy}{dx} \quad \left(\because \frac{\omega}{K} = \frac{2\pi n \lambda}{2\pi} = V\right)$$

Hence proved.

4 B) i) ~~...~~ Superposition of two SHMs of the same period.

$$x_1 = A_1 \sin(\omega t + \phi_1)$$

$$x_2 = A_2 \sin(\omega t + \phi_2)$$

$$x = x_1 + x_2 = A \sin(\omega t + \varepsilon)$$

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos(\phi_1 - \phi_2)$$

$$\tan \varepsilon = \frac{A_1 \sin\phi_1 + A_2 \sin\phi_2}{A_1 \cos\phi_1 + A_2 \cos\phi_2}$$

lcm

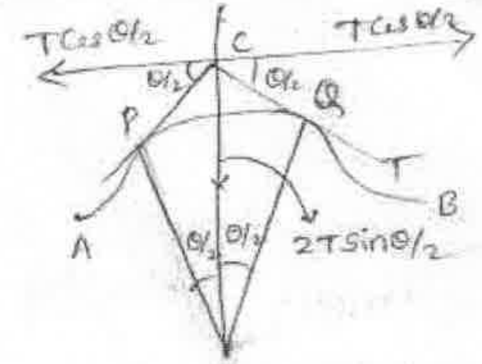
Sp. Cases :- 1) SHM's are in phase, $A = A_1 + A_2$

2) SHM's are in opposite phase, $A = |A_1 - A_2|$

3) when $A_1 = A_2$ and phase ϕ_1 and ϕ_2 are different $A = 2A_1 \cos\left(\frac{\phi_1 - \phi_2}{2}\right)$

if $\phi_1 - \phi_2 = 0$ $\quad \varepsilon_1 = \frac{1}{2}(\phi_1 + \phi_2)$

4B) ii) Velocity of a simple harmonic wave in a stretched string



$POQ = \theta$, Centripetal Acceleration $= \frac{m\delta x v^2}{R}$

Resultant tension $= 2T\frac{\theta}{2} = T\theta$.

$T\theta = \frac{m\delta x \cdot v^2}{R}$ $\quad \therefore \theta = \frac{\delta x}{R}$

$T\frac{\delta x}{R} = \frac{m\delta x v^2}{R}$

$v^2 = \frac{T}{m}$ $\quad \therefore v = \sqrt{T/m}$.

Q4 c) i) $x_1 = 12 \sin(\omega t)$, $x_2 = 5\cos\omega t = 5\sin(\omega t + \frac{\pi}{2})$

$A_1 = 12\,cm$, $A_2 = 5\,cm$.

$\phi_1 = 0$, $\phi_2 = \pi/2$, $\phi_2 - \phi_1 = \pi/2$.

$A^2 =$ Resultant amplitude $= A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_1 - \phi_2)$

$A^2 = 144 + 25 = 169$

$A = 13\,cm$

$\phi = \tan^{-1}\left(\frac{A_1\sin\phi_1 + A_2\sin\phi_2}{A_1\cos\phi_1 + A_2\cos\phi_2}\right)$

$= \tan^{-1}\left(\frac{12\times 0 + 5\times 1}{12\times 1 + 5\times 0}\right) = \tan^{-1}\left(\frac{5}{12}\right)$

$\phi =$

Q5 i) $y = 0.001 \cos(6.28t + 4x)$

$= 0.001 \sin(6.28t + 4x + \frac{\pi}{2})$

$\omega = 6.28 = 3.14 \times 2 = 2\pi$

Amplitude $A = 0.001$ units

$T = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi} = 1$ sec

$f = \frac{1}{T} = 1$ Hz ; wave number $k = 4$

~~velocity~~ $\cancel{v = y = 0.001 \cos(6.28t + 4x + \pi/2) \times 6.28}$

$\cancel{= \cos(6.28t + 4x + \pi/2)}$

$\cancel{v = f\lambda \therefore \lambda = \frac{v}{f} = \cos(6.28t + 4x + \pi/2)}$

wave number $K = \frac{2\pi}{\lambda} = \frac{\omega}{v}$

$4 = \frac{2\pi}{v}$ $\therefore v = \frac{2\pi}{4} = 1.57$

$\cancel{\frac{2\pi}{\lambda} = \frac{\omega}{v} = \frac{2\pi}{2\pi} \times 4}$ $\therefore \lambda = \frac{2\pi}{K} = \frac{2\pi}{4}$

$= 1.57$

Q5 ii) $\bar{A} = 4x^2\hat{i} - 3yz\hat{j} + xz^2\hat{k}$

$$\nabla \times \bar{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 4x^2 & -3yz & xz^2 \end{vmatrix}$$

$$= (0+3y)\hat{i} - (z^2 - 0)\hat{j} + (0-0)\hat{k}$$

$$= 3y\hat{i} - z^2\hat{j}$$

$\nabla \times \bar{A}$ at $(1,-1,1) = 3(-1)\hat{i} - (1)^2\hat{j} = -3\hat{i} - \hat{j}$

Q5 iii)

Progressive wave	Stationary wave
1) Motion of all particles is similar in nature	1) Different particles move with different velocities.
2) Disturbance propagates with definite velocity.	2) Disturbance is confined to a particular region
3) There is no particle which always remains at rest.	3) Particles at nodes always remain at rest
4) Energy is transmitted from one region to other.	4) Energy of one region is always confined in that region
5) Phases of nearby particles are always different	5) All particles are moving in phase

xiv) $A(1,3,2)$, $B(2,-1,1)$ $C(-1,2,3)$

$\bar{AB} = \hat{i} - 4\hat{j} - \hat{k}$

$\bar{AC} = -2\hat{i} - \hat{j} + \hat{k}$

Area of triangle with sides $\bar{AB}$ and $\bar{AC}$

is $= \frac{1}{2} |\bar{AB} \times \bar{AC}| = \frac{1}{2} |(\hat{i} - 4\hat{j} - \hat{k}) \times (-2\hat{i} - \hat{j} + \hat{k})|$

$= \frac{1}{2}\sqrt{107}$

Qs. v) $C = 15\mu F$

$E = 5V$

$R = 1M\Omega$

$\tau = RC = 1\times10^{6}\times15\times10^{-6}$

$= 15$ sec

Maximum charge $q_m = CE$

$= 15\times10^{-6}\times5 = 75\mu C$

$q = 50\%\ q_m$

$\frac{q}{q_m} = 50\% = \frac{1}{2} = 0.5$

$q = CE(1 - e^{-t/\tau}) = q_m(1 - e^{-t/\tau})$

$\frac{q}{q_m} = 1 - e^{-t/15}$

$0.5 = 1 - e^{-t/15}$

$e^{-t/15} = 1 - 0.5 = 0.5$

$e^{15/t} = \frac{1}{0.5}$

$\frac{15}{t} \log_e e = \ln \frac{1}{0.5}$

$\frac{15}{t} \times 1 =$

Qs. vi) Lissajous Figure

when a particle is subjected to two mutually perpendicular SHMs. It traces a path on a plane that depends upon the frequencies, amplitudes and phases of the Component SHMs. If the frequencies of the two component SHMs are not equal, the path of the particle is no longer an ellipse but is a curve Lissajous figure. It depends on amplitude and frequence of SHMs.