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Paper II Sem I Solution

Q1a-1 False Mean of Poisson is m

2 False Continuous on the right

3 True 4 True 5 True

1-b 1 If an expt results into n MEE & equally likely outcomes of which m are favorable to the occurrence of event A then $P(A) = m/n$.

2 $F(x) = P(X \leq x)$ is known as cdf of X.

3 S = { HHH TTT HHT HTH THH THT TTH HTT }

4 $X \sim P(m)$ $P(X=x) = e^{-m}m^{x}/x!$ $x \geq 0$

~~5 Outcomes where one is not preferred over the other are known as equally likely outcomes Ex. Outcomes Head & Tail are equally likely.~~

5 Independent events

Q2a (Probability is a measure of uncertainty. It deals with the study of random experiments & it is the backbone of statistical inference & decision theory)

Addition Th^m $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(Proof in usual notation) - Def^n of prob.

b Experiment results into random outcomes. Example A die is tossed. The S.S (1, 2 3 4 5 6) are mutually exclusive, exhaustive & equally likely.

A = (1 3 5) B = A^c = (2 4 6). If event A occurs B does not occur Hence A & B are complementary events. Certain event.

c (i) Bayes' Theorem — $P(A_i/B) = \dfrac{P(A_i) \times P(B/A_i)}{\sum_{i=1}^{n} P(A_i) \times P(B/A_i)}$

Proof in usual Notation (ii) Example

d $P(A \cap B) = P(A) \cdot P(B/A) = P(B) \cdot P(A/B)$

(Multiplication Theorem)

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$P(B) = 1/5$, $P(A \cap B) = 4/10$ $P(A|B) = 1/2$ $P(A \cup B) = 7/20$

Q3 a $P(X=x)$ is probability associated with value x of the random variable X $P(x)$ is pmf if $0 \le P(x) \le 1$ &

$\sum_x P(x) = 1$

Properties of CDF = $0 \le F(x) \le 1$

$F(-\infty) = 0$, $F(+\infty) = 1$

$F(x)$ is monotonically non decreasing fn of x

$F(a) \le F(b)$ for $a \le b$

$P(a \le X \le b) = F(b) - F(a)$

b $E(X+Y) = E(X) + E(Y)$

$E(XY) = E(X) \cdot E(Y)$ Proofs in usual notation

c $V(aX+bY) = a^2V(X) + b^2V(Y) + 2ab\,cov(XY)$

$V(aX+b) = a^2V(X)$

d J prob mass function $P(xy) = P(X=x, Y=y)$ $\forall (x,y)$

$P(xy)$ is $0 \le P(xy) \le 1$ & $\sum_x \sum_y P(xy) = 1$

$P_1(x)$ & $P_2 y$ are marginal probability distributions

$P(x/y)$ & $P(y/x)$ are conditional probability distribution

$P(x/y) = \frac{P(xy)}{P_2 y}$ $P(y|x) = \frac{P(xy)}{P_1 x}$

4a $P(X=x) = \frac{1}{n}$ $x = 1, 2 \ldots n$

$= 0$ ow

$E(X) = \frac{n+1}{2}$ $V(X) = n^2 - 1/12$

b $X \sim Bin(n\,p)$ $P(X=x) = \binom{n}{x} p^x q^{n-x}$ Bin → Poisson

R. Rel^n: $P(x+1) = \frac{n-x}{x+1} \times \frac{p}{q} P(x)$

c $X \sim Poisson\,(m)$ $P(X=x) = \frac{e^{-m} m^x}{x!}$

$E(X) = V(X) = m$

Number of errors on a page, Number of defective items per box, Number of radioactive atoms decaying in specific time all follow the Poisson probability.

(c) Condition : $n \to \infty$ & $p \to 0$ in a binomial distribution such that $np = m$ (finite) then binomial distbn tends to Poisson distribution (Proof in normal notations).

(d) Mean & Variance of binomial. Define Bin.

Q5 a (1 1 ... 1 6) Two dice tossed

(2 1 ... 2, 6)

Sample points = 36.

(6 1 ... 6 6)

b. P(both are aces) = 0.0045

P(1 Ace & 1 Queen) = 0.02413

c. $E(X) = \sum x \cdot p(x)$

$E(aX + b) = aE(X) + b$

d. $P(X = x) = \frac{1}{10}$ $x = 0, 1, \ldots 9$

= 0 o.w

$E(X) = 9/2$ $V(X) = 33/4$

e. $Cov(X, Y) = E(XY) - E(X) \cdot E(Y)$

Corrl^n coefficient $\rho = \frac{Cov(X, Y)}{\sqrt{V(X) V(Y)}}$

f. Sum of independent (iid) Bernoulli variates is Bin(n, p)

$X \sim B(n_1, p)$

$Y \sim B(n_2, p)$ $X + Y \sim Bin(n_1 + n_2, p)$

g. $p = \frac{1}{2}$, distb^n is symmetric about median

h. Poisson Recurrence: $P(x+1) = \frac{m}{x+1} \times P(x)$, $x = 0, 1, 2$